8 Quadrilaterals

TEXAS

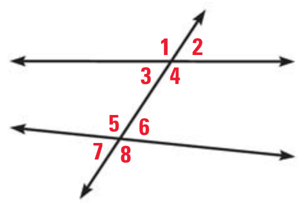
Before

In previous chapters, you learned the following skills, which you'll use in Chapter 8: identifying angle pairs, using the Triangle Sum Theorem, and using parallel lines.

Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement.



1. $\angle 1$ and _?_ are vertical angles.

2. $\angle 3$ and _?_ are consecutive interior angles.

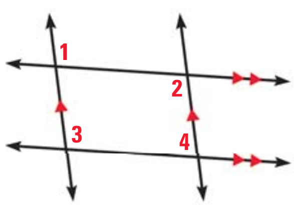
3. $\angle 7$ and _?_ are corresponding angles.

4. $\angle 5$ and _?_ are alternate interior angles.

SKILLS AND ALGEBRA CHECK

5. In $\triangle ABC$, $m\angle A = x°$, $m\angle B = 3x°$, and $m\angle C = (4x - 12)°$. Find the measures of the three angles. *(Review p. 217 for 8.1.)*

Find the measure of the indicated angle. *(Review p. 154 for 8.2–8.5.)*



6. If $m\angle 3 = 105°$, then $m\angle 2 =$ _?_.

7. If $m\angle 1 = 98°$, then $m\angle 3 =$ _?_.

8. If $m\angle 4 = 82°$, then $m\angle 1 =$ _?_.

9. If $m\angle 2 = 102°$, then $m\angle 4 =$ _?_.

TEXAS @HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 8, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 559. You will also use the key vocabulary listed below.

Big Ideas

1. Using angle relationships in polygons
2. Using properties of parallelograms
3. Classifying quadrilaterals by their properties

KEY VOCABULARY

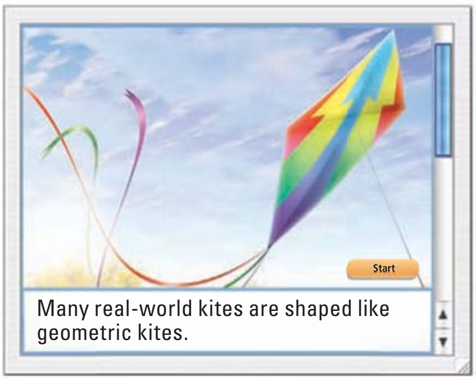
- diagonal, *p. 507*
- parallelogram, *p. 515*
- rhombus, *p. 533*
- rectangle, *p. 533*
- square, *p. 533*
- trapezoid, *p. 542*
  bases, base angles, legs
- isosceles trapezoid, *p. 543*
- midsegment of a trapezoid, *p. 544*
- kite, *p. 545*

Why?

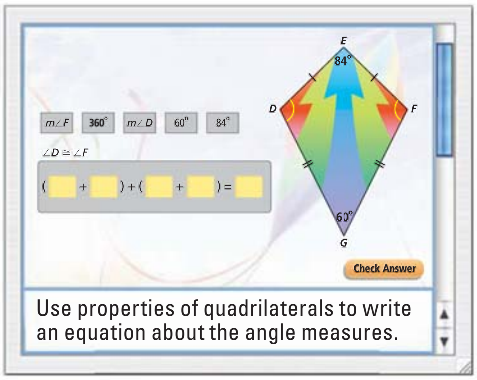
You can use properties of quadrilaterals and other polygons to find side lengths and angle measures.

Animated Geometry

The animation illustrated below for Example 4 on page 545 helps you answer this question: How can classifying a quadrilateral help you draw conclusions about its sides and angles?



Many real-world kites are shaped like geometric kites.



Use properties of quadrilaterals to write an equation about the angle measures.

Animated Geometry at classzone.com

Other animations for Chapter 8: pages 509, 519, 527, 535, 551, and 553

Investigating Geometry ACTIVITY Use before Lesson 8.1

8.1 Investigate Angle Sums in Polygons

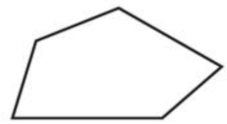
MATERIALS • straightedge • ruler TEKS G.3.D, G.5.B, G.9.B

QUESTION **What is the sum of the measures of the interior angles of a convex *n*-gon?**

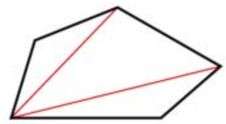
Recall from page 43 that an *n*-gon is a polygon with *n* sides and *n* vertices.

EXPLORE **Find sums of interior angle measures**

STEP 1 ***Draw polygons*** Use a straightedge to draw convex polygons with three sides, four sides, five sides, and six sides. An example is shown.



STEP 2 ***Draw diagonals*** In each polygon, draw all the diagonals from one vertex. A *diagonal* is a segment that joins two nonconsecutive vertices. Notice that the diagonals divide the polygon into triangles.



STEP 3 ***Make a table*** Copy the table below. By the Triangle Sum Theorem, the sum of the measures of the interior angles of a triangle is 180°. Use this theorem to complete the table.

Polygon	Number of sides	Number of triangles	Sum of measures of interior angles
Triangle	3	1	$1 \cdot 180° = 180°$
Quadrilateral	?	?	$2 \cdot 180° = 360°$
Pentagon	?	?	?
Hexagon	?	?	?

DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. Look for a pattern in the last column of the table. What is the sum of the measures of the interior angles of a convex heptagon? a convex octagon? *Explain* your reasoning.
2. Write an expression for the sum of the measures of the interior angles of a convex *n*-gon.
3. Measure the side lengths in the hexagon you drew. Compare the lengths with those in hexagons drawn by other students. Do the side lengths affect the sum of the interior angle measures of a hexagon? *Explain*.

8.1 Find Angle Measures in Polygons

TEKS a.2, G.5.A, G.8.A, G.9.B

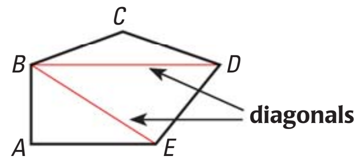
Before You classified polygons.

Now You will find angle measures in polygons.

Why? So you can describe a baseball park, as in Exs. 28–29.

Key Vocabulary
- **diagonal**
- **interior angle,** *p. 218*
- **exterior angle,** *p. 218*

In a polygon, two vertices that are endpoints of the same side are called *consecutive vertices*. A **diagonal** of a polygon is a segment that joins two *nonconsecutive vertices*. Polygon *ABCDE* has two diagonals from vertex *B*, $\overline{BD}$ and $\overline{BE}$.



As you can see, the diagonals from one vertex form triangles. In the Activity on page 506, you used these triangles to find the sum of the interior angle measures of a polygon. Your results support the following theorem and corollary.

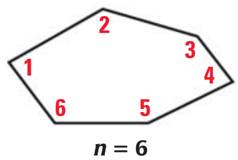
THEOREMS — *For Your Notebook*

THEOREM 8.1 Polygon Interior Angles Theorem

The sum of the measures of the interior angles of a convex n-gon is $(n - 2) \cdot 180°$.

$$m\angle 1 + m\angle 2 + \cdots + m\angle n = (n - 2) \cdot 180°$$

Proof: Ex. 33, p. 512 (for pentagons)



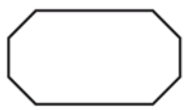
COROLLARY TO THEOREM 8.1 Interior Angles of a Quadrilateral

The sum of the measures of the interior angles of a quadrilateral is 360°.

Proof: Ex. 34, p. 512

EXAMPLE 1 Find the sum of angle measures in a polygon

Find the sum of the measures of the interior angles of a convex octagon.



Solution

An octagon has 8 sides. Use the Polygon Interior Angles Theorem.

$(n - 2) \cdot 180° = (8 - 2) \cdot 180°$ **Substitute 8 for *n*.**

$= 6 \cdot 180°$ **Subtract.**

$= 1080°$ **Multiply.**

▶ The sum of the measures of the interior angles of an octagon is 1080°.

EXAMPLE 2 Find the number of sides of a polygon

The sum of the measures of the interior angles of a convex polygon is 900°. Classify the polygon by the number of sides.

Solution

Use the Polygon Interior Angles Theorem to write an equation involving the number of sides n. Then solve the equation to find the number of sides.

$(n - 2) \cdot 180° = 900°$ — Polygon Interior Angles Theorem

$n - 2 = 5$ — Divide each side by 180°.

$n = 7$ — Add 2 to each side.

▶ The polygon has 7 sides. It is a heptagon.

✓ GUIDED PRACTICE for Examples 1 and 2

1. The coin shown is in the shape of a regular 11-gon. Find the sum of the measures of the interior angles.

2. The sum of the measures of the interior angles of a convex polygon is 1440°. Classify the polygon by the number of sides.

EXAMPLE 3 Find an unknown interior angle measure

xy ALGEBRA Find the value of x in the diagram shown.

Solution

The polygon is a quadrilateral. Use the Corollary to the Polygon Interior Angles Theorem to write an equation involving x. Then solve the equation.

$x° + 108° + 121° + 59° = 360°$ — Corollary to Theorem 8.1

$x + 288 = 360$ — Combine like terms.

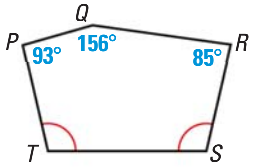
$x = 72$ — Subtract 288 from each side.

▶ The value of x is 72.

✓ GUIDED PRACTICE for Example 3

3. Use the diagram at the right. Find $m\angle S$ and $m\angle T$.

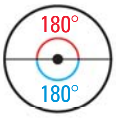
4. The measures of three of the interior angles of a quadrilateral are 89°, 110°, and 46°. Find the measure of the fourth interior angle.

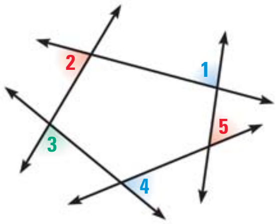


EXTERIOR ANGLES Unlike the sum of the interior angle measures of a convex polygon, the sum of the exterior angle measures does *not* depend on the number of sides of the polygon. The diagrams below suggest that the sum of the measures of the exterior angles, one at each vertex, of a pentagon is 360°. In general, this sum is 360° for any convex polygon.

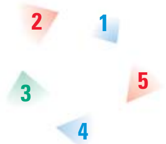
VISUALIZE IT

A circle contains two straight angles. So, there are 180° + 180°, or 360°, in a circle.

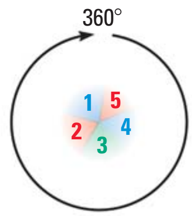




STEP 1 **Shade** one exterior angle at each vertex.



STEP 2 **Cut** out the exterior angles.



STEP 3 **Arrange** the exterior angles to form 360°.

Animated Geometry at classzone.com

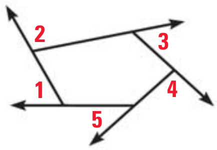
THEOREM *For Your Notebook*

THEOREM 8.2 Polygon Exterior Angles Theorem

The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is 360°.

$m\angle 1 + m\angle 2 + \cdots + m\angle n = 360°$

Proof: Ex. 35, p. 512



***n* = 5**

TAKS

EXAMPLE 4 TAKS PRACTICE: Multiple Choice

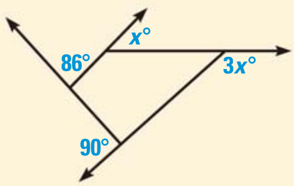
ELIMINATE CHOICES

You can quickly eliminate choice *D*. If *x* were equal to 138, then the sum of only two of the angle measures ($x°$ and $3x°$) would be greater than 360°.

What is the value of *x* in the diagram shown?

Ⓐ 45 Ⓑ 46

Ⓒ 89 Ⓓ 138



Solution

Use the Polygon Exterior Angles Theorem to write and solve an equation.

$x° + 3x° + 90° + 86° = 360°$	**Polygon Exterior Angles Theorem**
$4x + 176 = 360$	**Combine like terms.**
$4x = 184$	**Subtract 176 from each side.**
$x = 46$	**Divide each side by 4.**

▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Example 4

5. A convex hexagon has exterior angles with measures 34°, 49°, 58°, 67°, and 75°. What is the measure of an exterior angle at the sixth vertex?

EXAMPLE 5 Find angle measures in regular polygons

READ VOCABULARY
Recall that a *dodecagon* is a polygon with 12 sides and 12 vertices.

TRAMPOLINE The trampoline shown is shaped like a regular dodecagon. Find (a) the measure of each interior angle and (b) the measure of each exterior angle.

Solution

a. Use the Polygon Interior Angles Theorem to find the sum of the measures of the interior angles.

$(n - 2) \cdot 180° = (12 - 2) \cdot 180° = 1800°$

Then find the measure of one interior angle. A regular dodecagon has 12 congruent interior angles. Divide 1800° by 12: $1800° \div 12 = 150°$.

▶ The measure of each interior angle in the dodecagon is 150°.

b. By the Polygon Exterior Angles Theorem, the sum of the measures of the exterior angles, one angle at each vertex, is 360°. Divide 360° by 12 to find the measure of one of the 12 congruent exterior angles: $360° \div 12 = 30°$.

▶ The measure of each exterior angle in the dodecagon is 30°.

✓ GUIDED PRACTICE for Example 5

6. An interior angle and an adjacent exterior angle of a polygon form a linear pair. How can you use this fact as another method to find the exterior angle measure in Example 5?

8.1 EXERCISES

HOMEWORK KEY
- ○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 11, and 29
- ★ = **TAKS PRACTICE AND REASONING** Exs. 18, 23, 37, 39, 40, and 41
- ◆ = **MULTIPLE REPRESENTATIONS** Ex. 36

SKILL PRACTICE

1. VOCABULARY Sketch a convex hexagon. Draw all of its diagonals.

2. WRITING How many exterior angles are there in an n-gon? Are all the exterior angles considered when you use the Polygon Exterior Angles Theorem? *Explain.*

EXAMPLES 1 and 2
on pp. 507–508 for Exs. 3–10

INTERIOR ANGLE SUMS **Find the sum of the measures of the interior angles of the indicated convex polygon.**

3. Nonagon **4.** 14-gon **5.** 16-gon **6.** 20-gon

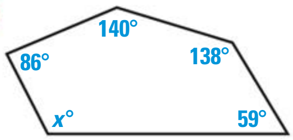
FINDING NUMBER OF SIDES **The sum of the measures of the interior angles of a convex polygon is given. Classify the polygon by the number of sides.**

7. 360° **8.** 720° **9.** 1980° **10.** 2340°

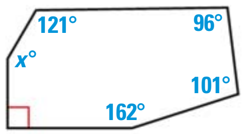
EXAMPLES 3 and 4
on pp. 508–509
for Exs. 11–18

ALGEBRA **Find the value of *x*.**

11.



12.



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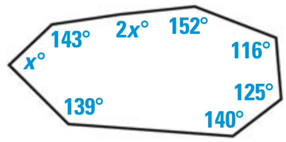

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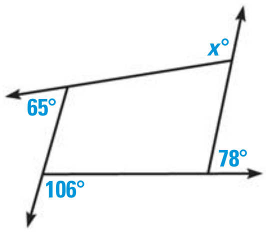
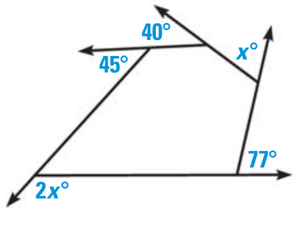
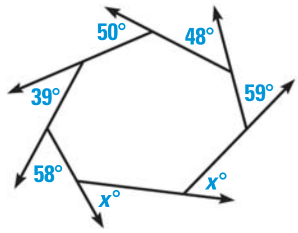


15.



16.



17. ERROR ANALYSIS A student claims that the sum of the measures of the exterior angles of an octagon is greater than the sum of the measures of the exterior angles of a hexagon. The student justifies this claim by saying that an octagon has two more sides than a hexagon. *Describe* and correct the error the student is making.

18. TAKS REASONING The measures of the interior angles of a quadrilateral are $x°$, $2x°$, $3x°$, and $4x°$. What is the measure of the largest interior angle?

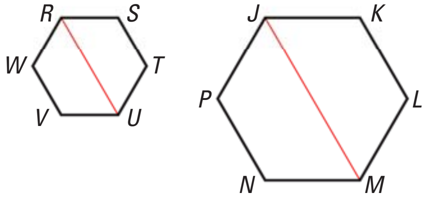
Ⓐ 120° Ⓑ 144° Ⓒ 160° Ⓓ 360°

EXAMPLE 5
on p. 510
for Exs. 19–21

REGULAR POLYGONS **Find the measures of an interior angle and an exterior angle of the indicated regular polygon.**

19. Regular pentagon **20.** Regular 18-gon **21.** Regular 90-gon

22. DIAGONALS OF SIMILAR FIGURES Hexagons *RSTUVW* and *JKLMNP* are similar. $\overline{RU}$ and $\overline{JM}$ are diagonals. Given $ST = 6$, $KL = 10$, and $RU = 12$, find JM.



23. TAKS REASONING *Explain* why any two regular pentagons are similar.

REGULAR POLYGONS **Find the value of *n* for each regular *n*-gon described.**

24. Each interior angle of the regular *n*-gon has a measure of 156°.

25. Each exterior angle of the regular *n*-gon has a measure of 9°.

26. POSSIBLE POLYGONS Determine if it is possible for a regular polygon to have an interior angle with the given angle measure. *Explain* your reasoning.

a. 165° **b.** 171° **c.** 75° **d.** 40°

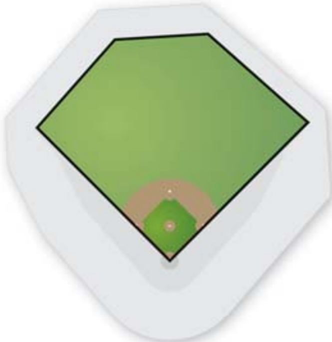
27. CHALLENGE Sides are added to a convex polygon so that the sum of its interior angle measures is increased by 540°. How many sides are added to the polygon? *Explain* your reasoning.

PROBLEM SOLVING

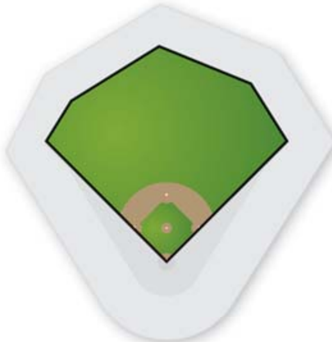
EXAMPLE 1
on p. 507
for Exs. 28–29

BASEBALL The outline of the playing field at a baseball park is a polygon, as shown. Find the sum of the measures of the interior angles of the polygon.

28.



29.



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EXAMPLE 5
on p. 510
for Exs. 30–31

30. **JEWELRY BOX** The base of a jewelry box is shaped like a regular hexagon. What is the measure of each interior angle of the hexagon?

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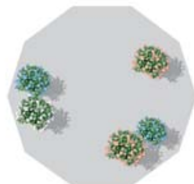
31. **GREENHOUSE** The floor of the greenhouse shown is a shaped like a regular decagon. Find the measure of an interior angle of the regular decagon. Then find the measure of an exterior angle.



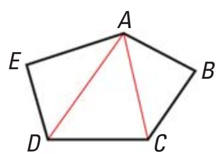
32. **MULTI-STEP PROBLEM** In pentagon *PQRST*, $\angle P$, $\angle Q$, and $\angle S$ are right angles, and $\angle R \cong \angle T$.

a. **Draw a Diagram** Sketch pentagon *PQRST*. Mark the right angles and the congruent angles.

b. **Calculate** Find the sum of the interior angle measures of *PQRST*.

c. **Calculate** Find $m\angle R$ and $m\angle T$.

33. **PROVING THEOREM 8.1 FOR PENTAGONS** The Polygon Interior Angles Theorem states that the sum of the measures of the interior angles of an n-gon is $(n - 2) \cdot 180°$. Write a paragraph proof of this theorem for the case when $n = 5$.



34. **PROVING A COROLLARY** Write a paragraph proof of the Corollary to the Polygon Interior Angles Theorem.

35. **PROVING THEOREM 8.2** Use the plan below to write a paragraph proof of the Polygon Exterior Angles Theorem.

Plan for Proof In a convex n-gon, the sum of the measures of an interior angle and an adjacent exterior angle at any vertex is 180°. Multiply by n to get the sum of all such sums at each vertex. Then subtract the sum of the interior angles derived by using the Polygon Interior Angles Theorem.

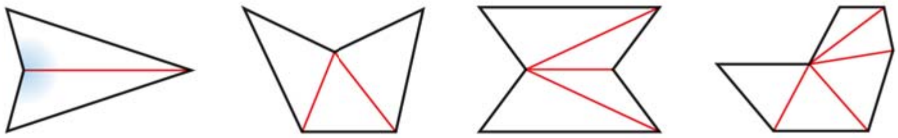
36. **MULTIPLE REPRESENTATIONS** The formula for the measure of each interior angle in a regular polygon can be written in function notation.

a. Writing a Function Write a function $h(n)$, where n is the number of sides in a regular polygon and $h(n)$ is the measure of any interior angle in the regular polygon.

b. Using a Function Use the function from part (a) to find $h(9)$. Then use the function to find n if $h(n) = 150°$.

c. Graphing a Function Graph the function from part (a) for $n = 3, 4, 5, 6, 7$, and 8. Based on your graph, *describe* what happens to the value of $h(n)$ as n increases. *Explain* your reasoning.

37. **TAKS REASONING** In a concave polygon, at least one interior angle measure is greater than 180°. For example, the measure of the shaded angle in the concave quadrilateral below is 210°.



a. In the diagrams above, the interiors of a concave quadrilateral, pentagon, hexagon, and heptagon are divided into triangles. Make a table like the one in the Activity on page 506. For each of the polygons shown above, record the number of sides, the number of triangles, and the sum of the measures of the interior angles.

b. Write an algebraic expression that you can use to find the sum of the measures of the interior angles of a concave polygon. *Explain*.

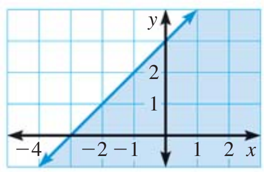
38. **CHALLENGE** Polygon *ABCDEFGH* is a regular octagon. Suppose sides $\overline{AB}$ and $\overline{CD}$ are extended to meet at a point *P*. Find $m\angle BPC$. *Explain* your reasoning. Include a diagram with your answer.

