# 6 Similarity



## Before

In previous courses and in Chapters 1–5, you learned the following skills, which you'll use in Chapter 6: using properties of parallel lines, using properties of triangles, simplifying expressions, and finding perimeter.

## Prerequisite Skills

### VOCABULARY CHECK

1. The alternate interior angles formed when a transversal intersects two __?__ lines are congruent.

2. Two triangles are congruent if and only if their corresponding parts are __?__.

### SKILLS AND ALGEBRA CHECK

**Simplify the expression.** *(Review pp. 870, 874 for 6.1.)*

**3.** $\frac{9 \cdot 20}{15}$ **4.** $\frac{15}{25}$ **5.** $\frac{3+4+5}{6+8+10}$ **6.** $\sqrt{5(5 \cdot 7)}$

**Find the perimeter of the rectangle with the given dimensions.** *(Review p. 49 for 6.1, 6.2.)*

**7.** $\ell = 5$ in., $w = 12$ in. **8.** $\ell = 30$ ft, $w = 10$ ft **9.** $A = 56\text{ m}^2$, $\ell = 8$ m

**10.** Find the slope of a line parallel to the line whose equation is $y - 4 = 7(x + 2)$. *(Review p. 171 for 6.5.)*

TEXAS @HomeTutor Prerequisite skills practice at classzone.com

## Now

In Chapter 6, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 417. You will also use the key vocabulary listed below.

## Big Ideas

1. Using ratios and proportions to solve geometry problems
2. Showing that triangles are similar
3. Using indirect measurement and similarity

### KEY VOCABULARY

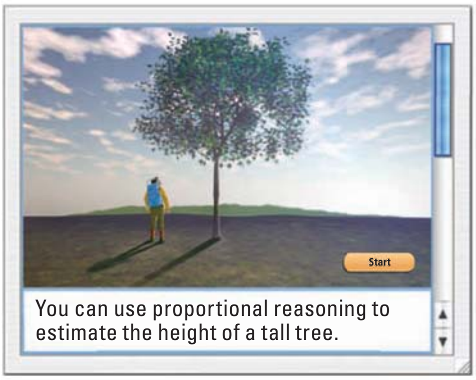
- ratio, *p. 356*
- proportion, *p. 358*
  means, extremes
- geometric mean, *p. 359*
- scale drawing, *p. 365*
- scale, *p. 365*
- similar polygons, *p. 372*
- scale factor of two similar polygons, *p. 373*
- dilation, *p. 409*
- center of dilation, *p. 409*
- scale factor of a dilation, *p. 409*
- reduction, *p. 409*
- enlargement, *p. 409*

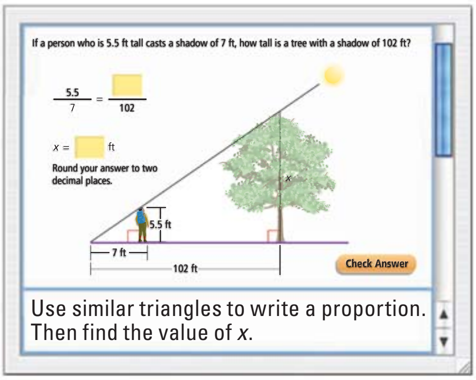
## Why?

You can use similarity to measure lengths indirectly. For example, you can use similar triangles to find the height of a tree.

## Animated Geometry

The animation illustrated below for Exercise 33 on page 394 helps you answer this question: What is the height of the tree?




**Geometry** at classzone.com

**Animated Geometry** at classzone.com

**Other animations for Chapter 6:** pages 365, 375, 391, 407, and 414

Other animations for Chapter 1 appear on pages 7, 9, 14, 21, 37, and 50.

# 6.1 Ratios, Proportions, and the Geometric Mean

TEKS a.1, G.8.A, G.11.B, G.11.C

**Before** You solved problems by writing and solving equations.

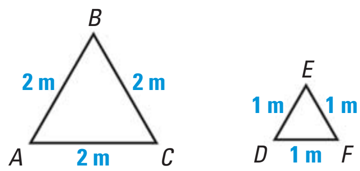
**Now** You will solve problems by writing and solving proportions.

**Why?** So you can estimate bird populations, as in Ex. 62.

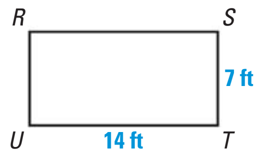
**Key Vocabulary**
- **ratio**
- **proportion**
  means, extremes
- **geometric mean**

If $a$ and $b$ are two numbers or quantities and $b \neq 0$, then the **ratio of *a* to *b*** is $\frac{a}{b}$. The ratio of $a$ to $b$ can also be written as $a:b$.