TAKS PRACTICE at classzone.com

MIXED REVIEW FOR TAKS

REVIEW
Skills Review Handbook p. 881; TAKS Workbook

39. **TAKS PRACTICE** Which inequality best describes the graph at the right? *TAKS Obj. 1*



Ⓐ $y \le x + 3$ Ⓑ $y \le x - 3$

Ⓒ $y \le -x + 3$ Ⓓ $y \le -x - 3$

REVIEW
Skills Review Handbook p. 894; TAKS Workbook

40. **TAKS PRACTICE** Don has 9 more movies than Will has. Kyle has 3 times as many movies as Don has. Together they have 46 movies. Which equation can be used to find how many movies Will has? *TAKS Obj. 10*

Ⓕ $9x + 5x + 3x = 46$ Ⓖ $x + (x + 9) + 3x = 46$

Ⓗ $x + (x + 9) + 3(5x) = 46$ Ⓙ $x + (x + 9) + 3(x + 9) = 46$

REVIEW
Lesson 6.2; TAKS Workbook

41. **TAKS PRACTICE** In 2001 the value of an investment account was $3500. In 2003 the value had increased linearly to $4750. If the value continues to increase at this rate, what is a reasonable projection for the value in 2007? *TAKS Obj. 4*

Ⓐ $6000 Ⓑ $7250 Ⓒ $8500 Ⓓ $11,750

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Keystrokes

8.2 Investigate Parallelograms

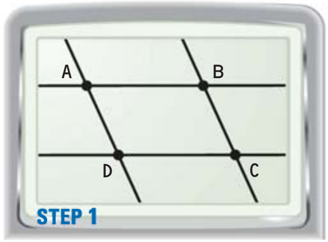
MATERIALS • graphing calculator or computer

TEKS *a.5, G.2.B, G.3.D, G.7.B*

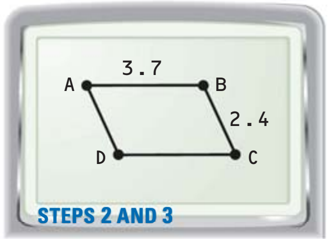
QUESTION What are some of the properties of a parallelogram?

You can use geometry drawing software to investigate relationships in special quadrilaterals.

EXPLORE Draw a quadrilateral



STEP 1 ***Draw parallel lines*** Construct $\overleftrightarrow{AB}$ and a line parallel to $\overleftrightarrow{AB}$ through point C. Then construct $\overleftrightarrow{BC}$ and a line parallel to $\overleftrightarrow{BC}$ through point A. Finally, construct a point D at the intersection of the line drawn parallel to $\overleftrightarrow{AB}$ and the line drawn parallel to $\overleftrightarrow{BC}$.



STEP 2 ***Draw quadrilateral*** Construct segments to form the sides of quadrilateral $ABCD$. After you construct $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$, hide the parallel lines that you drew in Step 1.

STEP 3 ***Measure side lengths*** Measure the side lengths AB, BC, CD, and DA. Drag point A or point B to change the side lengths of $ABCD$. What do you notice about the side lengths?

STEP 4 ***Measure angles*** Find the measures of $\angle A$, $\angle B$, $\angle C$, and $\angle D$. Drag point A or point B to change the angle measures of $ABCD$. What do you notice about the angle measures?

DRAW CONCLUSIONS Use your observations to complete these exercises

1. The quadrilateral you drew in the Explore is called a *parallelogram*. Why do you think this type of quadrilateral has this name?
2. Based on your observations, make a conjecture about the side lengths of a parallelogram and a conjecture about the angle measures of a parallelogram.
3. **REASONING** Draw a parallelogram and its diagonals. Measure the distance from the intersection of the diagonals to each vertex of the parallelogram. Make and test a conjecture about the diagonals of a parallelogram.

8.2 Use Properties of Parallelograms

TEKS G.1.A, G.3.E, G.5.A, G.7.C

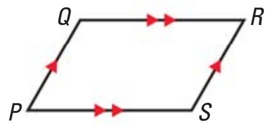
Before You used a property of polygons to find angle measures.

Now You will find angle and side measures in parallelograms.

Why? So you can solve a problem about airplanes, as in Ex. 38.

Key Vocabulary
- **parallelogram**

A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel. The term "parallelogram *PQRS*" can be written as ▱*PQRS*. In ▱*PQRS*, $\overline{PQ} \parallel \overline{RS}$ and $\overline{QR} \parallel \overline{PS}$ by definition. The theorems below describe other properties of parallelograms.

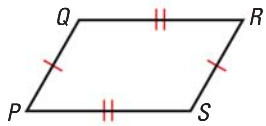


THEOREMS — *For Your Notebook*

THEOREM 8.3

If a quadrilateral is a parallelogram, then its opposite sides are congruent.

If *PQRS* is a parallelogram, then $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{PS}$.

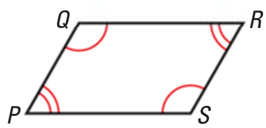


Proof: p. 516

THEOREM 8.4

If a quadrilateral is a parallelogram, then its opposite angles are congruent.

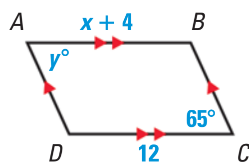
If *PQRS* is a parallelogram, then $\angle P \cong \angle R$ and $\angle Q \cong \angle S$.



Proof: Ex. 42, p. 520

EXAMPLE 1 Use properties of parallelograms

ALGEBRA **Find the values of x and y.**



ABCD is a parallelogram by the definition of a parallelogram. Use Theorem 8.3 to find the value of x.

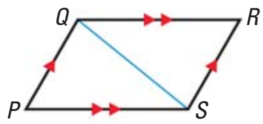
$AB = CD$	Opposite sides of a ▱ are ≅.
$x + 4 = 12$	Substitute $x + 4$ for AB and 12 for CD.
$x = 8$	Subtract 4 from each side.

By Theorem 8.4, $\angle A \cong \angle C$, or $m\angle A = m\angle C$. So, $y° = 65°$.

▶ In ▱*ABCD*, $x = 8$ and $y = 65$.

PROOF Theorem 8.3

If a quadrilateral is a parallelogram, then its opposite sides are congruent.



GIVEN ▶ *PQRS* is a parallelogram.

PROVE ▶ $\overline{PQ} \cong \overline{RS}$, $\overline{QR} \cong \overline{PS}$

Plan for Proof

a. Draw diagonal $\overline{QS}$ to form $\triangle PQS$ and $\triangle RSQ$.

b. Use the ASA Congruence Postulate to show that $\triangle PQS \cong \triangle RSQ$.

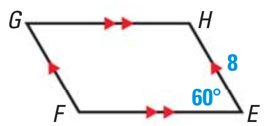
c. Use congruent triangles to show that $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{PS}$.

Plan in Action

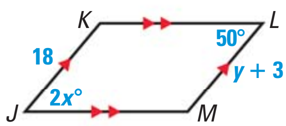
	STATEMENTS	REASONS
a.	**1.** *PQRS* is a ▱.	**1.** Given
	2. Draw $\overline{QS}$.	**2.** Through any 2 points there exists exactly 1 line.
	3. $\overline{PQ} \parallel \overline{RS}$, $\overline{QR} \parallel \overline{PS}$	**3.** Definition of parallelogram
b.	**4.** $\angle PQS \cong \angle RSQ$, $\angle PSQ \cong \angle RQS$	**4.** Alternate Interior Angles Theorem
	5. $\overline{QS} \cong \overline{QS}$	**5.** Reflexive Property of Congruence
	6. $\triangle PQS \cong \triangle RSQ$	**6.** ASA Congruence Postulate
c.	**7.** $\overline{PQ} \cong \overline{RS}$, $\overline{QR} \cong \overline{PS}$	**7.** Corresp. parts of ≅ ⚠ are ≅.

✓ GUIDED PRACTICE for Example 1

1. Find *FG* and $m\angle G$.

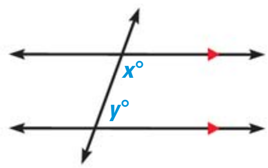


2. Find the values of *x* and *y*.



INTERIOR ANGLES The Consecutive Interior Angles Theorem (page 155) states that if two parallel lines are cut by a transversal, then the pairs of consecutive interior angles formed are supplementary.

A pair of consecutive angles in a parallelogram are like a pair of consecutive interior angles between parallel lines. This similarity suggests Theorem 8.5.



$x° + y° = 180°$

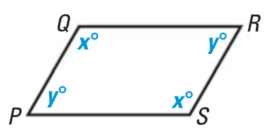
THEOREM *For Your Notebook*

THEOREM 8.5

If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

If *PQRS* is a parallelogram, then $x° + y° = 180°$.

Proof: Ex. 43, p. 520



EXAMPLE 2 Use properties of a parallelogram

DESK LAMP As shown, part of the extending arm of a desk lamp is a parallelogram. The angles of the parallelogram change as the lamp is raised and lowered. Find $m\angle BCD$ when $m\angle ADC = 110°$.

Solution

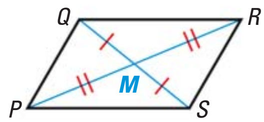
By Theorem 8.5, the consecutive angle pairs in $\square ABCD$ are supplementary. So, $m\angle ADC + m\angle BCD = 180°$. Because $m\angle ADC = 110°$, $m\angle BCD = 180° - 110° = 70°$.

THEOREM *For Your Notebook*

THEOREM 8.6

If a quadrilateral is a parallelogram, then its diagonals bisect each other.

Proof: Ex. 44, p. 521



$\overline{QM} \cong \overline{SM}$ and $\overline{PM} \cong \overline{RM}$

EXAMPLE 3 TAKS PRACTICE: Multiple Choice

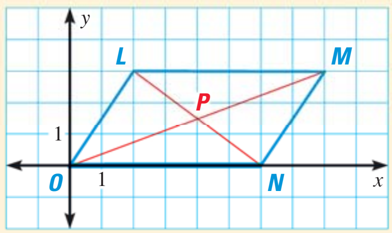
SIMPLIFY CALCULATIONS

In Example 3, you can use either diagonal to find the coordinates of *P*. Using $\overline{OM}$ simplifies calculations because one endpoint is (0, 0).

The diagonals of $\square LMNO$ intersect at point *P*. What are the coordinates of *P*?

Ⓐ $\left(4, \frac{5}{2}\right)$ Ⓑ $\left(\frac{5}{2}, 4\right)$

Ⓒ $\left(4, \frac{3}{2}\right)$ Ⓓ $\left(\frac{3}{2}, 4\right)$



Solution

By Theorem 8.6, the diagonals of a parallelogram bisect each other. So, *P* is the midpoint of diagonals $\overline{LN}$ and $\overline{OM}$. Use the Midpoint Formula.

Coordinates of midpoint *P* of $\overline{OM} = \left(\frac{8+0}{2}, \frac{3+0}{2}\right) = \left(4, \frac{3}{2}\right)$

▶ The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Examples 2 and 3

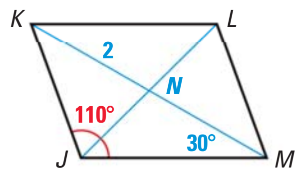
Find the indicated measure in $\square JKLM$.

3. NM

4. KM

5. $m\angle JML$

6. $m\angle KML$



8.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 13, and 39

★ = **TAKS PRACTICE AND REASONING** Exs. 16, 29, 35, 41, 46, 47, and 48

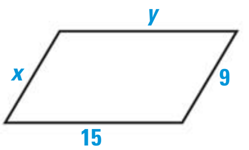
SKILL PRACTICE

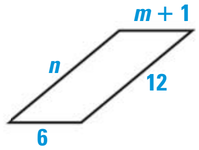
1. **VOCABULARY** What property of a parallelogram is included in the definition of a parallelogram? What properties are described by the theorems in this lesson?

2. **WRITING** In parallelogram $ABCD$, $m\angle A = 65°$. *Explain* how you would find the other angle measures of $\square ABCD$.

EXAMPLE 1 on p. 515 for Exs. 3–8

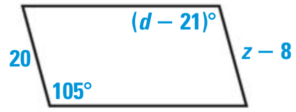
ALGEBRA **Find the value of each variable in the parallelogram.**

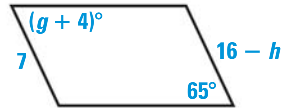
3.


4.


5.

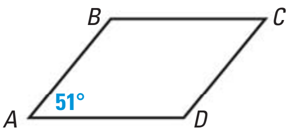

6.

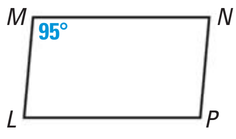

7.


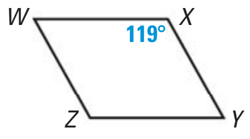
8.


EXAMPLE 2 on p. 517 for Exs. 9–12

FINDING ANGLE MEASURES **Find the measure of the indicated angle in the parallelogram.**

9. Find $m\angle B$.


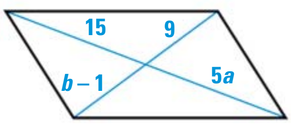
10. Find $m\angle L$.


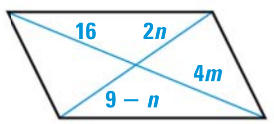
11. Find $m\angle Y$.


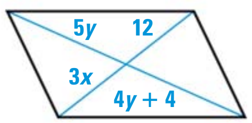
12. **SKETCHING** In $\square PQRS$, $m\angle R$ is 24 degrees more than $m\angle S$. Sketch $\square PQRS$. Find the measure of each interior angle. Then label each angle with its measure.

EXAMPLE 3 on p. 517 for Exs. 13–16

ALGEBRA **Find the value of each variable in the parallelogram.**

13.


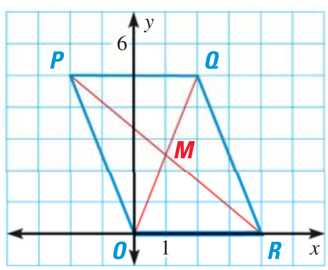
14.


15.


16. ★ **TAKS REASONING** The diagonals of parallelogram $OPQR$ intersect at point M. What are the coordinates of point M?

Ⓐ $\left(1, \frac{5}{2}\right)$ Ⓑ $\left(2, \frac{5}{2}\right)$

Ⓒ $\left(1, \frac{3}{2}\right)$ Ⓓ $\left(2, \frac{3}{2}\right)$



REASONING **Use the photo to copy and complete the statement. *Explain*.**

17. $\overline{AD} \cong$ __?__

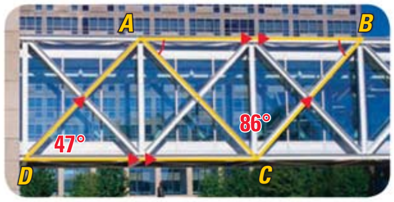
18. $\angle DAB \cong$ __?__

19. $\angle BCA \cong$ __?__

20. $m\angle ABC =$ __?__

21. $m\angle CAB =$ __?__

22. $m\angle CAD =$ __?__



USING A DIAGRAM **Find the indicated measure in $\square EFGH$. *Explain*.**

23. $m\angle EJF$

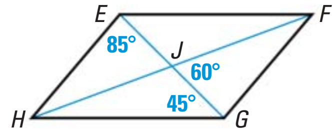
24. $m\angle EGF$

25. $m\angle HFG$

26. $m\angle GEF$

27. $m\angle HGF$

28. $m\angle EHG$



Animated Geometry at classzone.com

29. **TAKS REASONING** In parallelogram *ABCD*, $AB = 14$ inches and $BC = 20$ inches. What is the perimeter (in inches) of $\square ABCD$?

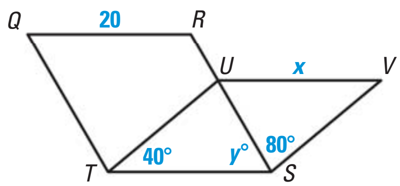
Ⓐ 28 Ⓑ 40 Ⓒ 68 Ⓓ 280

30. **ALGEBRA** The measure of one interior angle of a parallelogram is 0.25 times the measure of another angle. Find the measure of each angle.

31. **ALGEBRA** The measure of one interior angle of a parallelogram is 50 degrees more than 4 times the measure of another angle. Find the measure of each angle.

32. **ERROR ANALYSIS** In $\square ABCD$, $m\angle B = 50°$. A student says that $m\angle A = 50°$. *Explain* why this statement is incorrect.

33. **USING A DIAGRAM** In the diagram, *QRST* and *STUV* are parallelograms. Find the values of *x* and *y*. *Explain* your reasoning.

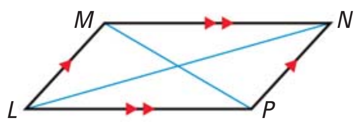


34. **FINDING A PERIMETER** The sides of $\square MNPQ$ are represented by the expressions below. Sketch $\square MNPQ$ and find its perimeter.

$MQ = -2x + 37$ $QP = y + 14$ $NP = x - 5$ $MN = 4y + 5$

35. **TAKS REASONING** In *ABCD*, $m\angle B = 124°$, $m\angle A = 66°$, and $m\angle C = 124°$. *Explain* why *ABCD* cannot be a parallelogram.

36. **FINDING ANGLE MEASURES** In $\square LMNP$ shown at the right, $m\angle MLN = 32°$, $m\angle NLP = (x^2)°$, $m\angle MNP = 12x°$, and $\angle MNP$ is an acute angle. Find $m\angle NLP$.

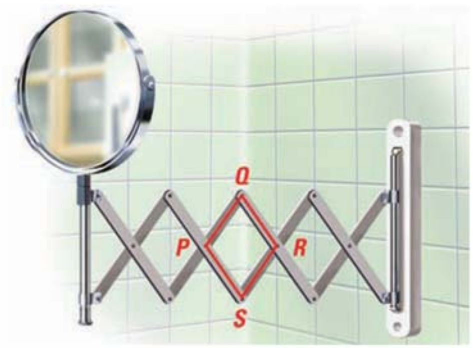


37. **CHALLENGE** Points $A(1, 2)$, $B(3, 6)$, and $C(6, 4)$ are three vertices of $\square ABCD$. Find the coordinates of each point that could be vertex *D*. Sketch each possible parallelogram in a separate coordinate plane. *Justify* your answers.

EXAMPLE 2 on p. 517 for Ex. 38

38. AIRPLANE The diagram shows the mechanism for opening the canopy on a small airplane. Two pivot arms attach at four pivot points A, B, C, and D. These points form the vertices of a parallelogram. Find $m\angle D$ when $m\angle C = 40°$. *Explain* your reasoning.

39. MIRROR The mirror shown is attached to the wall by an arm that can extend away from the wall. In the figure, points P, Q, R, and S are the vertices of a parallelogram. This parallelogram is one of several that change shape as the mirror is extended.



a. If $PQ = 3$ inches, find RS.

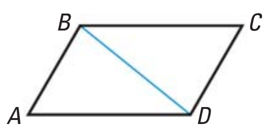
b. If $m\angle Q = 70°$, what is $m\angle S$?

c. What happens to $m\angle P$ as $m\angle Q$ increases? What happens to QS as $m\angle Q$ decreases? *Explain*.

40. USING RATIOS In $\square LMNO$, the ratio of LM to MN is 4 : 3. Find LM if the perimeter of $LMNO$ is 28.

41. TAKS REASONING Draw a triangle. Copy the triangle and combine the two triangles to form a quadrilateral. Show that the quadrilateral is a parallelogram. Then show how you can make additional copies of the triangle to form a larger parallelogram that is similar to the first parallelogram. *Justify* your method.

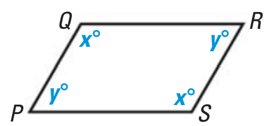
42. PROVING THEOREM 8.4 Use the diagram of quadrilateral $ABCD$ with the auxiliary line segment drawn to write a two-column proof of Theorem 8.4.



GIVEN ▶ $ABCD$ is a parallelogram.

PROVE ▶ $\angle A \cong \angle C$, $\angle B \cong \angle D$

43. PROVING THEOREM 8.5 Use properties of parallel lines to prove Theorem 8.5.



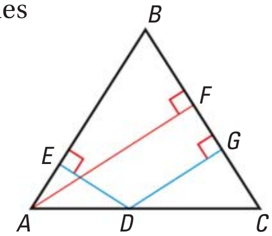
GIVEN ▶ $PQRS$ is a parallelogram.

PROVE ▶ $x° + y° = 180°$

○ = WORKED-OUT SOLUTIONS on p. WS1 = TAKS PRACTICE AND REASONING

44. PROVING THEOREM 8.6 Theorem 8.6 states that if a quadrilateral is a parallelogram, then its diagonals bisect each other. Write a two-column proof of Theorem 8.6.

45. CHALLENGE Suppose you choose a point on the base of an isosceles triangle. You draw segments from that point perpendicular to the legs of the triangle. Prove that the sum of the lengths of those segments is equal to the length of the altitude drawn to one leg.



GIVEN ▶ $\triangle ABC$ is isosceles with base $\overline{AC}$,
$\overline{AF}$ is the altitude drawn to $\overline{BC}$,
$\overline{DE} \perp \overline{AB}$, $\overline{DG} \perp \overline{BC}$

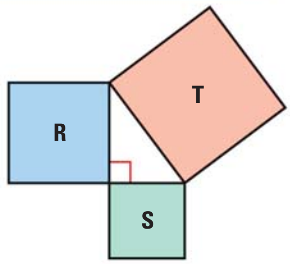
PROVE ▶ For D anywhere on $\overline{AC}$, $DE + DG = AF$.

TAKS PRACTICE at classzone.com

MIXED REVIEW FOR TAKS

REVIEW
Lesson 7.3;
TAKS Workbook

46. TAKS PRACTICE Which statement is true? *TAKS Obj. 7*



Ⓐ The perimeter of R plus the perimeter of S equals the perimeter of T.

Ⓑ The perimeter of R minus the perimeter of S equals the perimeter of T.

Ⓒ The area of R plus the area of S equals the area of T.

Ⓓ The area of R minus the area of S equals the area of T.

REVIEW
Skill Review Handbook p. 884;
TAKS Workbook

47. TAKS PRACTICE The equation $y = 2500(1.01)^x$ shows the relationship between x, the number of years since 2005, and y, the population of a town. What is the approximate population of the town in 2008? *TAKS Obj. 2*

Ⓐ 2550 Ⓑ 2576 Ⓒ 2654 Ⓓ 7575

REVIEW
Lesson 3.4;
TAKS Workbook

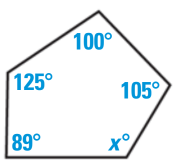
48. TAKS PRACTICE What is the slope of the line that contains the points (0, 1), (−2, 0), and (−4, −1)? *TAKS Obj. 3*

Ⓐ −2 Ⓑ −0.5 Ⓒ 0.5 Ⓓ 2

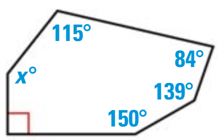
QUIZ *for Lessons 8.1–8.2*

Find the value of x. *(p. 507)*

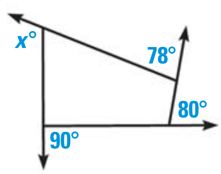
1.



2.

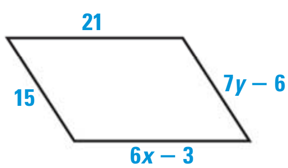


3.

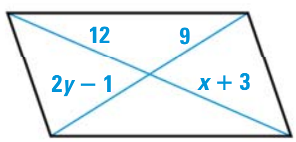


Find the value of each variable in the parallelogram. *(p. 515)*

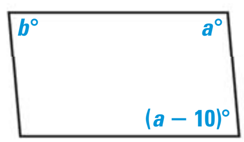
4.



5.



6.



8.3 Show that a Quadrilateral is a Parallelogram

TEKS G.1.A, G.3.C, G.7.A, G.7.C

Before You identified properties of parallelograms.

Now You will use properties to identify parallelograms.

Why? So you can describe how a music stand works, as in Ex. 32.

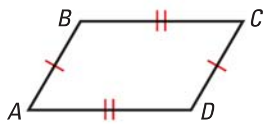
Key Vocabulary
- **parallelogram,** *p. 515*

Given a parallelogram, you can use Theorem 8.3 and Theorem 8.4 to prove statements about the angles and sides of the parallelogram. The converses of Theorem 8.3 and Theorem 8.4 are stated below. You can use these and other theorems in this lesson to prove that a quadrilateral with certain properties is a parallelogram.

THEOREMS *For Your Notebook*

THEOREM 8.7

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

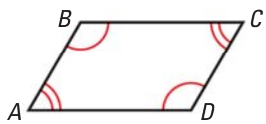


If $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$, then $ABCD$ is a parallelogram.

Proof: below

THEOREM 8.8

If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



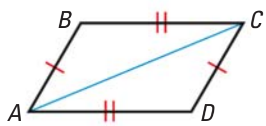
If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then $ABCD$ is a parallelogram.

Proof: Ex. 38, p. 529

PROOF Theorem 8.7

GIVEN ▶ $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{AD}$

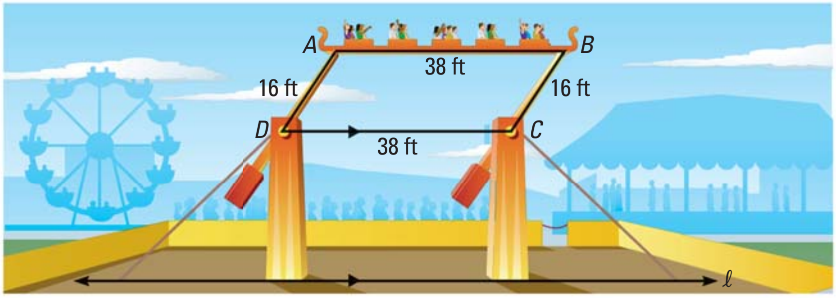
PROVE ▶ $ABCD$ is a parallelogram.



Proof Draw $\overline{AC}$, forming $\triangle ABC$ and $\triangle CDA$. You are given that $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$. Also, $\overline{AC} \cong \overline{AC}$ by the Reflexive Property of Congruence. So, $\triangle ABC \cong \triangle CDA$ by the SSS Congruence Postulate. Because corresponding parts of congruent triangles are congruent, $\angle BAC \cong \angle DCA$ and $\angle BCA \cong DAC$. Then, by the Alternate Interior Angles Converse, $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$. By definition, $ABCD$ is a parallelogram.

EXAMPLE 1 Solve a real-world problem

RIDE An amusement park ride has a moving platform attached to four swinging arms. The platform swings back and forth, higher and higher, until it goes over the top and around in a circular motion. In the diagram below, $\overline{AD}$ and $\overline{BC}$ represent two of the swinging arms, and $\overline{DC}$ is parallel to the ground (line ℓ). *Explain* why the moving platform $\overline{AB}$ is always parallel to the ground.



Solution

The shape of quadrilateral $ABCD$ changes as the moving platform swings around, but its side lengths do not change. Both pairs of opposite sides are congruent, so $ABCD$ is a parallelogram by Theorem 8.7.

By the definition of a parallelogram, $\overline{AB} \parallel \overline{DC}$. Because $\overline{DC}$ is parallel to line ℓ, $\overline{AB}$ is also parallel to line ℓ by the Transitive Property of Parallel Lines. So, the moving platform is parallel to the ground.

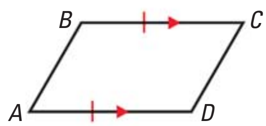
✓ GUIDED PRACTICE for Example 1

1. In quadrilateral $WXYZ$, $m\angle W = 42°$, $m\angle X = 138°$, $m\angle Y = 42°$. Find $m\angle Z$. Is $WXYZ$ a parallelogram? *Explain* your reasoning.

THEOREMS *For Your Notebook*

THEOREM 8.9

If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.

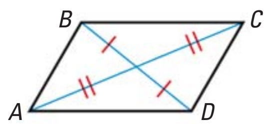


If $\overline{BC} \parallel \overline{AD}$ and $\overline{BC} \cong \overline{AD}$, then $ABCD$ is a parallelogram.

Proof: Ex. 33, p. 528

THEOREM 8.10

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.



If $\overline{BD}$ and $\overline{AC}$ bisect each other, then $ABCD$ is a parallelogram.

Proof: Ex. 39, p. 529

EXAMPLE 2 Identify a parallelogram

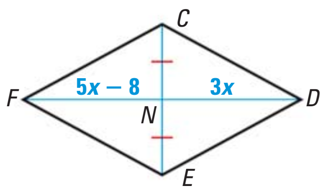
ARCHITECTURE The doorway shown is part of a building in England. Over time, the building has leaned sideways. *Explain* how you know that $SV = TU$.

Solution

In the photograph, $\overline{ST} \parallel \overline{UV}$ and $\overline{ST} \cong \overline{UV}$. By Theorem 8.9, quadrilateral *STUV* is a parallelogram. By Theorem 8.3, you know that opposite sides of a parallelogram are congruent. So, $SV = TU$.

EXAMPLE 3 Use algebra with parallelograms

ALGEBRA For what value of x is quadrilateral *CDEF* a parallelogram?



Solution

By Theorem 8.10, if the diagonals of *CDEF* bisect each other, then it is a parallelogram. You are given that $\overline{CN} \cong \overline{EN}$. Find x so that $\overline{FN} \cong \overline{DN}$.

$FN = DN$	**Set the segment lengths equal.**
$5x - 8 = 3x$	**Substitute $5x - 8$ for *FN* and $3x$ for *DN*.**
$2x - 8 = 0$	**Subtract $3x$ from each side.**
$2x = 8$	**Add 8 to each side.**
$x = 4$	**Divide each side by 2.**

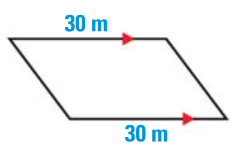
When $x = 4$, $FN = 5(4) - 8 = 12$ and $DN = 3(4) = 12$.

▶ Quadrilateral *CDEF* is a parallelogram when $x = 4$.

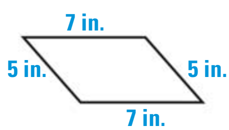
✓ GUIDED PRACTICE for Examples 2 and 3

What theorem can you use to show that the quadrilateral is a parallelogram?

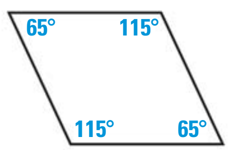
2.



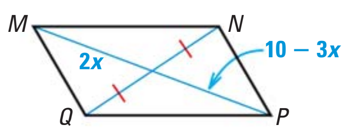
3.



4.



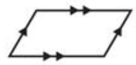
5. For what value of x is quadrilateral *MNPQ* a parallelogram? *Explain* your reasoning.



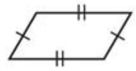
CONCEPT SUMMARY *For Your Notebook*

Ways to Prove a Quadrilateral is a Parallelogram

1. Show both pairs of opposite sides are parallel.
(DEFINITION)

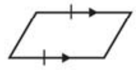


2. Show both pairs of opposite sides are congruent.
(THEOREM 8.7)



3. Show both pairs of opposite angles are congruent.
(THEOREM 8.8)

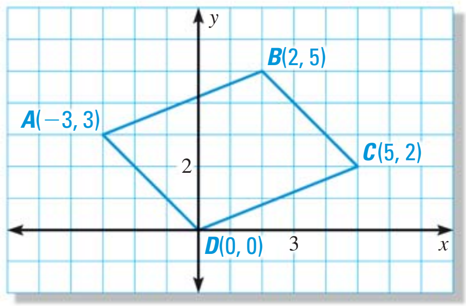
4. Show one pair of opposite sides are congruent and parallel.
(THEOREM 8.9)



5. Show the diagonals bisect each other.
(THEOREM 8.10)

EXAMPLE 4 Use coordinate geometry

Show that quadrilateral *ABCD* is a parallelogram.



ANOTHER WAY
For alternative methods for solving the problem in Example 4, turn to page 530 for the **Problem Solving Workshop**.

Solution

One way is to show that a pair of sides are congruent and parallel. Then apply Theorem 8.9.

First use the Distance Formula to show that $\overline{AB}$ and $\overline{CD}$ are congruent.

$AB = \sqrt{[2 - (-3)]^2 + (5 - 3)^2} = \sqrt{29}$ $\qquad CD = \sqrt{(5 - 0)^2 + (2 - 0)^2} = \sqrt{29}$

Because $AB = CD = \sqrt{29}$, $\overline{AB} \cong \overline{CD}$.

Then use the slope formula to show that $\overline{AB} \parallel \overline{CD}$.

Slope of $\overline{AB} = \frac{5 - (3)}{2 - (-3)} = \frac{2}{5}$ $\qquad$ Slope of $\overline{CD} = \frac{2 - 0}{5 - 0} = \frac{2}{5}$

Because $\overline{AB}$ and $\overline{CD}$ have the same slope, they are parallel.

▶ $\overline{AB}$ and $\overline{CD}$ are congruent and parallel. So, *ABCD* is a parallelogram by Theorem 8.9.

✓ GUIDED PRACTICE for Example 4

6. Refer to the Concept Summary above. *Explain* how other methods can be used to show that quadrilateral *ABCD* in Example 4 is a parallelogram.

8.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 15, and 35

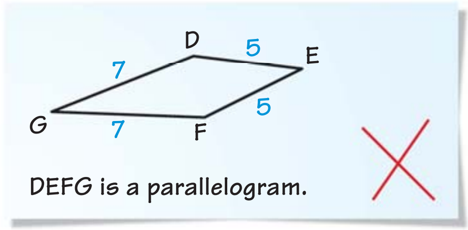
★ = **TAKS PRACTICE AND REASONING** Exs. 7, 18, 37, and 43

SKILL PRACTICE

1. **VOCABULARY** *Explain* how knowing that $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$ allows you to show that quadrilateral *ABCD* is a parallelogram.

2. **WRITING** A quadrilateral has four congruent sides. Is the quadrilateral a parallelogram? *Justify* your answer.

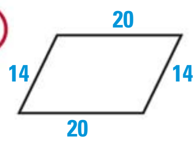
3. **ERROR ANALYSIS** A student claims that because two pairs of sides are congruent, quadrilateral *DEFG* shown at the right is a parallelogram. *Describe* the error that the student is making.

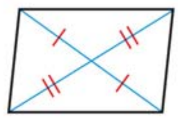


EXAMPLES 1 and 2 on pp. 523–524 for Exs. 4–7

REASONING **What theorem can you use to show that the quadrilateral is a parallelogram?**

4. 

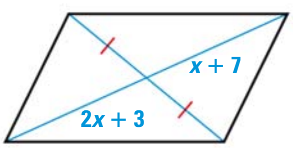
5. 

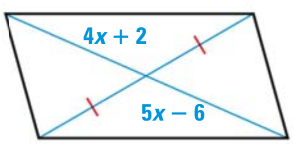
6. 

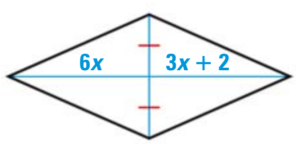
7. ★ **TAKS REASONING** When you shift gears on a bicycle, a mechanism called a *derailleur* moves the chain to a new gear. For the derailleur shown below, $JK = 5.5$ cm, $KL = 2$ cm, $ML = 5.5$ cm, and $MJ = 2$ cm. *Explain* why $\overline{JK}$ and $\overline{ML}$ are always parallel as the derailleur moves.

EXAMPLE 3 on p. 524 for Exs. 8–10

ALGEBRA **For what value of *x* is the quadrilateral a parallelogram?**

8. 

9. 

10. 

EXAMPLE 4 on p. 525 for Exs. 11–14

COORDINATE GEOMETRY **The vertices of quadrilateral *ABCD* are given. Draw *ABCD* in a coordinate plane and show that it is a parallelogram.**

11. $A(0, 1)$, $B(4, 4)$, $C(12, 4)$, $D(8, 1)$

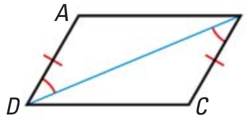
12. $A(-3, 0)$, $B(-3, 4)$, $C(3, -1)$, $D(3, -5)$

13. $A(-2, 3)$, $B(-5, 7)$, $C(3, 6)$, $D(6, 2)$

14. $A(-5, 0)$, $B(0, 4)$, $C(3, 0)$, $D(-2, -4)$

REASONING ***Describe*** **how to prove that *ABCD* is a parallelogram.**

15.



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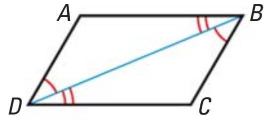

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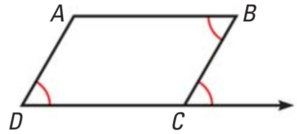


Animated Geometry at classzone.com

18. **TAKS REASONING** In quadrilateral *WXYZ*, $\overline{WZ}$ and $\overline{XY}$ are congruent and parallel. Which statement below is not necessarily true?

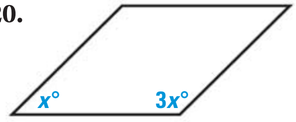
Ⓐ $m\angle Y + m\angle W = 180°$ Ⓑ $\angle X \cong \angle Z$

Ⓒ $\overline{WX} \cong \overline{ZY}$ Ⓓ $\overline{WX} \parallel \overline{ZY}$

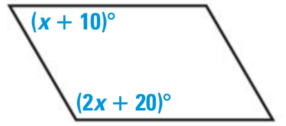
ALGEBRA **For what value of *x* is the quadrilateral a parallelogram?**

19.

20.



21.

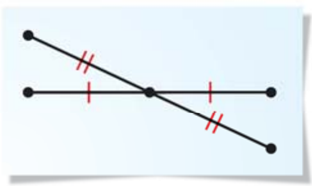


BICONDITIONALS **Write the indicated theorems as a biconditional statement.**

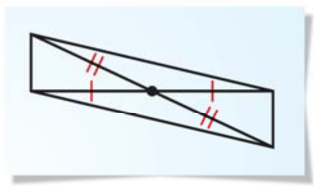
22. Theorem 8.3, page 515 and Theorem 8.7, page 522

23. Theorem 8.4, page 515 and Theorem 8.8, page 522

24. **REASONING** Follow the steps below to draw a parallelogram. *Explain* why this method works. State a theorem to support your answer.



STEP 1 Use a ruler to draw two segments that intersect at their midpoints.



STEP 2 Connect the endpoints of the segments to form a quadrilateral.

COORDINATE GEOMETRY **Three of the vertices of ▱*ABCD* are given. Find the coordinates of point *D*. Show your method.**

25. $A(-2, -3)$, $B(4, -3)$, $C(3, 2)$, $D(x, y)$

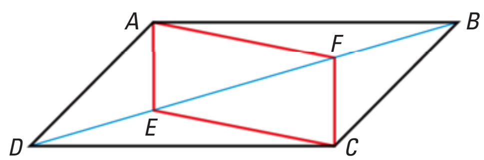
26. $A(-4, 1)$, $B(-1, 5)$, $C(6, 5)$, $D(x, y)$

27. $A(-4, 4)$, $B(4, 6)$, $C(3, -1)$, $D(x, y)$

28. $A(-1, 0)$, $B(0, -4)$, $C(8, -6)$, $D(x, y)$

29. **CONSTRUCTION** There is more than one way to use a compass and a straightedge to construct a parallelogram. *Describe* a method that uses Theorem 8.7 or Theorem 8.9. Then use your method to construct a parallelogram.

30. **CHALLENGE** In the diagram, *ABCD* is a parallelogram, $BF = DE = 12$, and $CF = 8$. Find *AE*. *Explain* your reasoning.



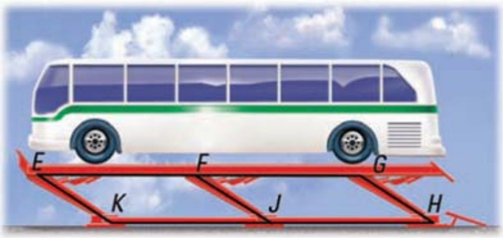
PROBLEM SOLVING

EXAMPLES 1 and 2
on pp. 523–524
for Exs. 31–32

31. **AUTOMOBILE REPAIR** The diagram shows an automobile lift. A bus drives on to the ramp ($\overline{EG}$). Levers ($\overline{EK}$, $\overline{FJ}$, and $\overline{GH}$) raise the bus. In the diagram, $\overline{EG} \cong \overline{KH}$ and $EK = FJ = GH$. Also, F is the midpoint of $\overline{EG}$, and J is the midpoint of $\overline{KH}$.

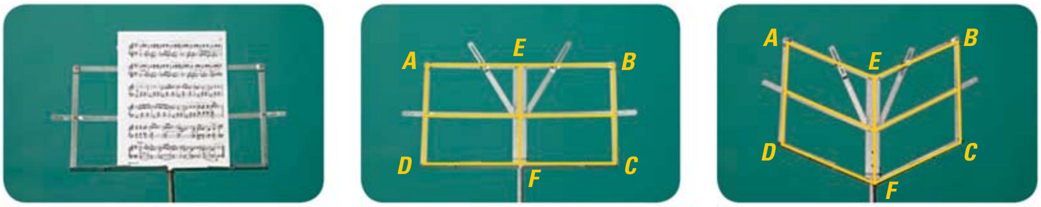
a. Identify all the quadrilaterals in the automobile lift. *Explain* how you know that each one is a parallelogram.

b. *Explain* why $\overline{EG}$ is always parallel to $\overline{KH}$.



TEXAS @HomeTutor for problem solving help at classzone.com

32. **MUSIC STAND** A music stand can be folded up, as shown below. In the diagrams, $\angle A \cong \angle EFD$, $\angle D \cong \angle AEF$, $\angle C \cong \angle BEF$, and $\angle B \cong \angle CFE$. *Explain* why $\overline{AD}$ and $\overline{BC}$ remain parallel as the stand is folded up. Which other labeled segments remain parallel?



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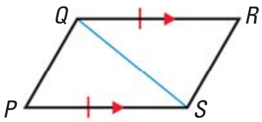
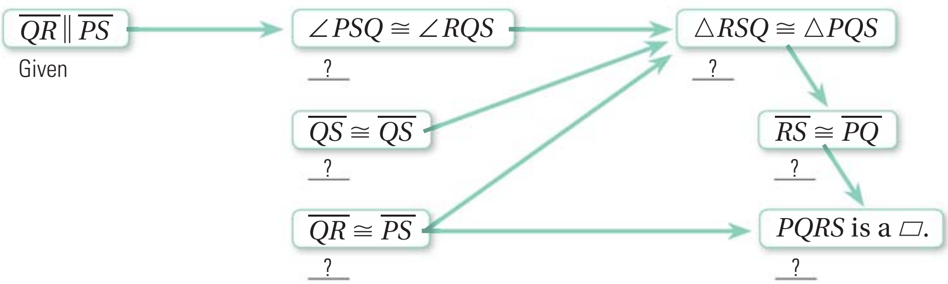
33. **PROVING THEOREM 8.9** Use the diagram of $PQRS$ with the auxiliary line segment drawn. Copy and complete the flow proof of Theorem 8.9.



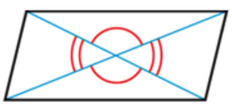
GIVEN ▶ $\overline{QR} \parallel \overline{PS}$, $\overline{QR} \cong \overline{PS}$

PROVE ▶ $PQRS$ is a parallelogram.



REASONING **A student claims incorrectly that the marked information can be used to show that the figure is a parallelogram. Draw a quadrilateral with the marked properties that is clearly *not* a parallelogram. *Explain*.**

34.



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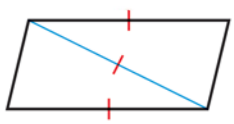
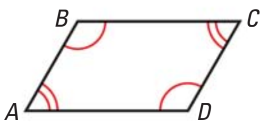


36.

37. **TAKS REASONING** Theorem 8.5 states that if a quadrilateral is a parallelogram, then its consecutive angles are supplementary. Write the converse of Theorem 8.5. Then write a plan for proving the converse of Theorem 8.5. Include a diagram.

38. **PROVING THEOREM 8.8** Prove Theorem 8.8.

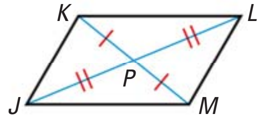


GIVEN ▶ $\angle A \cong \angle C$, $\angle B \cong \angle D$

PROVE ▶ $ABCD$ is a parallelogram.

Hint: Let $x°$ represent $m\angle A$ and $m\angle C$, and let $y°$ represent $m\angle B$ and $m\angle D$. Write and simplify an equation involving x and y.

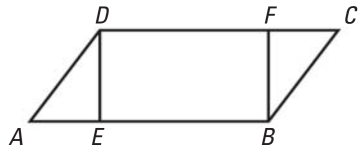
39. **PROVING THEOREM 8.10** Prove Theorem 8.10.



GIVEN ▶ Diagonals $\overline{JL}$ and $\overline{KM}$ bisect each other.

PROVE ▶ $JKLM$ is a parallelogram.

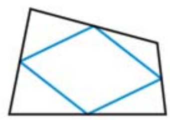
40. **PROOF** Use the diagram at the right.



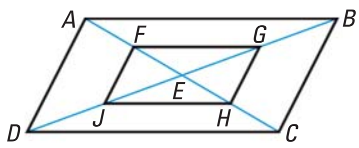
GIVEN ▶ $DEBF$ is a parallelogram, $AE = CF$

PROVE ▶ $ABCD$ is a parallelogram.

41. **REASONING** In the diagram, the midpoints of the sides of a quadrilateral have been joined to form what appears to be a parallelogram. Show that a quadrilateral formed by connecting the midpoints of the sides of any quadrilateral is *always* a parallelogram. (*Hint:* Draw a diagram. Include a diagonal of the larger quadrilateral. Show how two sides of the smaller quadrilateral are related to the diagonal.)



42. **CHALLENGE** Show that if $ABCD$ is a parallelogram with its diagonals intersecting at E, then you can connect the midpoints F, G, H, and J of $\overline{AE}$, $\overline{BE}$, $\overline{CE}$, and $\overline{DE}$, respectively, to form another parallelogram, $FGHJ$.



MIXED REVIEW FOR TAKS

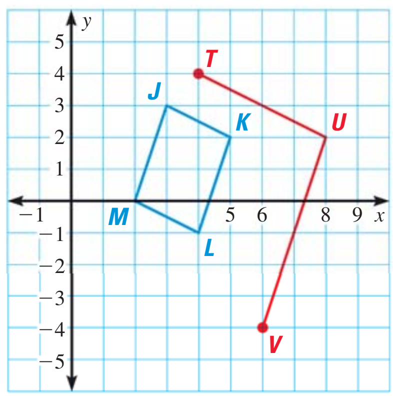
TAKS PRACTICE at classzone.com

REVIEW
Lesson 6.7;
TAKS Workbook

43. **TAKS PRACTICE** At what coordinates should vertex W be placed to create a quadrilateral $TUVW$ that is similar to quadrilateral $JKLM$? ***TAKS Obj. 6***

Ⓐ (2, −3)　　Ⓑ (3, −3)

Ⓒ (2, −2)　　Ⓓ (3, −2)



PROBLEM SOLVING WORKSHOP
LESSON 8.3

Using ALTERNATIVE METHODS

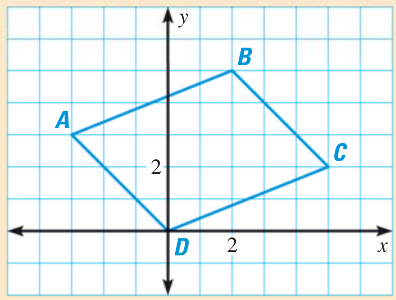
TEKS G.4, G.7.A, G.7.C, G.9.A

Another Way to Solve Example 4, page 525

MULTIPLE REPRESENTATIONS In Example 4 on page 525, the problem is solved by showing that one pair of opposite sides are congruent and parallel using the Distance Formula and the slope formula. There are other ways to show that a quadrilateral is a parallelogram.