For example, the ratio of a side length in $\triangle ABC$ to a side length in $\triangle DEF$ can be written as $\frac{2}{1}$ or $2:1$.




Ratios are usually expressed in simplest form. Two ratios that have the same simplified form are called *equivalent ratios*. The ratios $7:14$ and $1:2$ in the example below are *equivalent*.




$$\frac{\text{width of } RSTU}{\text{length of } RSTU} = \frac{7 \text{ ft}}{14 \text{ ft}} = \frac{1}{2}$$

## EXAMPLE 1 Simplify ratios

**Simplify the ratio.**

**a.** $64 \text{ m} : 6 \text{ m}$

**b.** $\frac{5 \text{ ft}}{20 \text{ in.}}$

**Solution**

**a.** Write $64 \text{ m} : 6 \text{ m}$ as $\frac{64 \text{ m}}{6 \text{ m}}$. Then divide out the units and simplify.

$$\frac{64 \text{ m}}{6 \text{ m}} = \frac{32}{3} = 32:3$$

**b.** To simplify a ratio with unlike units, multiply by a conversion factor.

$$\frac{5 \text{ ft}}{20 \text{ in.}} = \frac{5 \text{ ft}}{20 \text{ in.}} \cdot \frac{12 \text{ in.}}{1 \text{ ft}} = \frac{60}{20} = \frac{3}{1}$$

**REVIEW UNIT ANALYSIS**

For help with measures and conversion factors, see p. 886 and the Table of Measures on p. 921.

## ✓ GUIDED PRACTICE for Example 1

**Simplify the ratio.**

**1.** 24 yards to 3 yards

**2.** $150 \text{ cm} : 6 \text{ m}$

## EXAMPLE 2 Use a ratio to find a dimension

**PAINTING** You are planning to paint a mural on a rectangular wall. You know that the perimeter of the wall is 484 feet and that the ratio of its length to its width is 9 : 2. Find the area of the wall.

**Solution**

**WRITE EXPRESSIONS**

Because the ratio in Example 2 is 9 : 2, you can write an equivalent ratio to find expressions for the length and width.

$$\frac{\text{length}}{\text{width}} = \frac{9}{2} = \frac{9}{2} \cdot \frac{x}{x} = \frac{9x}{2x}$$

**STEP 1** **Write** expressions for the length and width. Because the ratio of length to width is 9 : 2, you can represent the length by $9x$ and the width by $2x$.

**STEP 2** **Solve** an equation to find $x$.

| | |
|---|---|
| $2\ell + 2w = P$ | Formula for perimeter of rectangle |
| $2(9x) + 2(2x) = 484$ | Substitute for $\ell$, $w$, and $P$. |
| $22x = 484$ | Multiply and combine like terms. |
| $x = 22$ | Divide each side by 22. |

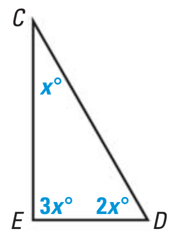
**STEP 3** **Evaluate** the expressions for the length and width. Substitute the value of $x$ into each expression.

Length $= 9x = 9(22) = 198$ Width $= 2x = 2(22) = 44$

▸ The wall is 198 feet long and 44 feet wide, so its area is 198 ft • 44 ft = 8712 $ft^2$.

## EXAMPLE 3 Use extended ratios

**ALGEBRA** **The measures of the angles in △*CDE* are in the *extended ratio* of 1 : 2 : 3. Find the measures of the angles.**




**Solution**

Begin by sketching the triangle. Then use the extended ratio of 1 : 2 : 3 to label the measures as $x°$, $2x°$, and $3x°$.

| | |
|---|---|
| $x° + 2x° + 3x° = 180°$ | Triangle Sum Theorem |
| $6x = 180$ | Combine like terms. |
| $x = 30$ | Divide each side by 6. |

▸ The angle measures are 30°, 2(30°) = 60°, and 3(30°) = 90°.

## ✓ GUIDED PRACTICE for Examples 2 and 3

3. The perimeter of a room is 48 feet and the ratio of its length to its width is 7 : 5. Find the length and width of the room.

4. A triangle's angle measures are in the extended ratio of 1 : 3 : 5. Find the measures of the angles.

**PROPORTIONS** An equation that states that two ratios are equal is called a **proportion**.

$$\text{extreme} \rightarrow \frac{a}{b} = \frac{c}{d} \leftarrow \text{mean}$$
$$\text{mean} \rightarrow b \qquad d \leftarrow \text{extreme}$$

The numbers $b$ and $c$ are the **means** of the proportion. The numbers $a$ and $d$ are the **extremes** of the proportion.

The property below can be used to solve proportions. To *solve a proportion*, you find the value of any variable in the proportion.

**PROPORTIONS**
You will learn more properties of proportions on p. 364.

## KEY CONCEPT — *For Your Notebook*

### A Property of Proportions

1. **Cross Products Property** In a proportion, the product of the extremes equals the product of the means.

If $\frac{a}{b} = \frac{c}{d}$ where $b \neq 0$ and $d \neq 0$, then $ad = bc$.

$$\frac{2}{3} = \frac{4}{6} \qquad 3 \cdot 4 = 12 \qquad 2 \cdot 6 = 12$$

## EXAMPLE 4 Solve proportions

**ALGEBRA** **Solve the proportion.**

**a.** $\frac{5}{10} = \frac{x}{16}$ **b.** $\frac{1}{y+1} = \frac{2}{3y}$

**Solution**

**a.**

| | |
|---|---|
| $\frac{5}{10} = \frac{x}{16}$ | Write original proportion. |
| $5 \cdot 16 = 10 \cdot x$ | Cross Products Property |
| $80 = 10x$ | Multiply. |
| $8 = x$ | Divide each side by 10. |

**b.**

| | |
|---|---|
| $\frac{1}{y+1} = \frac{2}{3y}$ | Write original proportion. |
| $1 \cdot 3y = 2(y+1)$ | Cross Products Property |
| $3y = 2y + 2$ | Distributive Property |
| $y = 2$ | Subtract $2y$ from each side. |

**ANOTHER WAY**
In part (a), you could multiply each side by the denominator, 16. Then $16 \cdot \frac{5}{10} = 16 \cdot \frac{x}{16}$, so $8 = x$.

## ✓ GUIDED PRACTICE for Example 4

**Solve the proportion.**

**5.** $\frac{2}{x} = \frac{5}{8}$ **6.** $\frac{1}{x-3} = \frac{4}{3x}$ **7.** $\frac{y-3}{7} = \frac{y}{14}$

## EXAMPLE 5 Solve a real-world problem

**SCIENCE** As part of an environmental study, you need to estimate the number of trees in a 150 acre area. You count 270 trees in a 2 acre area and you notice that the trees seem to be evenly distributed. Estimate the total number of trees.

**Solution**

Write and solve a proportion involving two ratios that compare the number of trees with the area of the land.

$\frac{270}{2} = \frac{n}{150}$ ← number of trees / ← area in acres — **Write proportion.**

$270 \cdot 150 = 2 \cdot n$ — **Cross Products Property**

$20{,}250 = n$ — **Simplify.**

▶ There are about 20,250 trees in the 150 acre area.

### KEY CONCEPT — *For Your Notebook*

**Geometric Mean**

The **geometric mean** of two positive numbers $a$ and $b$ is the positive number $x$ that satisfies $\frac{a}{x} = \frac{x}{b}$. So, $x^2 = ab$ and $x = \sqrt{ab}$.

## EXAMPLE 6 Find a geometric mean

**Find the geometric mean of 24 and 48.**

**Solution**

$x = \sqrt{ab}$ — **Definition of geometric mean**

$= \sqrt{24 \cdot 48}$ — **Substitute 24 for *a* and 48 for *b*.**

$= \sqrt{24 \cdot 24 \cdot 2}$ — **Factor.**

$= 24\sqrt{2}$ — **Simplify.**

▶ The geometric mean of 24 and 48 is $24\sqrt{2} \approx 33.9$.

### ✓ GUIDED PRACTICE for Examples 5 and 6

8. **WHAT IF?** In Example 5, suppose you count 390 trees in a 3 acre area of the 150 acre area. Make a new estimate of the total number of trees.

**Find the geometric mean of the two numbers.**

9. 12 and 27
10. 18 and 54
11. 16 and 18

# 6.1 EXERCISES

**HOMEWORK KEY**

- ◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 27, and 59
- ★ = **TAKS PRACTICE AND REASONING** Exs. 47, 48, 52, 63, 72, and 73
- ◆ = **MULTIPLE REPRESENTATIONS** Ex. 66

## SKILL PRACTICE

1. **VOCABULARY** Copy the proportion $\frac{m}{n} = \frac{p}{q}$. Identify the means of the proportion and the extremes of the proportion.

2. **WRITING** Write three ratios that are equivalent to the ratio 3 : 4. *Explain* how you found the ratios.