PROBLEM

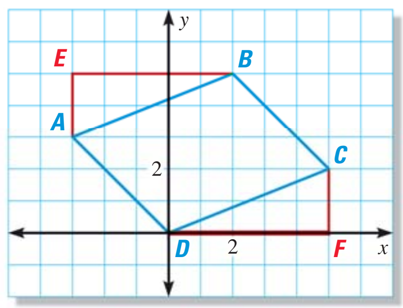
Show that quadrilateral *ABCD* is a parallelogram.



METHOD 1

Use Opposite Sides You can show that both pairs of opposite sides are congruent.

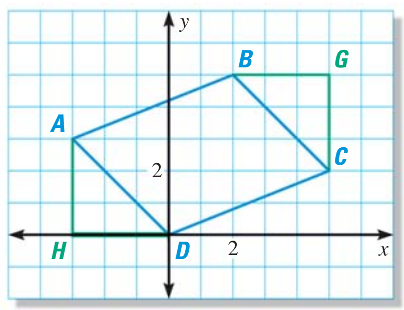
STEP 1 **Draw** two right triangles. Use $\overline{AB}$ as the hypotenuse of $\triangle AEB$ and $\overline{CD}$ as the hypotenuse of $\triangle CFD$.



STEP 2 **Show** that $\triangle AEB \cong \triangle CFD$. From the graph, $AE = 2$, $BE = 5$, and $\angle E$ is a right angle. Similarly, $CF = 2$, $DF = 5$, and $\angle F$ is a right angle. So, $\triangle AEB \cong \triangle CFD$ by the SAS Congruence Postulate.

STEP 3 **Use** the fact that corresponding parts of congruent triangles are congruent to show that $\overline{AB} \cong \overline{CD}$.

STEP 4 **Repeat** Steps 1–3 for sides $\overline{AD}$ and $\overline{BC}$. You can prove that $\triangle AHD \cong \triangle CGB$. So, $\overline{AD} \cong \overline{CB}$.



▸ The pairs of opposite sides, $\overline{AB}$ and $\overline{CD}$ and $\overline{AD}$ and $\overline{CB}$, are congruent. So, *ABCD* is a parallelogram by Theorem 8.7.

METHOD 2 **Use Diagonals** You can show that the diagonals bisect each other.

STEP 1 **Use** the Midpoint Formula to find the midpoint of diagonal $\overline{AC}$.

The coordinates of the endpoints of $\overline{AC}$ are $A(-3, 3)$ and $C(5, 2)$.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-3 + 5}{2}, \frac{3 + 2}{2}\right) = \left(\frac{2}{2}, \frac{5}{2}\right) = \left(1, \frac{5}{2}\right)$$

STEP 2 **Use** the Midpoint Formula to find the midpoint of diagonal $\overline{BD}$.

The coordinates of the endpoints of $\overline{BD}$ are $B(2, 5)$ and $D(0, 0)$.

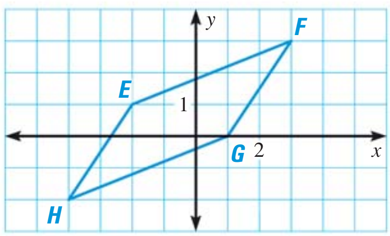
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2 + 0}{2}, \frac{5 + 0}{2}\right) = \left(\frac{2}{2}, \frac{5}{2}\right) = M\left(1, \frac{5}{2}\right)$$

▶ Because the midpoints of both diagonals are the same point, the diagonals bisect each other. So, *ABCD* is a parallelogram by Theorem 8.10.

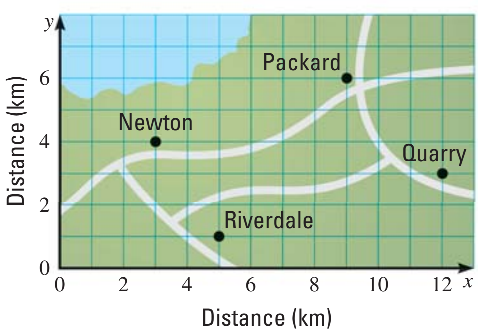
PRACTICE

1. **SLOPE** Show that quadrilateral *ABCD* in the problem on page 530 is a parallelogram by showing that both pairs of opposite sides are parallel.

2. **PARALLELOGRAMS** Use two methods to show that *EFGH* is a parallelogram.



3. **MAP** Do the four towns on the map form the vertices of a parallelogram? *Explain.*



4. **QUADRILATERALS** Is the quadrilateral a parallelogram? *Justify* your answer.

   a. $A(1, 0)$, $B(5, 0)$, $C(7, 2)$, $D(3, 2)$

   b. $E(3, 4)$ $F(9, 5)$, $G(6, 8)$, $H(6, 0)$

   c. $J(-1, 0)$, $K(2, -2)$, $L(2, 2)$, $M(-1, 4)$

5. **ERROR ANALYSIS** Quadrilateral *PQRS* has vertices $P(2, 2)$, $Q(3, 4)$, $R(6, 5)$, and $S(5, 3)$. A student makes the conclusion below. *Describe* and correct the error(s) made by the student.

   $\overline{PQ}$ and $\overline{QR}$ are opposite sides, so they should be congruent.

   $PQ = \sqrt{(3 - 2)^2 + (4 - 2)^2} = \sqrt{5}$

   $QR = \sqrt{(6 - 3)^2 + (5 - 4)^2} = \sqrt{10}$

   But $\overline{PQ} \not\cong \overline{QR}$. So, PQRS is not a parallelogram.

6. **WRITING** Points $O(0, 0)$, $P(3, 5)$, and $Q(4, 0)$ are vertices of $\triangle OPQ$, and are also vertices of a parallelogram. Find all points *R* that could be the other vertex of the parallelogram. *Explain* your reasoning.

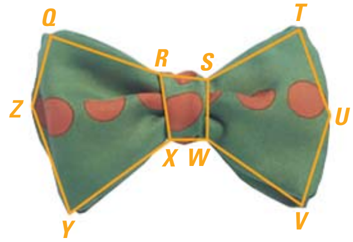
MIXED REVIEW FOR TEKS

TAKS PRACTICE
classzone.com

Lessons 8.1–8.3

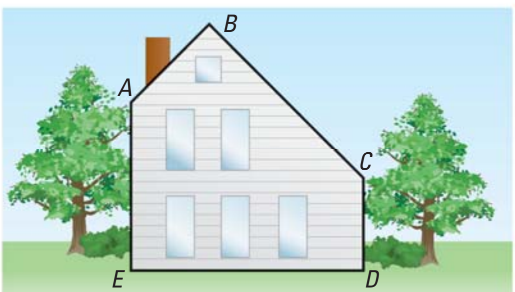
MULTIPLE CHOICE

1. **BOWTIE** The shape of a bowtie can be approximated by the composite of three convex polygons, as shown. What is the sum of the measures of the interior angles of the three polygons? *TEKS G.5.B*



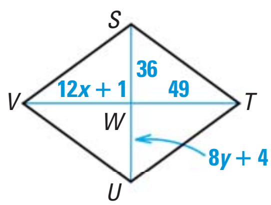
(A) 1080° (B) 1260°
(C) 1440° (D) 1800°

2. **GRAPHIC DESIGN** A graphic designer creates an electronic image of a house. In the drawing, $\angle B$, $\angle D$, and $\angle E$ are right angles, and $\angle A \cong \angle C$. What is $m\angle A$? *TEKS G.5.A*



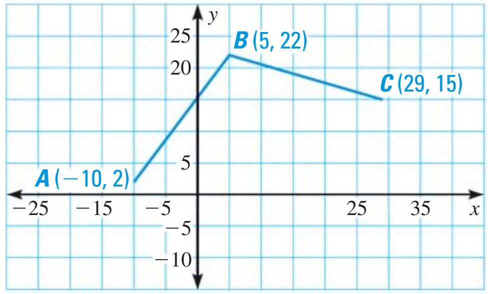
(F) 90° (G) 135°
(H) 165° (J) 270°

3. **PARALLELOGRAM** Quadrilateral *STUV* is a parallelogram. What is the value of x? *TEKS G.5.A*



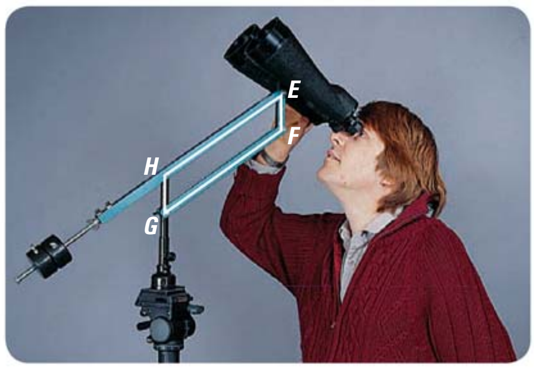
(A) 3 (B) 4
(C) 5 (D) 6

4. **PARALLELOGRAMS** Quadrilateral *ABCD* is a parallelogram. What are the coordinates of vertex *D*? *TEKS G.7.B*



(F) (5, −22)
(G) (−14, 5)
(H) (14, −5)
(J) (−5, 22)

5. **BINOCULARS STAND** A stand to hold binoculars uses a quadrilateral in its design, as shown below. Quadrilateral *EFGH* changes shape as the binoculars are moved, but $\overline{EF}$ and $\overline{GH}$ are always congruent and parallel. As *EFGH* changes shape, $m\angle F$ changes from 45° to 40°. What is the new measure of $\angle G$? *TEKS G.5.B*



(A) 40° (B) 45°
(C) 135° (D) 140°

GRIDDED ANSWER

6. **DECAGON** A convex decagon has interior angles with measures 157°, 128°, 115°, 162°, 169°, 131°, 155°, 168°, x°, and $2x$°. Find the value of x. *TEKS G.5.A*

8.4 Properties of Rhombuses, Rectangles, and Squares

TEKS *a.4, G.3.A, G.9.A, G.9.B*

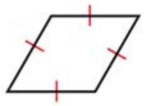
Before You used properties of parallelograms.

Now You will use properties of rhombuses, rectangles, and squares.

Why? So you can solve a carpentry problem, as in Example 4.

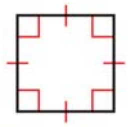
Key Vocabulary
- rhombus
- rectangle
- square

In this lesson, you will learn about three special types of parallelograms: *rhombuses*, *rectangles*, and *squares*.



A rhombus is a parallelogram with four congruent sides.

A rectangle is a parallelogram with four right angles.



A square is a parallelogram with four congruent sides and four right angles.

You can use the corollaries below to prove that a quadrilateral is a rhombus, rectangle, or square, without first proving that the quadrilateral is a parallelogram.

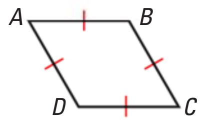
COROLLARIES — *For Your Notebook*

RHOMBUS COROLLARY

A quadrilateral is a rhombus if and only if it has four congruent sides.

ABCD is a rhombus if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$.

Proof: Ex. 57, p. 539

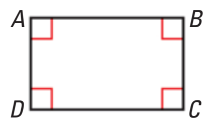


RECTANGLE COROLLARY

A quadrilateral is a rectangle if and only if it has four right angles.

ABCD is a rectangle if and only if $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

Proof: Ex. 58, p. 539

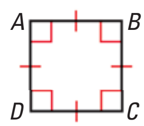


SQUARE COROLLARY

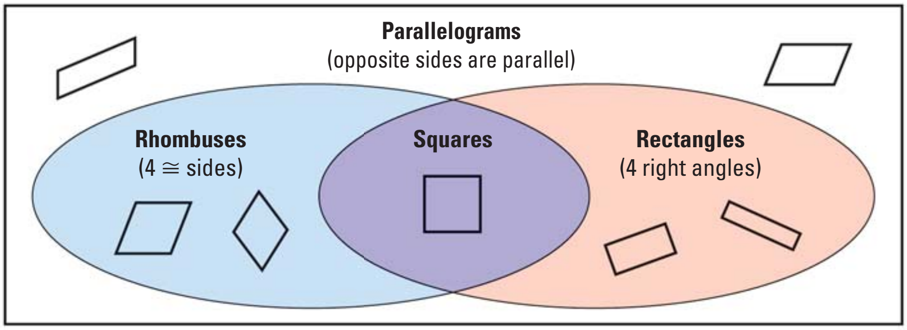
A quadrilateral is a square if and only if it is a rhombus and a rectangle.

ABCD is a square if and only if $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{AD}$ and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are right angles.

Proof: Ex. 59, p. 539



The *Venn diagram* below illustrates some important relationships among parallelograms, rhombuses, rectangles, and squares. For example, you can see that a square is a rhombus because it is a parallelogram with four congruent sides. Because it has four right angles, a square is also a rectangle.



EXAMPLE 1 Use properties of special quadrilaterals

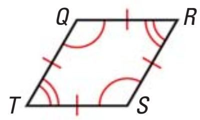
For any rhombus *QRST*, decide whether the statement is *always* or *sometimes* true. Draw a sketch and explain your reasoning.

a. $\angle Q \cong \angle S$

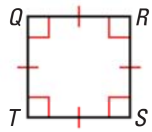
b. $\angle Q \cong \angle R$

Solution

a. By definition, a rhombus is a parallelogram with four congruent sides. By Theorem 8.4, opposite angles of a parallelogram are congruent. So, $\angle Q \cong \angle S$. The statement is *always* true.

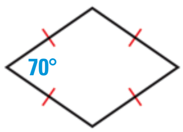


b. If rhombus *QRST* is a square, then all four angles are congruent right angles. So, $\angle Q \cong \angle R$ if *QRST* is a square. Because not all rhombuses are also squares, the statement is *sometimes* true.



EXAMPLE 2 Classify special quadrilaterals

Classify the special quadrilateral. Explain your reasoning.



Solution

The quadrilateral has four congruent sides. One of the angles is not a right angle, so the rhombus is not also a square. By the Rhombus Corollary, the quadrilateral is a rhombus.

✓ GUIDED PRACTICE for Examples 1 and 2

1. For any rectangle *EFGH*, is it *always* or *sometimes* true that $\overline{FG} \cong \overline{GH}$? *Explain* your reasoning.
2. A quadrilateral has four congruent sides and four congruent angles. Sketch the quadrilateral and classify it.

DIAGONALS The theorems below describe some properties of the diagonals of rhombuses and rectangles.

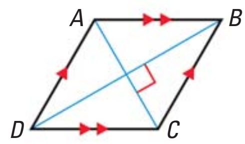
THEOREMS *For Your Notebook*

THEOREM 8.11

A parallelogram is a rhombus if and only if its diagonals are perpendicular.

▱*ABCD* is a rhombus if and only if $\overline{AC} \perp \overline{BD}$.

Proof: p. 536; Ex. 56, p. 539

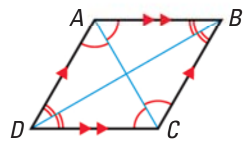


THEOREM 8.12

A parallelogram is a rhombus if and only if each diagonal bisects a pair of opposite angles.

▱*ABCD* is a rhombus if and only if $\overline{AC}$ bisects $\angle BCD$ and $\angle BAD$ and $\overline{BD}$ bisects $\angle ABC$ and $\angle ADC$.

Proof: Exs. 60–61, p. 539

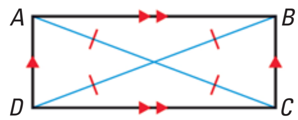


THEOREM 8.13

A parallelogram is a rectangle if and only if its diagonals are congruent.

▱*ABCD* is a rectangle if and only if $\overline{AC} \cong \overline{BD}$.

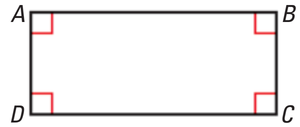
Proof: Exs. 63–64, p. 540



EXAMPLE 3 List properties of special parallelograms

Sketch rectangle *ABCD*. List everything that you know about it.

Solution



By definition, you need to draw a figure with the following properties:

- The figure is a parallelogram.
- The figure has four right angles.

Because *ABCD* is a parallelogram, it also has these properties:

- Opposite sides are parallel and congruent.
- Opposite angles are congruent. Consecutive angles are supplementary.
- Diagonals bisect each other.

By Theorem 8.13, the diagonals of *ABCD* are congruent.

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✓ GUIDED PRACTICE for Example 3

3. Sketch square *PQRS*. List everything you know about the square.

BICONDITIONALS Recall that biconditionals such as Theorem 8.11 can be rewritten as two parts. To prove a biconditional, you must prove both parts.

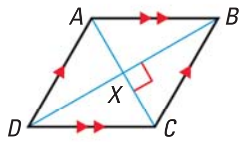
Conditional statement If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

Converse If a parallelogram is a rhombus, then its diagonals are perpendicular.

PROOF Part of Theorem 8.11

PROVE THEOREMS
You will prove the other part of Theorem 8.11 in Exercise 56 on page 539.

If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.



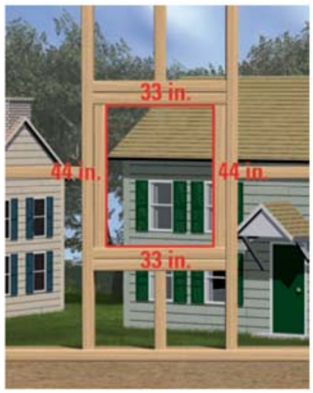
GIVEN ▶ $ABCD$ is a parallelogram; $\overline{AC} \perp \overline{BD}$

PROVE ▶ $ABCD$ is a rhombus.

Proof $ABCD$ is a parallelogram, so $\overline{AC}$ and $\overline{BD}$ bisect each other, and $\overline{BX} \cong \overline{DX}$. Also, $\angle BXC$ and $\angle CXD$ are congruent right angles, and $\overline{CX} \cong \overline{CX}$. So, $\triangle BXC \cong \triangle DXC$ by the SAS Congruence Postulate. Corresponding parts of congruent triangles are congruent, so $\overline{BC} \cong \overline{DC}$. Opposite sides of a $\square ABCD$ are congruent, so $\overline{AD} \cong \overline{BC} \cong \overline{DC} \cong \overline{AB}$. By definition, $ABCD$ is a rhombus.

EXAMPLE 4 Solve a real-world problem

CARPENTRY You are building a frame for a window. The window will be installed in the opening shown in the diagram.



a. The opening must be a rectangle. Given the measurements in the diagram, can you assume that it is? *Explain.*

b. You measure the diagonals of the opening. The diagonals are 54.8 inches and 55.3 inches. What can you conclude about the shape of the opening?

Solution

a. No, you cannot. The boards on opposite sides are the same length, so they form a parallelogram. But you do not know whether the angles are right angles.

b. By Theorem 8.13, the diagonals of a rectangle are congruent. The diagonals of the quadrilateral formed by the boards are not congruent, so the boards do not form a rectangle.

✓ GUIDED PRACTICE for Example 4

4. Suppose you measure only the diagonals of a window opening. If the diagonals have the same measure, can you conclude that the opening is a rectangle? *Explain.*

8.4 EXERCISES

HOMEWORK KEY

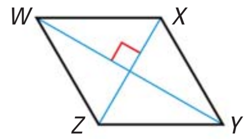
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 15, and 55

★ = **TAKS PRACTICE AND REASONING** Exs. 30, 31, 62, 65, and 66

SKILL PRACTICE

1. **VOCABULARY** What is another name for an equilateral rectangle?

2. **WRITING** Do you have enough information to identify the figure at the right as a rhombus? *Explain.*



EXAMPLES 1, 2, and 3 on pp. 534–535 for Exs. 3–25

RHOMBUSES For any rhombus *JKLM*, decide whether the statement is *always* or *sometimes* true. Draw a diagram and *explain* your reasoning.

3. $\angle L \cong \angle M$
4. $\angle K \cong \angle M$
5. $\overline{JK} \cong \overline{KL}$
6. $\overline{JM} \cong \overline{KL}$
7. $\overline{JL} \cong \overline{KM}$
8. $\angle JKM \cong \angle LKM$

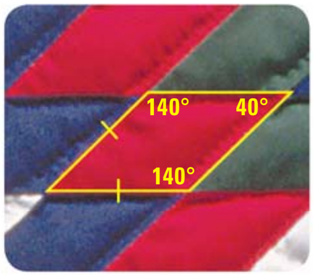
RECTANGLES For any rectangle *WXYZ*, decide whether the statement is *always* or *sometimes* true. Draw a diagram and *explain* your reasoning.

9. $\angle W \cong \angle X$
10. $\overline{WX} \cong \overline{YZ}$
11. $\overline{WX} \cong \overline{XY}$
12. $\overline{WY} \cong \overline{XZ}$
13. $\overline{WY} \perp \overline{XZ}$
14. $\angle WXZ \cong \angle YXZ$

CLASSIFYING Classify the quadrilateral. *Explain* your reasoning.

15. 

16. 

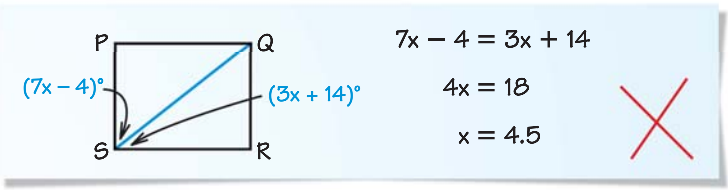
17. 

18. **USING PROPERTIES** Sketch rhombus *STUV*. *Describe* everything you know about the rhombus.

USING PROPERTIES Name each quadrilateral—*parallelogram, rectangle, rhombus,* and *square*—for which the statement is true.

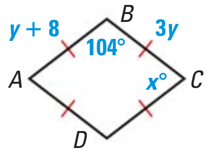
19. It is equiangular.
20. It is equiangular and equilateral.
21. Its diagonals are perpendicular.
22. Opposite sides are congruent.
23. The diagonals bisect each other.
24. The diagonals bisect opposite angles.

25. **ERROR ANALYSIS** Quadrilateral *PQRS* is a rectangle. *Describe* and correct the error made in finding the value of *x*.

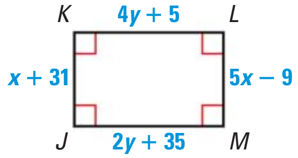


ALGEBRA **Classify the special quadrilateral. *Explain* your reasoning. Then find the values of x and y.**

26.



27.



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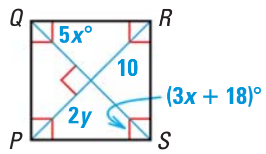
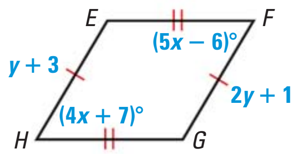


29.

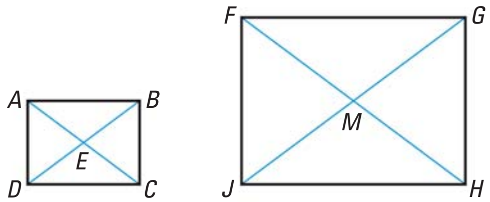


30. **TAKS REASONING** The diagonals of a rhombus are 6 inches and 8 inches. What is the perimeter of the rhombus? *Explain.*

31. **TAKS REASONING** Rectangle $ABCD$ is similar to rectangle $FGHJ$. If $AC = 5$, $CD = 4$, and $FM = 5$, what is HJ?

Ⓐ 4 Ⓑ 5

Ⓒ 8 Ⓓ 10

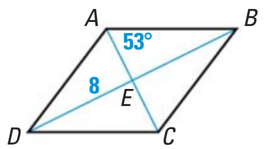


RHOMBUS **The diagonals of rhombus *ABCD* intersect at *E*. Given that $m\angle BAC = 53°$ and $DE = 8$, find the indicated measure.**

32. $m\angle DAC$ **33.** $m\angle AED$

34. $m\angle ADC$ **35.** DB

36. AE **37.** AC

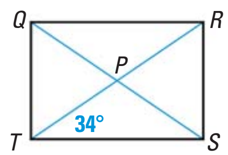


RECTANGLE **The diagonals of rectangle *QRST* intersect at *P*. Given that $m\angle PTS = 34°$ and $QS = 10$, find the indicated measure.**

38. $m\angle SRT$ **39.** $m\angle QPR$

40. QP **41.** RP

42. QR **43.** RS

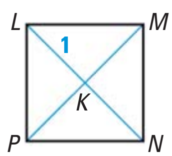


SQUARE **The diagonals of square *LMNP* intersect at *K*. Given that $LK = 1$, find the indicated measure.**

44. $m\angle MKN$ **45.** $m\angle LMK$

46. $m\angle LPK$ **47.** KN

48. MP **49.** LP

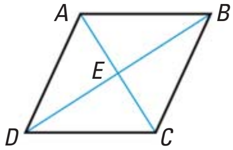


COORDINATE GEOMETRY **Use the given vertices to graph ▱*JKLM*. Classify ▱*JKLM* and *explain* your reasoning. Then find the perimeter of ▱*JKLM*.**

50. $J(-4, 2)$, $K(0, 3)$, $L(1, -1)$, $M(-3, -2)$ **51.** $J(-2, 7)$, $K(7, 2)$, $L(-2, -3)$, $M(-11, 2)$

52. **REASONING** Are all rhombuses similar? Are all squares similar? *Explain* your reasoning.

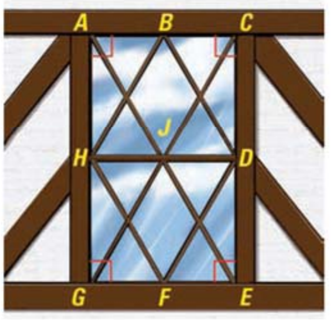
53. **CHALLENGE** Quadrilateral $ABCD$ shown at the right is a rhombus. Given that $AC = 10$ and $BD = 16$, find all side lengths and angle measures. *Explain* your reasoning.



PROBLEM SOLVING

EXAMPLE 2 on p. 534 for Ex. 54

54. **MULTI-STEP PROBLEM** In the window shown at the right, $\overline{BD} \cong \overline{DF} \cong \overline{BH} \cong \overline{HF}$. Also, $\angle HAB$, $\angle BCD$, $\angle DEF$, and $\angle FGH$ are right angles.



   a. Classify $HBDF$ and $ACEG$. *Explain* your reasoning.

   b. What can you conclude about the lengths of the diagonals $\overline{AE}$ and $\overline{GC}$? Given that these diagonals intersect at J, what can you conclude about the lengths of $\overline{AJ}$, $\overline{JE}$, $\overline{CJ}$, and $\overline{JG}$? *Explain*.

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EXAMPLE 4 on p. 536 for Ex. 55

55. **PATIO** You want to mark off a square region in your yard for a patio. You use a tape measure to mark off a quadrilateral on the ground. Each side of the quadrilateral is 2.5 meters long. *Explain* how you can use the tape measure to make sure that the quadrilateral you drew is a square.

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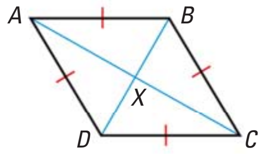
56. **PROVING THEOREM 8.11** Use the plan for proof below to write a paragraph proof for the converse statement of Theorem 8.11.



   GIVEN ▶ $ABCD$ is a rhombus.

   PROVE ▶ $\overline{AC} \perp \overline{BD}$

   Plan for Proof Because $ABCD$ is a parallelogram, its diagonals bisect each other at X. Show that $\triangle AXB \cong \triangle CXB$. Then show that $\overline{AC}$ and $\overline{BD}$ intersect to form congruent adjacent angles, $\angle AXB$ and $\angle CXB$.

PROVING COROLLARIES **Write the corollary as a conditional statement and its converse. Then *explain* why each statement is true.**

57. Rhombus Corollary

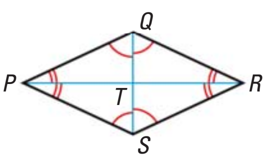
58. Rectangle Corollary

59. Square Corollary

PROVING THEOREM 8.12 **In Exercises 60 and 61, prove both parts of Theorem 8.12.**

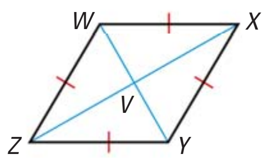
60. **GIVEN** ▶ $PQRS$ is a parallelogram.
$\overline{PR}$ bisects $\angle SPQ$ and $\angle QRS$.
$\overline{SQ}$ bisects $\angle PSR$ and $\angle RQP$.

   PROVE ▶ $PQRS$ is a rhombus.

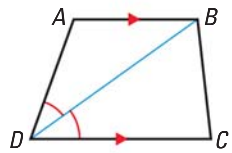


61. **GIVEN** ▶ $WXYZ$ is a rhombus.

   PROVE ▶ $\overline{WY}$ bisects $\angle ZWX$ and $\angle XYZ$.
$\overline{ZX}$ bisects $\angle WZY$ and $\angle YXW$.



62. **TAKS REASONING** In $ABCD$, $\overline{AB} \parallel \overline{CD}$, and $\overline{DB}$ bisects $\angle ADC$.



a. Show that $\angle ABD \cong \angle CDB$. What can you conclude about $\angle ADB$ and $\angle ABD$? What can you conclude about $\overline{AB}$ and $\overline{AD}$? *Explain*.

b. Suppose you also know that $\overline{AD} \cong \overline{BC}$. Classify $ABCD$. *Explain*.

63. **PROVING THEOREM 8.13** Write a coordinate proof of the following statement, which is part of Theorem 8.13.

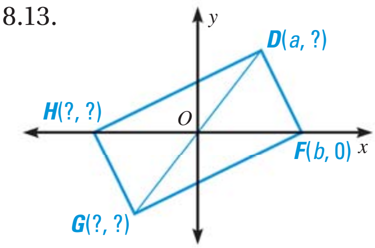
If a quadrilateral is a rectangle, then its diagonals are congruent.

64. **CHALLENGE** Write a coordinate proof of part of Theorem 8.13.

GIVEN ▶ $DFGH$ is a parallelogram, $\overline{DG} \cong \overline{HF}$

PROVE ▶ $DFGH$ is a rectangle.

Plan for Proof Write the coordinates of the vertices in terms of a and b. Find and compare the slopes of the sides.



TAKS PRACTICE at classzone.com

MIXED REVIEW FOR TAKS

REVIEW
Skills Review Handbook p. 894; TAKS Workbook

65. **TAKS PRACTICE** Katy and Laura go to a football game. Katy pays for the tickets, which cost \$7.40 each, and Laura pays for the food. Hot dogs are \$3.75 each and soft drinks are \$2.50 each. Laura buys a hot dog and a soft drink for each of them. How much money does Laura owe Katy so that they split the cost of the tickets and food equally? *TAKS Obj. 10*

Ⓐ \$1.15 Ⓑ \$2.30 Ⓒ \$4.60 Ⓓ Not here

REVIEW
Skills Review Handbook p. 882; TAKS Workbook

66. **TAKS PRACTICE** What is the effect on the graph of the equation $y = -3x^2$ when the equation is changed to $y = 3x^2$? *TAKS Obj. 5*

Ⓕ The graph of $y = 3x^2$ is translated 6 units down.

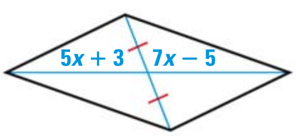
Ⓖ The graph of $y = 3x^2$ is translated 6 units up.

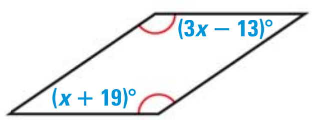
Ⓗ The graph of $y = 3x^2$ is a reflection of $y = -3x^2$ across the x-axis.

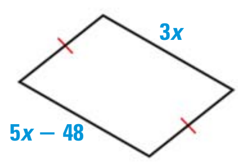
Ⓙ The graph of $y = 3x^2$ is a reflection of $y = -3x^2$ across the y-axis.

QUIZ *for Lessons 8.3–8.4*

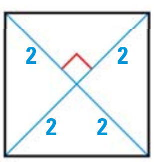
For what value of x is the quadrilateral a parallelogram? *(p. 522)*

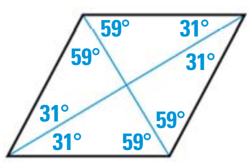
1. 

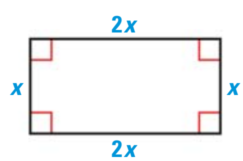
2. 

3. 

Classify the quadrilateral. *Explain* your reasoning. *(p. 533)*

4. 

5. 

6. 

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Keystrokes

8.5 Midsegment of a Trapezoid

MATERIALS • graphing calculator or computer

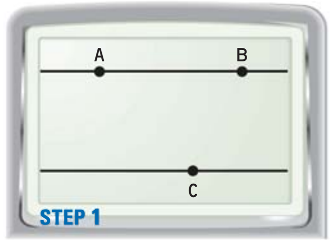
TEKS a.5, G.2.A, G.7.C, G.9.A

QUESTION What are the properties of the midsegment of a trapezoid?

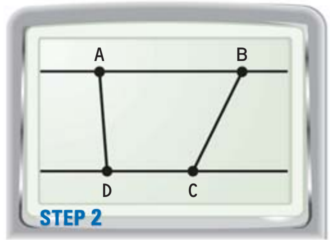
You can use geometry drawing software to investigate properties of trapezoids.

EXPLORE Draw a trapezoid and its midsegment

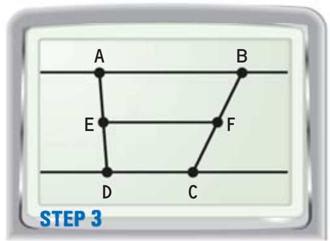
STEP 1 ***Draw parallel lines*** Draw $\overleftrightarrow{AB}$. Draw a point C not on $\overleftrightarrow{AB}$ and construct a line parallel to $\overleftrightarrow{AB}$ through point C.



STEP 2 ***Draw trapezoid*** Construct a point D on the same line as point C. Then draw $\overline{AD}$ and $\overline{BC}$ so that the segments are not parallel. Draw $\overline{AB}$ and $\overline{DC}$. Quadrilateral $ABCD$ is called a *trapezoid*. A trapezoid is a quadrilateral with exactly one pair of parallel sides.



STEP 3 ***Draw midsegment*** Construct the midpoints of $\overline{AD}$ and $\overline{BC}$. Label the points E and F. Draw $\overline{EF}$. $\overline{EF}$ is called a *midsegment* of trapezoid $ABCD$. The midsegment of a trapezoid connects the midpoints of its nonparallel sides.



STEP 4 ***Measure lengths*** Measure AB, DC, and EF.

STEP 5 ***Compare lengths*** The average of AB and DC is $\frac{AB + DC}{2}$. Calculate and compare this average to EF. What do you notice? Drag point A or point B to change the shape of trapezoid $ABCD$. Do not allow $\overline{AD}$ to intersect $\overline{BC}$. What do you notice about EF and $\frac{AB + DC}{2}$?

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Make a conjecture about the length of the midsegment of a trapezoid.
2. The midsegment of a trapezoid is parallel to the two parallel sides of the trapezoid. What measurements could you make to show that the midsegment in the *Explore* is parallel to $\overline{AB}$ and $\overline{CD}$? *Explain.*
3. In Lesson 5.1 (page 295), you learned a theorem about the midsegment of a triangle. How is the midsegment of a trapezoid similar to the midsegment of a triangle? How is it different?

8.5 Use Properties of Trapezoids and Kites

TEKS G.3.E, G.5.A, G.7.A, G.7.C

Before You used properties of special parallelograms.

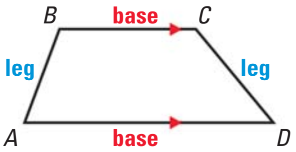
Now You will use properties of trapezoids and kites.

Why? So you can measure part of a building, as in Example 2.

Key Vocabulary
- **trapezoid**
  bases, base angles, legs
- **isosceles trapezoid**
- **midsegment of a trapezoid**
- **kite**

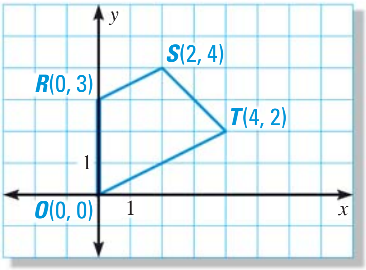
A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are the **bases**.

A trapezoid has two pairs of **base angles**. For example, in trapezoid *ABCD*, $\angle A$ and $\angle D$ are one pair of base angles, and $\angle B$ and $\angle C$ are the second pair. The nonparallel sides are the **legs** of the trapezoid.



EXAMPLE 1 Use a coordinate plane

Show that *ORST* is a trapezoid.



Solution

Compare the slopes of opposite sides.

Slope of $\overline{RS} = \frac{4-3}{2-0} = \frac{1}{2}$

Slope of $\overline{OT} = \frac{2-0}{4-0} = \frac{2}{4} = \frac{1}{2}$

The slopes of $\overline{RS}$ and $\overline{OT}$ are the same, so $\overline{RS} \parallel \overline{OT}$.

Slope of $\overline{ST} = \frac{2-4}{4-2} = \frac{-2}{2} = -1$

Slope of $\overline{OR} = \frac{3-0}{0-0} = \frac{3}{0}$, which is undefined.

The slopes of $\overline{ST}$ and $\overline{OR}$ are not the same, so $\overline{ST}$ is not parallel to $\overline{OR}$.

▶ Because quadrilateral *ORST* has exactly one pair of parallel sides, it is a trapezoid.

✓ GUIDED PRACTICE for Example 1

1. **WHAT IF?** In Example 1, suppose the coordinates of point *S* are (4, 5). What type of quadrilateral is *ORST*? *Explain.*

2. In Example 1, which of the interior angles of quadrilateral *ORST* are supplementary angles? *Explain* your reasoning.

ISOSCELES TRAPEZOIDS If the legs of a trapezoid are congruent, then the trapezoid is an **isosceles trapezoid**.


isosceles trapezoid

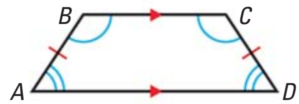
THEOREMS *For Your Notebook*

THEOREM 8.14

If a trapezoid is isosceles, then each pair of base angles is congruent.

If trapezoid $ABCD$ is isosceles, then $\angle A \cong \angle D$ and $\angle B \cong \angle C$.

Proof: Ex. 37, p. 548

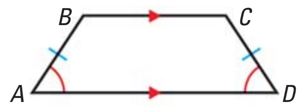


THEOREM 8.15

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

If $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$), then trapezoid $ABCD$ is isosceles.

Proof: Ex. 38, p. 548

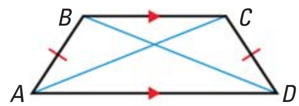


THEOREM 8.16

A trapezoid is isosceles if and only if its diagonals are congruent.

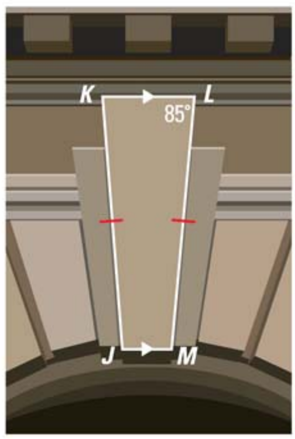
Trapezoid $ABCD$ is isosceles if and only if $\overline{AC} \cong \overline{BD}$.

Proof: Exs. 39 and 43, p. 549



EXAMPLE 2 Use properties of isosceles trapezoids

ARCH The stone above the arch in the diagram is an isosceles trapezoid. Find $m\angle K$, $m\angle M$, and $m\angle J$.



Solution

STEP 1 **Find** $m\angle K$. $JKLM$ is an isosceles trapezoid, so $\angle K$ and $\angle L$ are congruent base angles, and $m\angle K = m\angle L = 85°$.

STEP 2 **Find** $m\angle M$. Because $\angle L$ and $\angle M$ are consecutive interior angles formed by $\overleftrightarrow{LM}$ intersecting two parallel lines, they are supplementary. So, $m\angle M = 180° - 85° = 95°$.

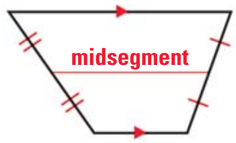
STEP 3 **Find** $m\angle J$. Because $\angle J$ and $\angle M$ are a pair of base angles, they are congruent, and $m\angle J = m\angle M = 95°$.

▶ So, $m\angle J = 95°$, $m\angle K = 85°$, and $m\angle M = 95°$.

READ VOCABULARY

The midsegment of a trapezoid is sometimes called the *median* of the trapezoid.

MIDSEGMENTS Recall that a midsegment of a triangle is a segment that connects the midpoints of two sides of the triangle. The **midsegment of a trapezoid** is the segment that connects the midpoints of its legs.



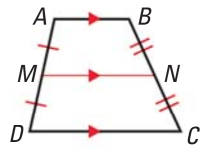
The theorem below is similar to the Midsegment Theorem for Triangles.

THEOREM *For Your Notebook*

THEOREM 8.17 Midsegment Theorem for Trapezoids

The midsegment of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases.

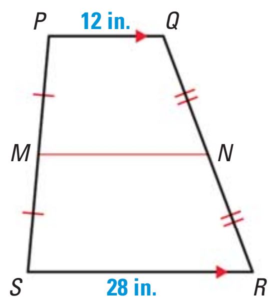
If $\overline{MN}$ is the midsegment of trapezoid $ABCD$, then $\overline{MN} \parallel \overline{AB}$, $\overline{MN} \parallel \overline{DC}$, and $MN = \frac{1}{2}(AB + CD)$.



Justification: Ex. 40, p. 549
Proof: p. 937

EXAMPLE 3 Use the midsegment of a trapezoid

In the diagram, $\overline{MN}$ is the midsegment of trapezoid $PQRS$. Find MN.



Solution

Use Theorem 8.17 to find MN.

$MN = \frac{1}{2}(PQ + SR)$ **Apply Theorem 8.17.**

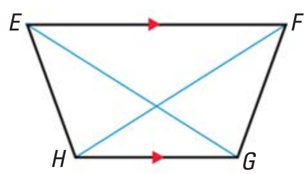
$= \frac{1}{2}(12 + 28)$ **Substitute 12 for *PQ* and 28 for *XU*.**

$= 20$ **Simplify.**

▶ The length MN is 20 inches.

✓ GUIDED PRACTICE for Examples 2 and 3

In Exercises 3 and 4, use the diagram of trapezoid *EFGH*.

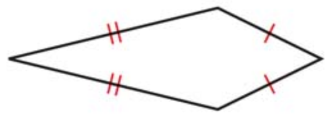


3. If $EG = FH$, is trapezoid $EFGH$ isosceles? *Explain.*

4. If $m\angle HEF = 70°$ and $m\angle FGH = 110°$, is trapezoid $EFGH$ isosceles? *Explain.*

5. In trapezoid $JKLM$, $\angle J$ and $\angle M$ are right angles, and $JK = 9$ cm. The length of the midsegment $\overline{NP}$ of trapezoid $JKLM$ is 12 cm. Sketch trapezoid $JKLM$ and its midsegment. Find ML. *Explain* your reasoning.

KITES A **kite** is a quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are not congruent.



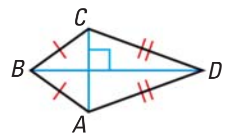
THEOREMS *For Your Notebook*

THEOREM 8.18

If a quadrilateral is a kite, then its diagonals are perpendicular.

If quadrilateral $ABCD$ is a kite, then $\overline{AC} \perp \overline{BD}$.

Proof: Ex. 41, p. 549

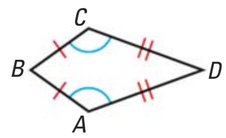


THEOREM 8.19

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

If quadrilateral $ABCD$ is a kite and $\overline{BC} \cong \overline{BA}$, then $\angle A \cong \angle C$ and $\angle B \not\cong \angle D$.

Proof: Ex. 42, p. 549



EXAMPLE 4 Apply Theorem 8.19

Find $m\angle D$ in the kite shown at the right.

Solution

By Theorem 8.19, $DEFG$ has exactly one pair of congruent opposite angles. Because $\angle E \not\cong \angle G$, $\angle D$ and $\angle F$ must be congruent. So, $m\angle D = m\angle F$. Write and solve an equation to find $m\angle D$.

$m\angle D + m\angle F + 124° + 80° = 360°$ — **Corollary to Theorem 8.1**

$m\angle D + m\angle D + 124° + 80° = 360°$ — **Substitute $m\angle D$ for $m\angle F$.**

$2(m\angle D) + 204° = 360°$ — **Combine like terms.**

$m\angle D = 78°$ — **Solve for $m\angle D$.**

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✓ GUIDED PRACTICE for Example 4

6. In a kite, the measures of the angles are $3x°$, $75°$, $90°$, and $120°$. Find the value of x. What are the measures of the angles that are congruent?

8.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 21, and 35

★ = **TAKS PRACTICE AND REASONING** Exs. 16, 28, 31, 36, 44, and 45

SKILL PRACTICE

1. **VOCABULARY** In trapezoid $PQRS$, $\overline{PQ} \parallel \overline{RS}$. Sketch $PQRS$ and identify its bases and its legs.

2. **WRITING** *Describe* the differences between a kite and a trapezoid.

EXAMPLES 1 and 2 on pp. 542–543 for Exs. 3–12

COORDINATE PLANE Points A, B, C, and D are the vertices of a quadrilateral. Determine whether $ABCD$ is a trapezoid.

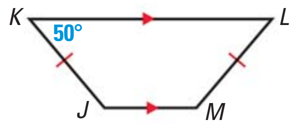
3. $A(0, 4)$, $B(4, 4)$, $C(8, -2)$, $D(2, 1)$

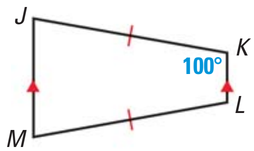
4. $A(-5, 0)$, $B(2, 3)$, $C(3, 1)$, $D(-2, -2)$

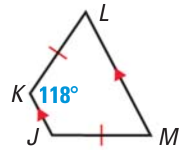
5. $A(2, 1)$, $B(6, 1)$, $C(3, -3)$, $D(-1, -4)$

6. $A(-3, 3)$, $B(-1, 1)$, $C(1, -4)$, $D(-3, 0)$

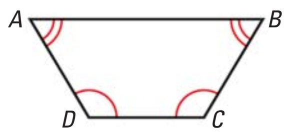
FINDING ANGLE MEASURES Find $m\angle J$, $m\angle L$, and $m\angle M$.

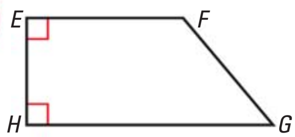
7. 

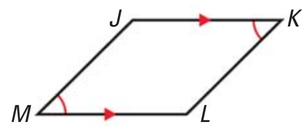
8. 

9. 

REASONING Determine whether the quadrilateral is a trapezoid. *Explain*.

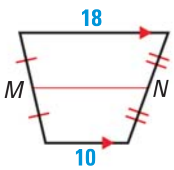
10. 

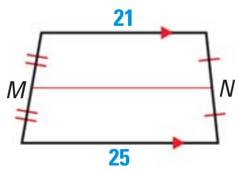
11. 

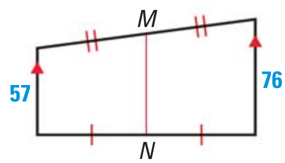
12. 

EXAMPLE 3 on p. 544 for Exs. 13–16

FINDING MIDSEGMENTS Find the length of the midsegment of the trapezoid.

13. 

14. 

15. 

16. ★ **TAKS REASONING** Which statement is not always true?

Ⓐ The base angles of an isosceles trapezoid are congruent.

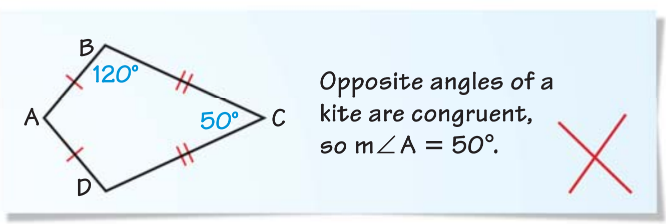
Ⓑ The midsegment of a trapezoid is parallel to the bases.

Ⓒ The bases of a trapezoid are parallel.

Ⓓ The legs of a trapezoid are congruent.

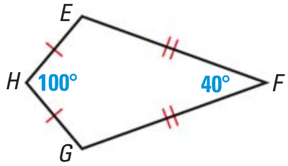
EXAMPLE 4 on p. 545 for Exs. 17–20

17. **ERROR ANALYSIS** *Describe* and correct the error made in finding $m\angle A$.

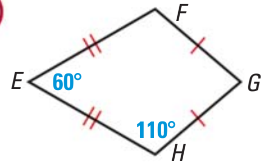


ANGLES OF KITES ***EFGH*** **is a kite. Find** $m\angle G$**.**

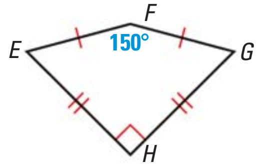
18.



19.

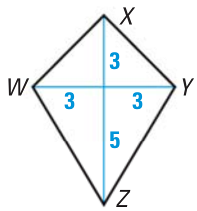


20.

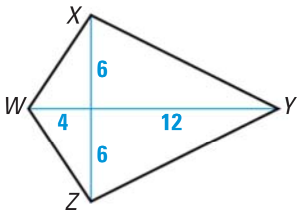


DIAGONALS OF KITES **Use Theorem 8.18 and the Pythagorean Theorem to find the side lengths of the kite. Write the lengths in simplest radical form.**

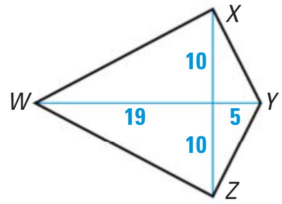
21.



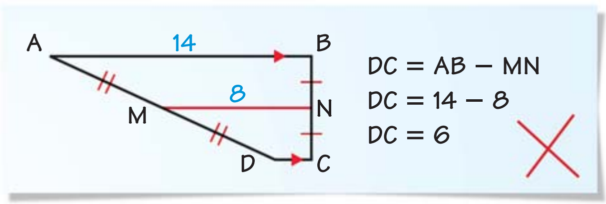
22.



23.

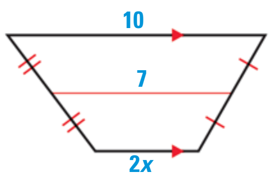


24. ERROR ANALYSIS In trapezoid *ABCD*, $\overline{MN}$ is the midsegment. *Describe* and correct the error made in finding *DC*.

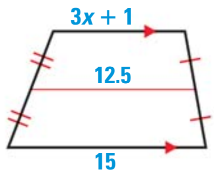


ALGEBRA **Find the value of** x**.**

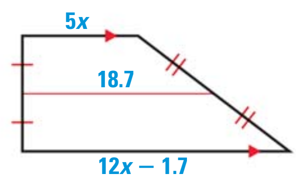
25.



26.



27.



28. TAKS REASONING The points $M(-3, 5)$, $N(-1, 5)$, $P(3, -1)$, and $Q(-5, -1)$ form the vertices of a trapezoid. Draw *MNPQ* and find *MP* and *NQ*. What do your results tell you about the trapezoid? *Explain*.

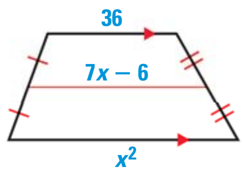
29. DRAWING In trapezoid *JKLM*, $\overline{JK} \parallel \overline{LM}$ and $JK = 17$. The midsegment of *JKLM* is $\overline{XY}$, and $XY = 37$. Sketch *JKLM* and its midsegment. Then find *LM*.

30. RATIOS The ratio of the lengths of the bases of a trapezoid is 1 : 3. The length of the midsegment is 24. Find the lengths of the bases.

31. TAKS REASONING In trapezoid *PQRS*, $\overline{PQ} \parallel \overline{RS}$ and $\overline{MN}$ is the midsegment of *PQRS*. If $RS = 5 \cdot PQ$, what is the ratio of *MN* to *RS*?

Ⓐ 3 : 5 Ⓑ 5 : 3 Ⓒ 2 : 1 Ⓓ 3 : 1

32. CHALLENGE The figure shown at the right is a trapezoid with its midsegment. Find all the possible values of x. What is the length of the midsegment? *Explain*. (The figure may not be drawn to scale.)

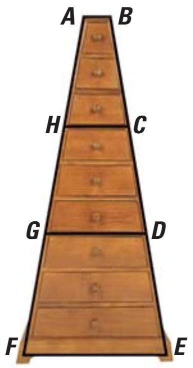


33. REASONING *Explain* why a kite and a general quadrilateral are the only quadrilaterals that can be concave.

PROBLEM SOLVING

EXAMPLES 3 and 4 on pp. 544–545 for Exs. 34–35

34. FURNITURE In the photograph of a chest of drawers, $\overline{HC}$ is the midsegment of trapezoid $ABDG$, $\overline{GD}$ is the midsegment of trapezoid $HCEF$, $AB = 13.9$ centimeters, and $GD = 50.5$ centimeters. Find HC. Then find FE.



TEXAS @HomeTutor for problem solving help at classzone.com

35. GRAPHIC DESIGN You design a logo in the shape of a convex kite. The measure of one angle of the kite is 90°. The measure of another angle is 30°. Sketch a kite that matches this description. Give the measures of all the angles and mark any congruent sides.

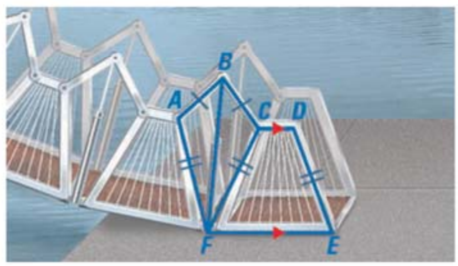
TEXAS @HomeTutor for problem solving help at classzone.com

36. **TAKS REASONING** The bridge below is designed to fold up into an octagon shape. The diagram shows a section of the bridge.

a. Classify the quadrilaterals shown in the diagram.

b. As the bridge folds up, what happens to the length of $\overline{BF}$? What happens to $m\angle BAF$, $m\angle ABC$, $m\angle BCF$, and $m\angle CFA$?

c. Given $m\angle CFE = 65°$, find $m\angle DEF$, $m\angle FCD$, and $m\angle CDE$. *Explain.*

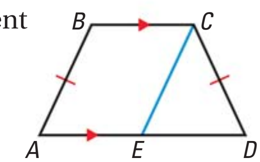


37. PROVING THEOREM 8.14 Use the diagram and the auxiliary segment to prove Theorem 8.14. In the diagram, $\overline{EC}$ is drawn parallel to $\overline{AB}$.

GIVEN ▶ $ABCD$ is an isosceles trapezoid, $\overline{BC} \parallel \overline{AD}$

PROVE ▶ $\angle A \cong \angle D$, $\angle B \cong \angle BCD$

Hint: Find a way to show that $\triangle ECD$ is an isosceles triangle.

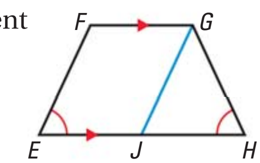


38. PROVING THEOREM 8.15 Use the diagram and the auxiliary segment to prove Theorem 8.15. In the diagram, $\overline{JG}$ is drawn parallel to $\overline{EF}$.

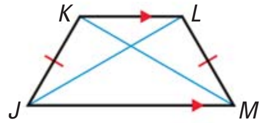
GIVEN ▶ $EFGH$ is a trapezoid, $\overline{FG} \parallel \overline{EH}$, $\angle E \cong \angle H$

PROVE ▶ $EFGH$ is an isosceles trapezoid.

Hint: Find a way to show that $\triangle JGH$ is an isosceles triangle.



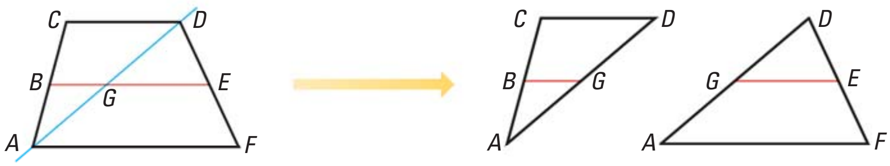
39. **PROVING THEOREM 8.16** Prove part of Theorem 8.16.



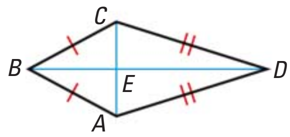
GIVEN ▶ *JKLM* is an isosceles trapezoid.
$\overline{KL} \parallel \overline{JM}$, $\overline{JK} \cong \overline{LM}$

PROVE ▶ $\overline{JL} \cong \overline{KM}$

40. **REASONING** In the diagram below, $\overline{BG}$ is the midsegment of $\triangle ACD$ and $\overline{GE}$ is the midsegment of $\triangle ADF$. *Explain* why the midsegment of trapezoid *ACDF* is parallel to each base and why its length is one half the sum of the lengths of the bases.



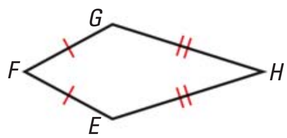
41. **PROVING THEOREM 8.18** Prove Theorem 8.18.



GIVEN ▶ *ABCD* is a kite.
$\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$

PROVE ▶ $\overline{AC} \perp \overline{BD}$

42. **PROVING THEOREM 8.19** Write a paragraph proof of Theorem 8.19.



GIVEN ▶ *EFGH* is a kite.
$\overline{EF} \cong \overline{GF}$, $\overline{EH} \cong \overline{GH}$

PROVE ▶ $\angle E \cong \angle G$, $\angle F \ncong \angle H$

Plan for Proof First show that $\angle E \cong \angle G$. Then use an indirect argument to show that $\angle F \ncong \angle H$: If $\angle F \cong \angle H$, then *EFGH* is a parallelogram. But opposite sides of a parallelogram are congruent. This result contradicts the definition of a kite.

43. **CHALLENGE** In Exercise 39, you proved that part of Theorem 8.16 is true. Write the other part of Theorem 8.16 as a conditional statement. Then prove that the statement is true.

TAKS PRACTICE at classzone.com

MIXED REVIEW FOR TAKS

REVIEW
Skills Review Handbook p. 873; TAKS Workbook

44. **TAKS PRACTICE** Simplify the algebraic expression $3(x + 1)(x + 3) - 2(x^2 + 5x + 2)$. *TAKS Obj. 2*

Ⓐ $x^2 + 5$ Ⓑ $x^2 - 6x - 1$ Ⓒ $x^2 + 2x + 5$ Ⓓ $x^2 + 22x + 5$

REVIEW
Lesson 5.5; TAKS Workbook

45. **TAKS PRACTICE** A triangular lawn with sides lengths of 15 feet, 16 feet, and 22 feet is used for a flower bed. A rose bush will be planted in the corner with the largest angle. Where will it be planted? *TAKS Obj. 7*

Ⓕ In the corner opposite the side that is 15 feet.

Ⓖ In the corner opposite the side that is 16 feet.

Ⓗ In the corner opposite the side that is 22 feet.

Ⓙ Along the longest side.

Extension

Use after Lesson 8.5

Draw Three-Dimensional Figures

TEKS a.2, a.3, G.6.C, G.9.D

GOAL Create isometric drawings and orthographic projections of three-dimensional figures.

Key Vocabulary
- **isometric drawing**
- **orthographic projection**

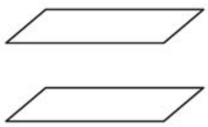
Technical drawings are drawings that show different viewpoints of an object. Engineers and architects create technical drawings of products and buildings before actually constructing the actual objects.

EXAMPLE 1 Draw a rectangular box

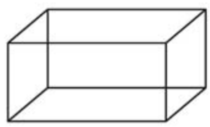
Draw a rectangular box.

Solution

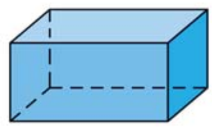
STEP 1 **Draw** the bases. They are rectangular, but you need to draw them tilted.



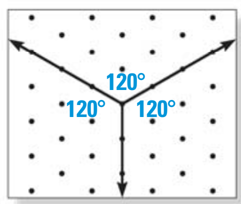
STEP 2 **Connect** the bases using vertical lines.



STEP 3 **Erase** parts of the hidden edges so that they are dashed lines.



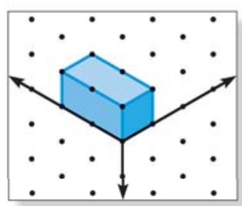
ISOMETRIC DRAWINGS Technical drawings may include **isometric drawings**. These drawings look three-dimensional and can be created on a grid of dots using three axes that intersect to form 120° angles.



EXAMPLE 2 Create an isometric drawing

Create an isometric drawing of the rectangular box in Example 1.

Solution



STEP 1 **Draw** three axes on isometric dot paper.

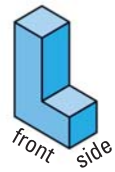
STEP 2 **Draw** the box so that the edges of the box are parallel to the three axes.

STEP 3 **Add** depth to the drawing by using different shading for the front, top, and sides.

ANOTHER VIEW Technical drawings may also include an *orthographic projection*. An **orthographic projection** is a two-dimensional drawing of the front, top, and side views of an object. The interior lines in these two-dimensional drawings represent edges of the object.

EXAMPLE 3 Create an orthographic projection

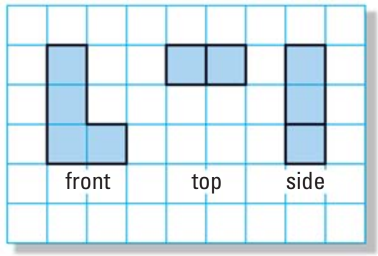
Create an orthographic projection of the solid.



VISUAL REASONING
In this Extension, you can think of the solids as being constructed from cubes. You can assume there are no cubes hidden from view except those needed to support the visible ones.

Solution

On graph paper, draw the front, top, and side views of the solid.



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PRACTICE

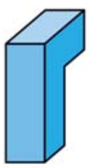
EXAMPLE 1
on p. 550
for Exs. 1–3

EXAMPLES 2 and 3
on pp. 550–551
for Exs. 4–12

DRAWING BOXES **Draw a box with the indicated base.**

1. Equilateral triangle **2.** Regular hexagon **3.** Square

DRAWING SOLIDS **Create an isometric drawing of the solid. Then create an orthographic projection of the solid.**

4.


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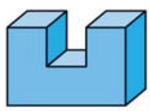
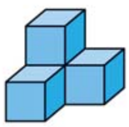
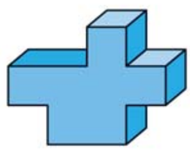


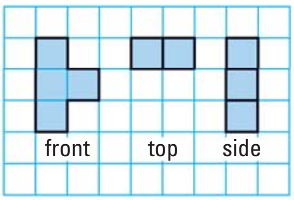
6.

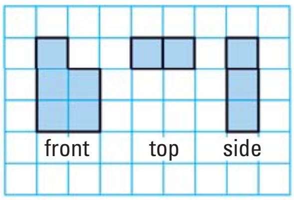

7.


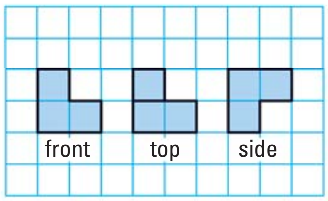
8.


9.


CREATING ISOMETRIC DRAWINGS **Create an isometric drawing of the orthographic projection.**

10.


11.


12.


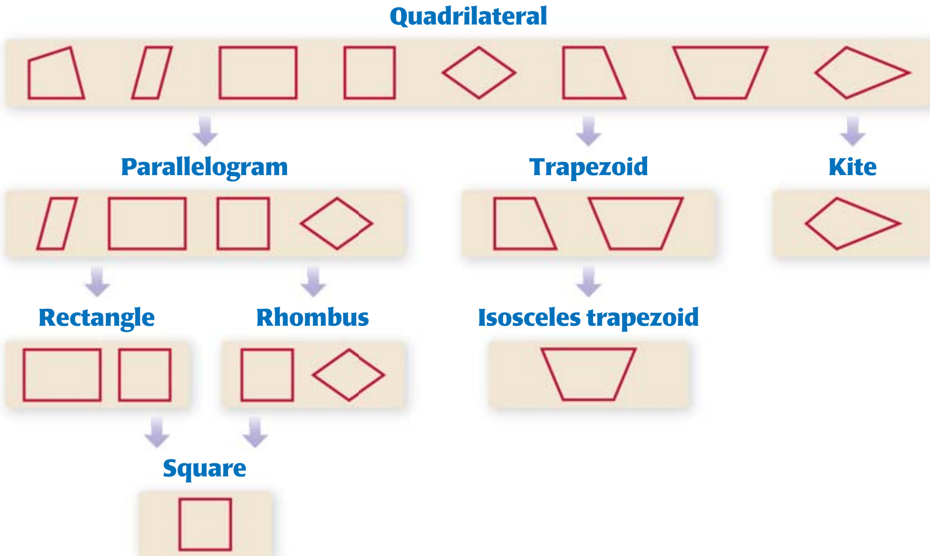
8.6 Identify Special Quadrilaterals

TEKS G.1.A, G.3.C, G.5.A, G.9.B

Before You identified polygons.
Now You will identify special quadrilaterals.
Why? So you can describe part of a pyramid, as in Ex. 36.

Key Vocabulary
- **parallelogram,** *p. 515*
- **rhombus,** *p. 533*
- **rectangle,** *p. 533*
- **square,** *p. 533*
- **trapezoid,** *p. 542*
- **kite,** *p. 545*

The diagram below shows relationships among the special quadrilaterals you have studied in Chapter 8. Each shape in the diagram has the properties of the shapes linked above it. For example, a rhombus has the properties of a parallelogram and a quadrilateral.

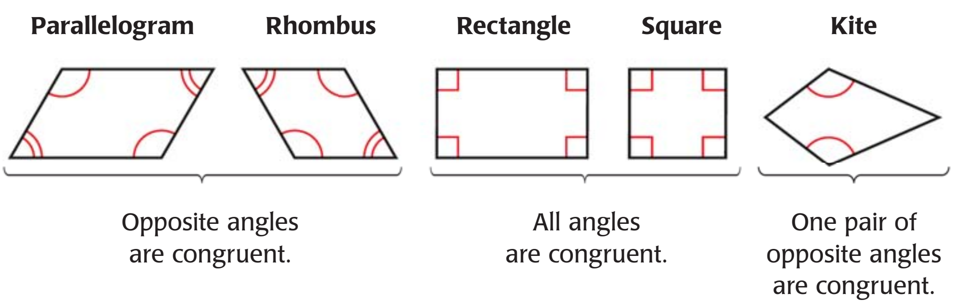


EXAMPLE 1 Identify quadrilaterals

Quadrilateral *ABCD* has at least one pair of opposite angles congruent. What types of quadrilaterals meet this condition?

Solution

There are many possibilities.



EXAMPLE 2 TAKS PRACTICE: Multiple Choice

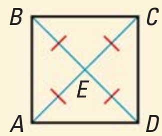
AVOID ERRORS

In Example 2, *ABCD* is shaped like a square. But you must rely only on marked information when you interpret a diagram.

What is the most specific name for quadrilateral *ABCD*?

Ⓐ Parallelogram Ⓑ Rhombus

Ⓒ Square Ⓓ Rectangle



Solution

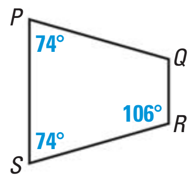
The diagram shows $\overline{AE} \cong \overline{CE}$ and $\overline{BE} \cong \overline{DE}$. So, the diagonals bisect each other. By Theorem 8.10, *ABCD* is a parallelogram. Also, the diagonals are congruent. So, by Theorem 8.13, the parallelogram is a rectangle.

A square is also a rectangle. However, there is no information given about the side lengths of *ABCD*. So, you cannot determine whether it is a square.

▶ The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

EXAMPLE 3 Identify a quadrilateral

Is enough information given in the diagram to show that quadrilateral *PQRS* is an isosceles trapezoid? Explain.



Solution

STEP 1 **Show** that *PQRS* is a trapezoid. ∠*R* and ∠*S* are supplementary, but ∠*P* and ∠*S* are not. So, $\overline{PS} \parallel \overline{QR}$, but $\overline{PQ}$ is not parallel to $\overline{SR}$. By definition, *PQRS* is a trapezoid.

STEP 2 **Show** that trapezoid *PQRS* is isosceles. ∠*P* and ∠*S* are a pair of congruent base angles. So, *PQRS* is an isosceles trapezoid by Theorem 8.15.

▶ Yes, the diagram is sufficient to show that *PQRS* is an isosceles trapezoid.

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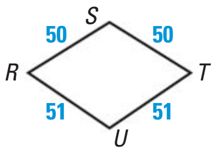
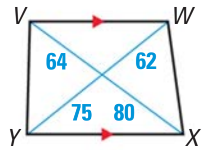
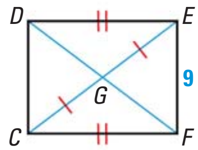
✓ GUIDED PRACTICE for Examples 1, 2, and 3

1. Quadrilateral *DEFG* has at least one pair of opposite sides congruent. What types of quadrilaterals meet this condition?

Give the most specific name for the quadrilateral. *Explain* your reasoning.

2. 

3. 

4. 

5. **ERROR ANALYSIS** A student knows the following information about quadrilateral *MNPQ*: $\overline{MN} \parallel \overline{PQ}$, $\overline{MP} \cong \overline{NQ}$, and ∠*P* ≅ ∠*Q*. The student concludes that *MNPQ* is an isosceles trapezoid. *Explain* why the student cannot make this conclusion.

8.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 21, and 33

★ = **TAKS PRACTICE AND REASONING** Exs. 13, 37, 38, 43, and 44

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A quadrilateral that has exactly one pair of parallel sides and diagonals that are congruent is a(n) __?__.

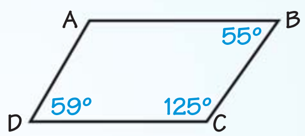
2. **WRITING** *Describe* three methods you could use to prove that a parallelogram is a rhombus.

EXAMPLE 1 on p. 552 for Exs. 3–12

PROPERTIES OF QUADRILATERALS **Copy the chart. Put an X in the box if the shape *always* has the given property.**

	Property	▱	Rectangle	Rhombus	Square	Kite	Trapezoid
3.	All sides are ≅.	?	?	?	?	?	?
4.	Both pairs of opp. sides are ≅.	?	?	?	?	?	?
5.	Both pairs of opp. sides are ∥.	?	?	?	?	?	?
6.	Exactly 1 pair of opp. sides are ∥.	?	?	?	?	?	?
7.	All ∠s are ≅.	?	?	?	?	?	?
8.	Exactly 1 pair of opp. ∠s are ≅.	?	?	?	?	?	?
9.	Diagonals are ⊥.	?	?	?	?	?	?
10.	Diagonals are ≅.	?	?	?	?	?	?
11.	Diagonals bisect each other.	?	?	?	?	?	?

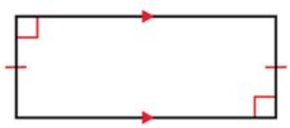
12. **ERROR ANALYSIS** *Describe* and correct the error in classifying the quadrilateral.



∠B and ∠C are supplements, so $\overline{AB} \parallel \overline{CD}$. So, ABCD is a parallelogram.

EXAMPLE 2 on p. 553 for Exs. 13–17

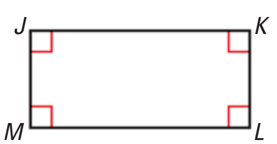
13. ★ **TAKS REASONING** What is the most specific name for the quadrilateral shown at the right?

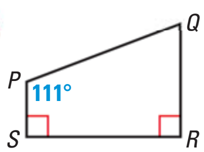


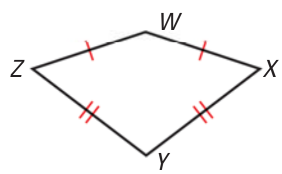
Ⓐ Rectangle Ⓑ Parallelogram

Ⓒ Trapezoid Ⓓ Isosceles trapezoid

CLASSIFYING QUADRILATERALS **Give the most specific name for the quadrilateral. *Explain.***

14. 

15. 

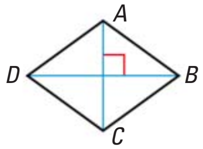
16. 

17. DRAWING Draw a quadrilateral with congruent diagonals and exactly one pair of congruent sides. What is the most specific name for this quadrilateral?

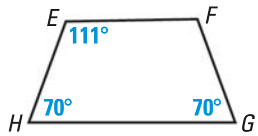
EXAMPLE 3
on p. 553
for Exs. 18–20

IDENTIFYING QUADRILATERALS Tell whether enough information is given in the diagram to classify the quadrilateral by the indicated name. *Explain.*

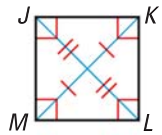
18. Rhombus



19. Isosceles trapezoid



20. Square



COORDINATE PLANE Points *P, Q, R,* and *S* are the vertices of a quadrilateral. Give the most specific name for *PQRS*. *Justify* your answer.

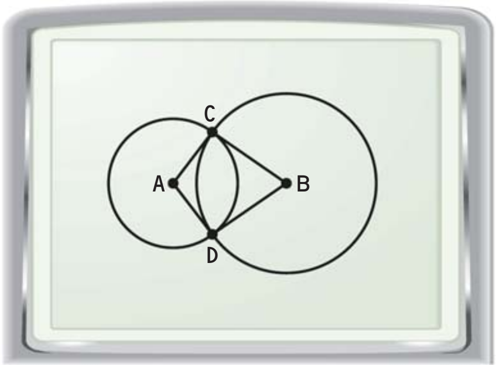
21. $P(1, 0)$, $Q(1, 2)$, $R(6, 5)$, $S(3, 0)$

22. $P(2, 1)$, $Q(6, 1)$, $R(5, 8)$, $S(3, 8)$

23. $P(2, 7)$, $Q(6, 9)$, $R(9, 3)$, $S(5, 1)$

24. $P(1, 7)$, $Q(5, 8)$, $R(6, 2)$, $S(2, 1)$

25. TECHNOLOGY Use geometry drawing software to draw points A, B, C, and segments AC and BC. Draw a circle with center A and radius AC. Draw a circle with center B and radius BC. Label the other intersection of the circles D. Draw $\overline{BD}$ and $\overline{AD}$.

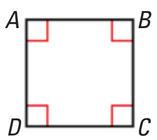


a. Drag point A, B, C, or D to change the shape of $ABCD$. What types of quadrilaterals can be formed?

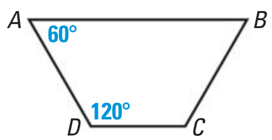
b. Are there types of quadrilaterals that cannot be formed? *Explain.*

DEVELOPING PROOF Which pairs of segments or angles must be congruent so that you can prove that *ABCD* is the indicated quadrilateral? *Explain.* There may be more than one right answer.

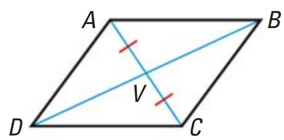
26. Square



27. Isosceles trapezoid



28. Parallelogram

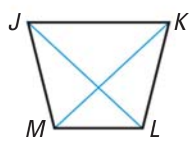


TRAPEZOIDS In Exercises 29–31, determine whether there is enough information to prove that *JKLM* is an isosceles trapezoid. *Explain.*

29. GIVEN ▶ $\overline{JK} \parallel \overline{LM}$, $\angle JKL \cong \angle KJM$

30. GIVEN ▶ $\overline{JK} \parallel \overline{LM}$, $\angle JML \cong \angle KLM$, $m\angle KLM \neq 90°$

31. GIVEN ▶ $\overline{JL} \cong \overline{KM}$, $\overline{JK} \parallel \overline{LM}$, $JK > LM$

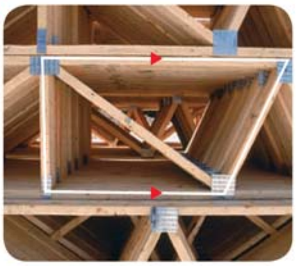


32. CHALLENGE Draw a rectangle and bisect its angles. What type of quadrilateral is formed by the intersecting bisectors? *Justify* your answer.

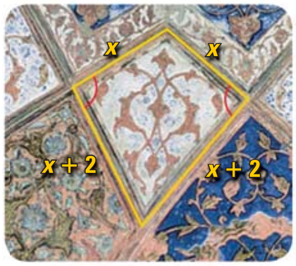
PROBLEM SOLVING

REAL-WORLD OBJECTS What type of special quadrilateral is outlined?

33.



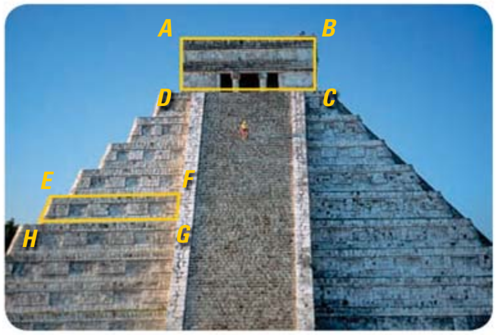
34.



35.

TEXAS @HomeTutor for problem solving help at classzone.com

36. PYRAMID Use the photo of the Pyramid of Kukulcan in Mexico.



a. $\overline{EF} \parallel \overline{HG}$, and $\overline{EH}$ and $\overline{FG}$ are not parallel. What shape is this part of the pyramid?

b. $\overline{AB} \parallel \overline{DC}$, $\overline{AD} \parallel \overline{BC}$, and $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are all congruent to each other. What shape is this part of the pyramid?

TEXAS @HomeTutor for problem solving help at classzone.com

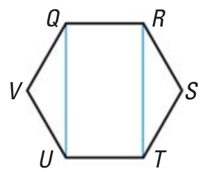
37. **TAKS REASONING** *Explain* why a parallelogram with one right angle must be a rectangle.

38. **TAKS REASONING** Segments *AC* and *BD* bisect each other.

a. Suppose that $\overline{AC}$ and $\overline{BD}$ are congruent, but not perpendicular. Draw quadrilateral *ABCD* and classify it. *Justify* your answer.

b. Suppose that $\overline{AC}$ and $\overline{BD}$ are perpendicular, but not congruent. Draw quadrilateral *ABCD* and classify it. *Justify* your answer.

39. MULTI-STEP PROBLEM Polygon *QRSTUV* shown at the right is a regular hexagon, and $\overline{QU}$ and $\overline{RT}$ are diagonals. Follow the steps below to classify quadrilateral *QRTU*. *Explain* your reasoning in each step.



a. Show that $\triangle QVU$ and $\triangle RST$ are congruent isosceles triangles.

b. Show that $\overline{QR} \cong \overline{UT}$ and that $\overline{QU} \cong \overline{RT}$.

c. Show that $\angle UQR \cong \angle QRT \cong \angle RTU \cong \angle TUQ$. Find the measure of each of these angles.

d. Classify quadrilateral *QRTU*.

40. REASONING In quadrilateral *WXYZ*, $\overline{WY}$ and $\overline{XZ}$ intersect each other at point *V*. $\overline{WV} \cong \overline{XV}$ and $\overline{YV} \cong \overline{ZV}$, but $\overline{WY}$ and $\overline{XZ}$ do not bisect each other. Draw $\overline{WY}$, $\overline{XY}$, and *WXYZ*. What special type of quadrilateral is *WXYZ*? Write a plan for a proof of your answer.

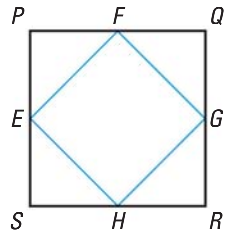
◯ = WORKED-OUT SOLUTIONS on p. WS1

= TAKS PRACTICE AND REASONING

CHALLENGE **What special type of quadrilateral is *EFGH*? Write a paragraph proof to show that your answer is correct.**

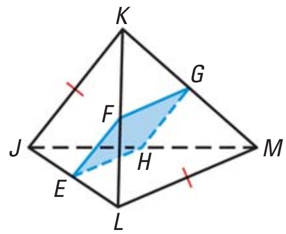
41. **GIVEN ▶** *PQRS* is a square. *E, F, G,* and *H* are midpoints of the sides of the square.

PROVE ▶ *EFGH* is a __?__.



42. **GIVEN ▶** In the three-dimensional figure, $\overline{JK} \cong \overline{LM}$; *E, F, G,* and *H* are the midpoints of $\overline{JL}$, $\overline{KL}$, $\overline{KM}$, and $\overline{JM}$.

PROVE ▶ *EFGH* is a __?__.

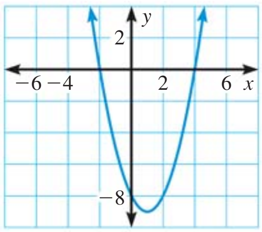


MIXED REVIEW FOR TAKS

TAKS PRACTICE at classzone.com

REVIEW
Skills Review Handbook p. 882; TAKS Workbook

43. **TAKS PRACTICE** What are the *x*-intercepts of the graph of the function at the right? ***TAKS Obj. 5***

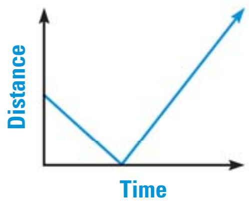


Ⓐ (0, 4) and (0, −2) Ⓑ (0, 4) and (−2, 0)

Ⓒ (4, 0) and (0, −2) Ⓓ (4, 0) and (−2, 0)

REVIEW
Skills Review Handbook p. 888; TAKS Workbook

44. **TAKS PRACTICE** The graph at the right best represents which of the following relationships between distance and time? ***TAKS Obj. 1***



Ⓕ Distance from a location while driving to it

Ⓖ Distance from a location while driving away from it

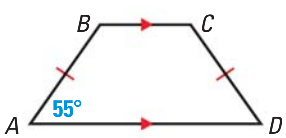
Ⓗ Distance from a location while driving to it and arriving

Ⓙ Distance from a location while driving to it, arriving, then leaving

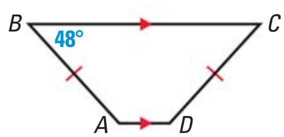
QUIZ *for Lessons 8.5–8.6*

Find the unknown angle measures. ***(p. 542)***

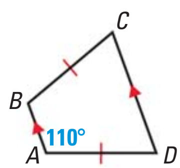
1.



2.



3.



4. The diagonals of quadrilateral *ABCD* are congruent and bisect each other. What types of quadrilaterals match this description? ***(p. 552)***

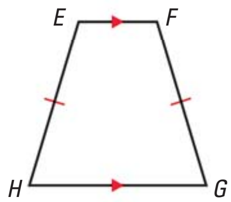
5. In quadrilateral *EFGH*, $\angle E \cong \angle G$, $\angle F \cong \angle H$, and $\overline{EF} \cong \overline{EH}$. What is the most specific name for quadrilateral *EFGH*? ***(p. 552)***

MIXED REVIEW FOR TEKS

TAKS PRACTICE
classzone.com

Lessons 8.4–8.6

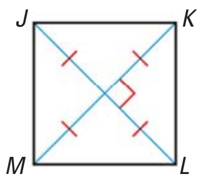
1. **ARCHITECTURE** In the photo of the installation of the courthouse steeple below, quadrilateral *EFGH* represents the front view of the steeple. $\angle H$ measures approximately 72°. Which is closest to the measure of $\angle F$? ***TEKS G.5.A***



Ⓐ 72° Ⓑ 108°

Ⓒ 144° Ⓓ 162°

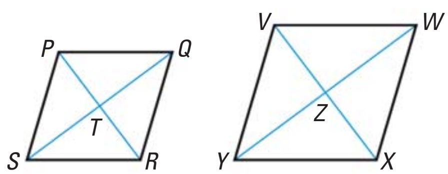
2. **QUADRILATERALS** What is the most specific name for quadrilateral *JKLM*? ***TEKS G.5.B***



Ⓕ Parallelogram Ⓖ Rhombus

Ⓗ Rectangle Ⓙ Square

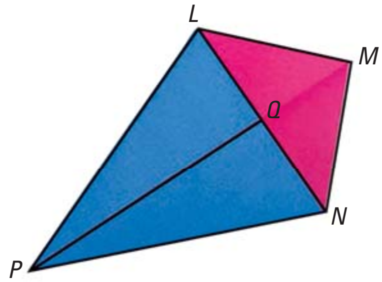
3. **SIMILARITY** In the diagram below, rhombus *PQRS* is similar to rhombus *VWXY*, and $QS = 32$, $QR = 20$, and $WZ = 20$. What is *WX*? ***TEKS G.5.B***



Ⓐ 12.5 Ⓑ 25

Ⓒ 36 Ⓓ 64

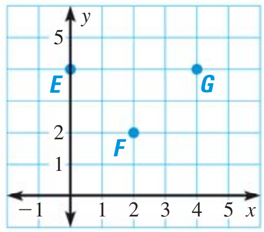
4. **PAPER FOLDING** A square piece of paper is folded into a kite shape as shown below. If $m\angle LMN = 90°$ and $m\angle NPL = 45°$, what is $m\angle PLM$? ***TEKS G.5.A***



Ⓕ 45° Ⓖ 56.25°

Ⓗ 90° Ⓙ 112.5°

5. **RHOMBUS** Three vertices of rhombus *EFGH* are shown below. What are the coordinates of *H*? ***TEKS G.7.C***

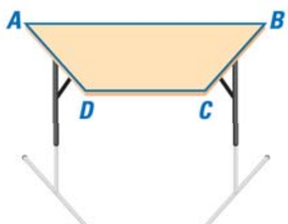


Ⓐ (2, 4) Ⓑ (2, 6)

Ⓒ (4, 2) Ⓓ (6, 2)

GRIDDED ANSWER ⓪①●③④⑤⑥⑦⑧⑨

6. **TABLE TOP** The top of the table shown is shaped like an isosceles trapezoid. In *ABCD*, $AB = 48$ inches, $BC = 18$ inches, $CD = 24$ inches, and $DA = 18$ inches. Find the length (in inches) of the midsegment of *ABCD*. ***TEKS G.5.B***



8 CHAPTER SUMMARY

BIG IDEAS *For Your Notebook*

Big Idea 1

TEKS G.5.B

Using Angle Relationships in Polygons

You can use theorems about the interior and exterior angles of convex polygons to solve problems.

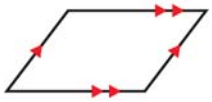
Polygon Interior Angles Theorem	Polygon Exterior Angles Theorem
The sum of the interior angle measures of a convex n-gon is $(n - 2) \cdot 180°$.	The sum of the exterior angle measures of a convex n-gon is $360°$.

Big Idea 2

TEKS G.3.C, G.9.A

Using Properties of Parallelograms

By definition, a parallelogram is a quadrilateral with both pairs of opposite sides parallel. Other properties of parallelograms:



- Opposite sides are congruent.
- Opposite angles are congruent.
- Diagonals bisect each other.
- Consecutive angles are supplementary.

Ways to show that a quadrilateral is a parallelogram:

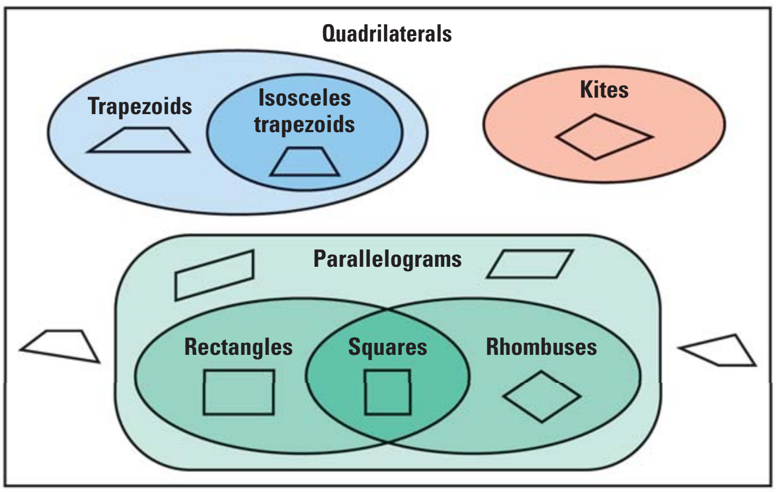
- Show both pairs of opposite sides are parallel.
- Show both pairs of opposite sides or opposite angles are congruent.
- Show one pair of opposite sides are congruent and parallel.
- Show the diagonals bisect each other.

Big Idea 3

TEKS G.9.B

Classifying Quadrilaterals by Their Properties

Special quadrilaterals can be classified by their properties. In a parallelogram, both pairs of opposite sides are parallel. In a trapezoid, only one pair of sides are parallel. A kite has two pairs of consecutive congruent sides, but opposite sides are not congruent.



8 CHAPTER REVIEW

TEXAS @HomeTutor
classzone.com
- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- diagonal, *p. 507*
- parallelogram, *p. 515*
- rhombus, *p. 533*
- rectangle, *p. 533*
- square, *p. 533*
- trapezoid, *p. 542*
- bases of a trapezoid, *p. 542*
- base angles of a trapezoid, *p. 542*
- legs of a trapezoid, *p. 542*
- isosceles trapezoid, *p. 543*
- midsegment of a trapezoid, *p. 544*
- kite, *p. 545*

VOCABULARY EXERCISES

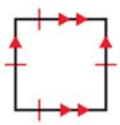
In Exercises 1 and 2, copy and complete the statement.

1. The _?_ of a trapezoid is parallel to the bases.

2. A(n) _?_ of a polygon is a segment whose endpoints are nonconsecutive vertices.

3. **WRITING** *Describe* the different ways you can show that a trapezoid is an isosceles trapezoid.

In Exercises 4–6, match the figure with the most specific name.

4.


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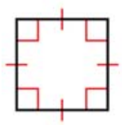

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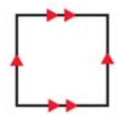


A. Square **B.** Parallelogram **C.** Rhombus

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 8.

8.1 Find Angle Measures in Polygons *pp. 507–513*

EXAMPLE

The sum of the measures of the interior angles of a convex regular polygon is 1080°. Classify the polygon by the number of sides. What is the measure of each interior angle?

Write and solve an equation for the number of sides n.

$(n - 2) \cdot 180° = 1080°$ **Polygon Interior Angles Theorem**

$n = 8$ **Solve for n.**

The polygon has 8 sides, so it is an octagon.

A regular octagon has 8 congruent interior angles, so divide to find the measure of each angle: $1080° \div 8 = 135°$. The measure of each interior angle is 135°.

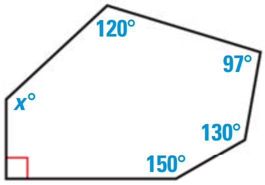
EXERCISES

EXAMPLES 2, 3, 4, and 5 on pp. 508–510 for Exs. 7–11

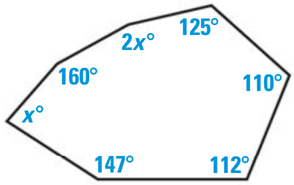
7. The sum of the measures of the interior angles of a convex regular polygon is 3960°. Classify the polygon by the number of sides. What is the measure of each interior angle?

In Exercises 8–10, find the value of x.

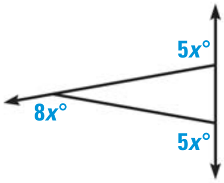
8.



9.



10.



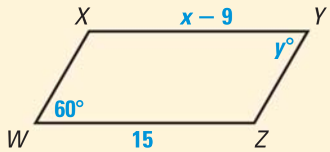
11. In a regular nonagon, the exterior angles are all congruent. What is the measure of one of the exterior angles? *Explain*.

8.2 Use Properties of Parallelograms

pp. 515–521

EXAMPLE

Quadrilateral $WXYZ$ is a parallelogram. Find the values of x and y.



To find the value of x, apply Theorem 8.3.

$XY = WZ$	**Opposite sides of a ▱ are ≅.**
$x - 9 = 15$	**Substitute.**
$x = 24$	**Add 9 to each side.**

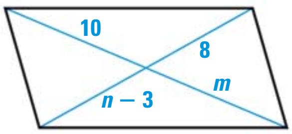
By Theorem 8.4, $\angle W \cong \angle Y$, or $m\angle W = m\angle Y$. So, $y = 60$.

EXERCISES

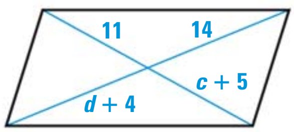
EXAMPLES 1, 2, and 3 on pp. 515, 517 for Exs. 12–17

Find the value of each variable in the parallelogram.

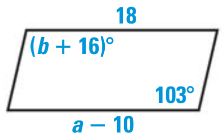
12.



13.



14.



15. In ▱$PQRS$, $PQ = 5$ centimeters, $QR = 10$ centimeters, and $m\angle PQR = 36°$. Sketch $PQRS$. Find and label all of its side lengths and interior angle measures.

16. The perimeter of ▱$EFGH$ is 16 inches. If EF is 5 inches, find the lengths of all the other sides of $EFGH$. *Explain* your reasoning.

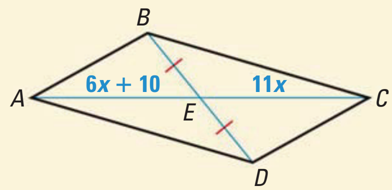
17. In ▱$JKLM$, the ratio of the measure of $\angle J$ to the measure of $\angle M$ is 5 : 4. Find $m\angle J$ and $m\angle M$. *Explain* your reasoning.

8 CHAPTER REVIEW

8.3 Show that a Quadrilateral is a Parallelogram *pp. 522–529*

EXAMPLE

For what value of x is quadrilateral $ABCD$ a parallelogram?



If the diagonals bisect each other, then $ABCD$ is a parallelogram. The diagram shows that $\overline{BE} \cong \overline{DE}$. You need to find the value of x that makes $\overline{AE} \cong \overline{CE}$.

$AE = CE$ **Set the segment lengths equal.**

$6x + 10 = 11x$ **Substitute expressions for the lengths.**

$x = 2$ **Solve for x.**

When $x = 2$, $AE = 6(2) + 10 = 22$ and $CE = 11(2) = 22$. So, $\overline{AE} \cong \overline{CE}$.

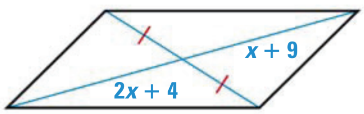
Quadrilateral $ABCD$ is a parallelogram when $x = 2$.

EXERCISES

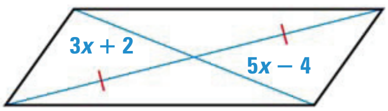
EXAMPLE 3 on p. 524 for Exs. 18–19

For what value of x is the quadrilateral a parallelogram?

18.



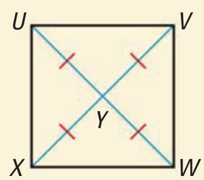
19.



8.4 Properties of Rhombuses, Rectangles, and Squares *pp. 533–540*

EXAMPLE

Classify the special quadrilateral.



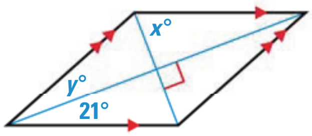
In quadrilateral $UVWX$, the diagonals bisect each other. So, $UVWX$ is a parallelogram. Also, $\overline{UY} \cong \overline{VY} \cong \overline{WY} \cong \overline{XY}$. So, $UY + YW = VY + XY$. Because $UY + YW = UW$, and $VY + XY = VX$, you can conclude that $\overline{UW} \cong \overline{VX}$. By Theorem 8.13, $UVWX$ is a rectangle.

EXERCISES

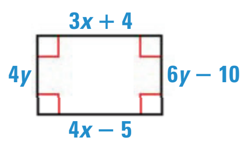
EXAMPLES 2 and 3 on pp. 534–535 for Exs. 20–22

Classify the special quadrilateral. Then find the values of x and y.

20.



21.

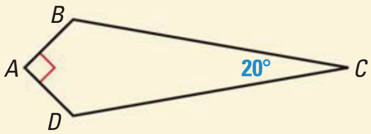


22. The diagonals of a rhombus are 10 centimeters and 24 centimeters. Find the length of a side. *Explain.*

8.5 Use Properties of Trapezoids and Kites *pp. 542–549*

EXAMPLE

Quadrilateral *ABCD* is a kite. Find $m\angle B$ and $m\angle D$.



A kite has exactly one pair of congruent opposite angles. Because $\angle A \not\cong \angle C$, $\angle B$ and $\angle D$ must be congruent. Write and solve an equation.

$90° + 20° + m\angle B + m\angle D = 360°$ Corollary to Theorem 8.1

$110° + m\angle B° + m\angle D = 360°$ Combine like terms.

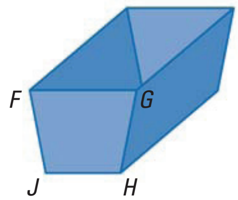
$m\angle B + m\angle D = 250°$ Subtract 110° from each side.

Because $\angle B \cong \angle D$, you can substitute $m\angle B$ for $m\angle D$ in the last equation. Then $m\angle B + m\angle B = 250°$, and $m\angle B = m\angle D = 125°$.

EXERCISES

EXAMPLES 2 and 3 on pp. 543–544 for Exs. 20–22

In Exercises 23 and 24, use the diagram of a recycling container. One end of the container is an isosceles trapezoid with $\overline{FG} \parallel \overline{JH}$ and $m\angle F = 79°$.



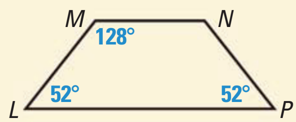
23. Find $m\angle G$, $m\angle H$, and $m\angle J$.

24. Copy trapezoid *FGHJ* and sketch its midsegment. If the midsegment is 16.5 inches long and $\overline{FG}$ is 19 inches long, find *JH*.

8.6 Identify Special Quadrilaterals *pp. 552–557*

EXAMPLE

Give the most specific name for quadrilateral *LMNP*.



In *LMNP*, $\angle L$ and $\angle M$ are supplementary, but $\angle L$ and $\angle P$ are not. So, $\overline{MN} \parallel \overline{LP}$, but $\overline{LM}$ is not parallel to $\overline{NP}$. By definition, *LMNP* is a trapezoid.

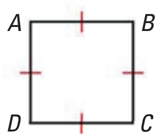
Also, $\angle L$ and $\angle P$ are a pair of base angles and $\angle L \cong \angle P$. So, *LMNP* is an isosceles trapezoid by Theorem 8.15.

EXERCISES

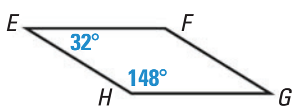
EXAMPLE 2 on p. 553 for Exs. 25–28

Give the most specific name for the quadrilateral. *Explain* your reasoning.

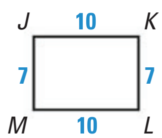
25.



26.



27.

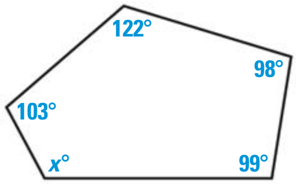


28. In quadrilateral *RSTU*, $\angle R$, $\angle T$, and $\angle U$ are right angles, and $RS = ST$. What is the most specific name for quadrilateral *RSTU*? *Explain.*

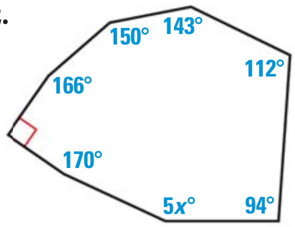
8 CHAPTER TEST

Find the value of *x*.

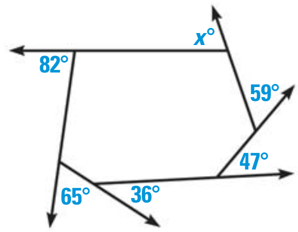
1.



2.



3.

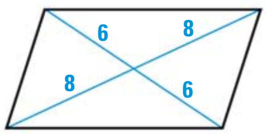


4. In ▱*EFGH*, $m\angle F$ is 40° greater than $m\angle G$. Sketch ▱*EFGH* and label each angle with its correct angle measure. *Explain* your reasoning.

Are you given enough information to determine whether the quadrilateral is a parallelogram? *Explain* your reasoning.

5.

6.



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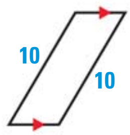


In Exercises 8–11, list each type of quadrilateral—*parallelogram*, *rectangle*, *rhombus*, and *square*—for which the statement is always true.

8. It is equilateral.

9. Its interior angles are all right angles.

10. The diagonals are congruent.

11. Opposite sides are parallel.

12. The vertices of quadrilateral *PQRS* are $P(-2, 0)$, $Q(0, 3)$, $R(6, -1)$, and $S(1, -2)$. Draw *PQRS* in a coordinate plane. Show that it is a trapezoid.

13. One side of a quadrilateral *JKLM* is longer than another side.

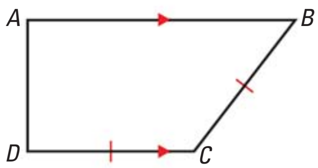
a. Suppose *JKLM* is an isosceles trapezoid. In a coordinate plane, find possible coordinates for the vertices of *JKLM*. *Justify* your answer.

b. Suppose *JKLM* is a kite. In a coordinate plane, find possible coordinates for the vertices of *JKLM*. *Justify* your answer.

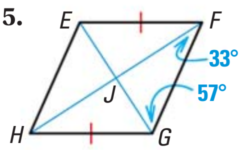
c. Name other special quadrilaterals that *JKLM* could be.

Give the most specific name for the quadrilateral. *Explain* your reasoning.

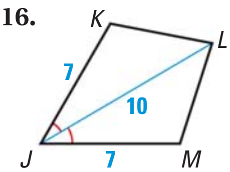
14.



15.



16.



17. In trapezoid *WXYZ*, $\overline{WX} \parallel \overline{YZ}$, and $YZ = 4.25$ centimeters. The midsegment of trapezoid *WXYZ* is 2.75 centimeters long. Find *WX*.

18. In ▱*RSTU*, $\overline{RS}$ is 3 centimeters shorter than $\overline{ST}$. The perimeter of ▱*RSTU* is 42 centimeters. Find *RS* and *ST*.

8 xy **ALGEBRA REVIEW**

Animated Algebra classzone.com

GRAPH NONLINEAR FUNCTIONS

EXAMPLE 1 Graph a quadratic function in vertex form

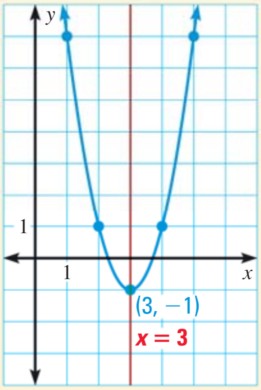
Graph $y = 2(x - 3)^2 - 1$.

The *vertex form* of a quadratic function is $y = a(x - h)^2 + k$. Its graph is a parabola with vertex at (h, k) and axis of symmetry $x = h$.

The given function is in vertex form. So, $a = 2$, $h = 3$, and $k = -1$. Because $a > 0$, the parabola opens up.

Graph the vertex at $(3, -1)$. Sketch the axis of symmetry, $x = 3$. Use a table of values to find points on each side of the axis of symmetry. Draw a parabola through the points.

x	3	1	2	4	5
y	−1	7	1	1	7

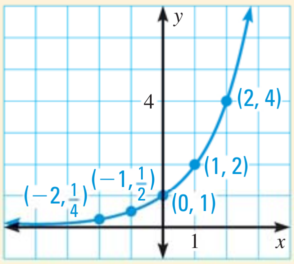


EXAMPLE 2 Graph an exponential function

Graph $y = 2^x$.

Make a table by choosing a few values for x and finding the values for y. Plot the points and connect them with a smooth curve.

x	−2	−1	0	1	2
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4



EXERCISES

EXAMPLE 1 for Exs. 1–6

Graph the quadratic function. Label the vertex and sketch the axis of symmetry.

1. $y = 3x^2 + 5$
2. $y = -2x^2 + 4$
3. $y = 0.5x^2 - 3$
4. $y = 3(x + 3)^2 - 3$
5. $y = -2(x - 4)^2 - 1$
6. $y = \frac{1}{2}(x - 4)^2 + 3$

EXAMPLE 2 for Exs. 7–10

Graph the exponential function.

7. $y = 3^x$
8. $y = 8^x$
9. $y = 2.2^x$
10. $y = \left(\frac{1}{3}\right)^x$

Use a table of values to graph the cubic or absolute value function.

11. $y = x^3$
12. $y = x^3 - 2$
13. $y = 3x^3 - 1$
14. $y = 2|x|$
15. $y = 2|x| - 4$
16. $y = -|x| - 1$

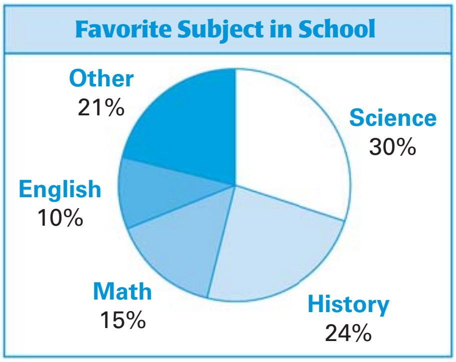
8 TAKS PREPARATION

TEXAS **TAKS Obj. 9**
TEKS 8.12.C, 8.13B

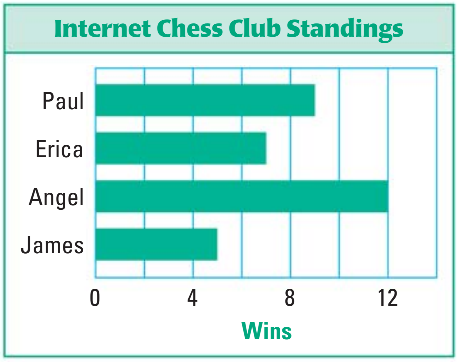
REVIEWING DATA DISPLAY PROBLEMS

In this TAKS Preparation, you will see how to represent data visually using circle graphs, bar graphs, and histograms.

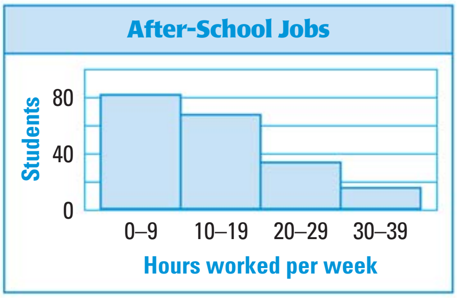
A *circle graph* displays data as sections of a circle. The whole circle represents all of the data.



A *bar graph* uses lengths of bars to represent and compare data. The bars can be horizontal or vertical.



A *histogram* displays data from a frequency table. A *frequency table* groups data values into intervals. The frequency of an interval is the number of values that lie in the interval. In a histogram, there is one bar for each interval, and the length of a bar indicates the frequency of the interval.



EXAMPLE

EXERCISE The frequency table at the right shows the results of a survey that asked 20 adults the average number of hours they exercise each week. Make a histogram of the data.

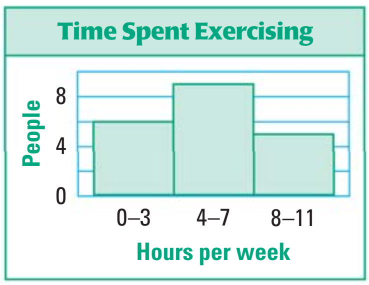
Hours	Frequency
0–3	6
4–7	9
8–11	5

Solution

STEP 1 **Show** the intervals in the frequency table on the horizontal axis and the frequencies on the vertical axis.

STEP 2 **Draw** a bar to represent the frequency of each interval.

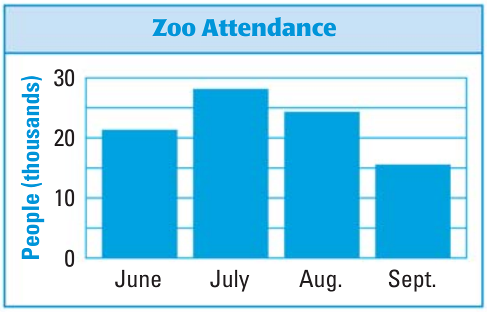
STEP 3 **Label** the histogram.



DATA DISPLAY PROBLEMS ON TAKS

Below are examples of data display problems in multiple choice format. Try solving the problems before looking at the solutions. (Cover the solutions with a piece of paper.) Then check your solutions against the ones given.

1. Which statement about the zoo is true?



A The zoo's attendance increased every month from June to September.

B The zoo's attendance decreased every month from June to September.

C About 5000 more people attended the zoo in June than in September.

D About 5000 more people attended the zoo in August than in July.

Solution

The zoo's attendance increased from June to July, and then decreased from July to August and August to September. So, neither A nor B is correct.

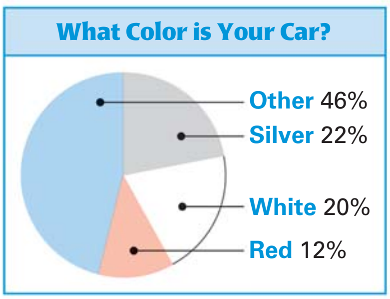
Choice D is incorrect because more people attended the zoo in July than in August.

About 21,000 people attended the zoo in June and about 16,000 people attended the zoo in September. Because $21{,}000 - 16{,}000 = 5000$, you can say that about 5000 more people attended the zoo in June than in September.

So, the correct answer is C.

(A) (B) (C) (D)

2. A survey asked 200 car owners the color of their car. Which statement is true?



F 11 people own a silver car.

G 20 people own a white car and 12 people own a red car.

H 44 people own a silver car and 24 people own a red car.

J 100 people own a white car.

Solution

To find the number of people in each group, write each percent as a decimal and multiply by the total number of people surveyed.

Silver	White
$0.22 \times 200 = 44$	$0.2 \times 200 = 40$
Red	**Other**
$0.12 \times 200 = 24$	$0.46 \times 200 = 92$

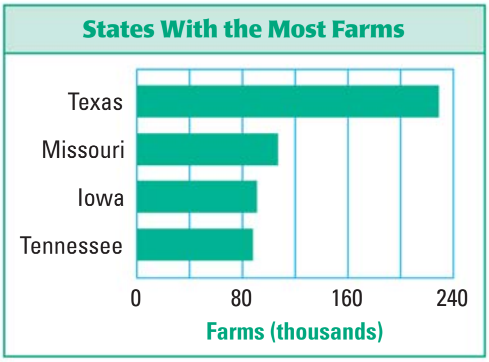
So, the correct answer is H.

(F) (G) (H) (J)

8 TAKS PRACTICE

PRACTICE FOR TAKS OBJECTIVE 9

1. The bar graph shows the four U.S. states with the most farms in 2002. Which statement is true?



A Tennessee has more farms than any other state.

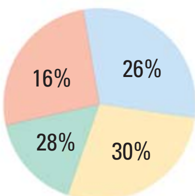
B Texas has about twice as many farms as Missouri.

C Iowa has about half as many farms as Missouri.

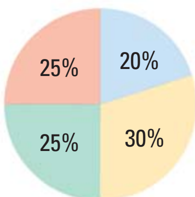
D Of the four states shown, Iowa has the fewest farms.

2. In a survey of 400 students, 96 have no pets, 120 have 1 pet, 108 have 2 pets, and the rest have 3 or more pets. Use the key shown. Which circle graph shows this information?

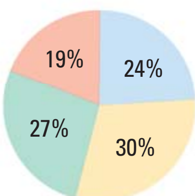
F



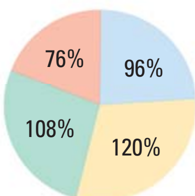
G



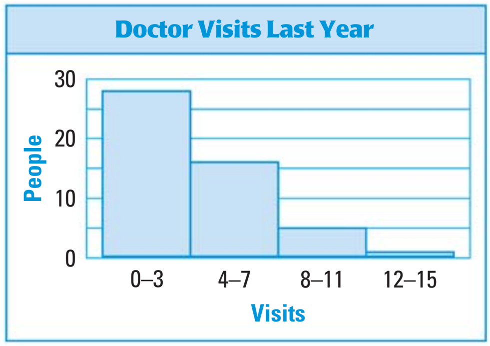
H



J



3. The histogram shows the results of a survey that asked 50 adults how many times they visited their physician last year. Which statement is true?



A Most of the people surveyed visited their physician about 28 times last year.

B Most of the people surveyed visited their physician less than 8 times last year.

C More than half of the people surveyed visited their physician 4 or more times last year.

D About 16 of the people surveyed visited their physician 4 or more times last year.

MIXED TAKS PRACTICE

4. Which expression can you use to find the values of $f(x)$ in the table below? ***TAKS Obj. 2***

x	-2	-1	0	1	2	3
$f(x)$	4	$\frac{9}{2}$	5	$\frac{11}{2}$	6	$\frac{13}{2}$

F $x + 6$

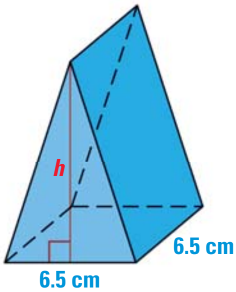
G $-2x$

H $\frac{1}{2}x + 5$

J $\frac{3}{2}x + 7$

MIXED TAKS PRACTICE

5. The triangular prism below has a volume of 211.25 square centimeters. What is the height, h, of the prism? ***TAKS Obj. 8***



A 5 cm

B 6.5 cm

C 10 cm

D 13 cm

6. Which expression is equivalent to $\frac{12a^3c^2}{30abc^{-7}}$? ***TAKS Obj. 5***

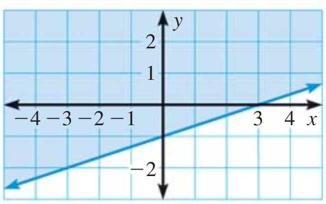
F $\frac{2a^2}{5bc^5}$

G $\frac{2a^2c^9}{5b}$

H $\frac{2a^4b}{5c^5}$

J $\frac{2a^2c^5}{5b}$

7. Which inequality best describes the shaded region of the graph? ***TAKS Obj. 1***



A $y \le \frac{1}{3}x - 1$

B $y \le 3x - 1$

C $y \ge \frac{1}{3}x - 1$

D $y \ge 3x - 1$

8. Which of the following dimensions do not represent an enlargement or reduction of the photo shown below? ***TAKS Obj. 6***

F 2.8 in. by 2 in.

G 5.25 in. by 3.5 in.

H 8.4 in. by 6 in.

J 17.5 in. by 12.5 in.

9. Which of the following are the side lengths of a right triangle? ***TAKS Obj. 7***

A 4, 5, 9

B 9, 24, 25

C 6, 9.1, 10.9

D 1.3, 1.4, 1.5

10. What is the solution of the system of linear equations? ***TAKS Obj. 4***

$$2x + y = -5$$
$$x - 3y = -6$$

F $(1, -3)$

G $(-3, 1)$

H $(-2, -1)$

J $(-1, -2)$

11. **GRIDDED ANSWER** Of the 640 girls in your school, 5% are on the softball team. Of those girls on the softball team, 12.5% are also on the soccer team. How many girls in your school are on both the softball and soccer teams? ***TAKS Obj. 9***

Record your answer and fill in the bubbles on your answer document. Be sure to use the correct place value.