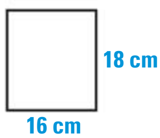
**EXAMPLE 1** on p. 356 for Exs. 3–17

**SIMPLIFYING RATIOS** **Simplify the ratio.**

3. \$20 : \$5
4. $\frac{15\text{ cm}^2}{12\text{ cm}^2}$
5. 6 L : 10 mL
6. $\frac{1\text{ mi}}{20\text{ ft}}$
7. $\frac{7\text{ ft}}{12\text{ in.}}$
8. $\frac{80\text{ cm}}{2\text{ m}}$
9. $\frac{3\text{ lb}}{10\text{ oz}}$
10. $\frac{2\text{ gallons}}{18\text{ quarts}}$

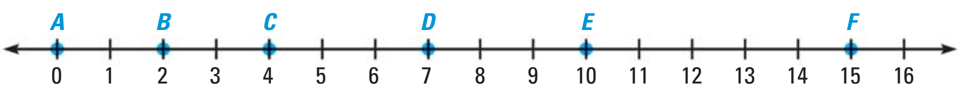
**WRITING RATIOS** **Find the ratio of the width to the length of the rectangle. Then simplify the ratio.**

11. 


12. 


13. 


**FINDING RATIOS** **Use the number line to find the ratio of the distances.**

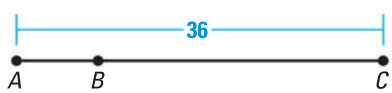



14. $\frac{AD}{CF}$
15. $\frac{BD}{AB}$
16. $\frac{CE}{EF}$
17. $\frac{BE}{CE}$

**EXAMPLE 2** on p. 357 for Exs. 18–19

18. **PERIMETER** The perimeter of a rectangle is 154 feet. The ratio of the length to the width is 10 : 1. Find the length and the width.

19. **SEGMENT LENGTHS** In the diagram, $AB : BC$ is 2 : 7 and $AC = 36$. Find $AB$ and $BC$.




**EXAMPLE 3** on p. 357 for Exs. 20–22

**USING EXTENDED RATIOS** **The measures of the angles of a triangle are in the extended ratio given. Find the measures of the angles of the triangle.**

20. 3 : 5 : 10
21. 2 : 7 : 9
22. 11 : 12 : 13

**EXAMPLE 4** on p. 358 for Exs. 23–30

**xy ALGEBRA** **Solve the proportion.**

23. $\frac{6}{x} = \frac{3}{2}$
24. $\frac{y}{20} = \frac{3}{10}$
25. $\frac{2}{7} = \frac{12}{z}$
26. $\frac{j+1}{5} = \frac{4}{10}$
27. $\frac{1}{c+5} = \frac{3}{24}$
28. $\frac{4}{a-3} = \frac{2}{5}$
29. $\frac{1+3b}{4} = \frac{5}{2}$
30. $\frac{3}{2p+5} = \frac{1}{9p}$

**EXAMPLE 6**
on p. 359
for Exs. 31–36

**GEOMETRIC MEAN** **Find the geometric mean of the two numbers.**

**31.** 2 and 18 **32.** 4 and 25 **33.** 32 and 8

**34.** 4 and 16 **35.** 2 and 25 **36.** 6 and 20

**37.** **ERROR ANALYSIS** A student incorrectly simplified the ratio. *Describe* and correct the student's error.

$$\frac{8 \text{ in.}}{3 \text{ ft}} = \frac{8 \text{ in.}}{3 \text{ ft}} \cdot \frac{12 \text{ in.}}{1 \text{ ft}} = \frac{96 \text{ in.}}{3 \text{ ft}} = \frac{32 \text{ in.}}{1 \text{ ft}}$$

**WRITING RATIOS** **Let $x = 10$, $y = 3$, and $z = 8$. Write the ratio in simplest form.**

**38.** $x : z$ **39.** $\frac{8y}{x}$ **40.** $\frac{4}{2x + 2z}$ **41.** $\frac{2x - z}{3y}$

**ALGEBRA** **Solve the proportion.**

**42.** $\frac{2x + 5}{3} = \frac{x - 5}{4}$ **43.** $\frac{2 - s}{3} = \frac{2s + 1}{5}$ **44.** $\frac{15}{m} = \frac{m}{5}$ **45.** $\frac{7}{q + 1} = \frac{q - 1}{5}$

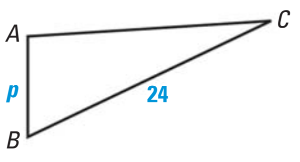
**46.** **ANGLE MEASURES** The ratio of the measures of two supplementary angles is 5 : 3. Find the measures of the angles.

**47.** **TAKS REASONING** The ratio of the measure of an exterior angle of a triangle to the measure of the adjacent interior angle is 1 : 4. Is the triangle *acute* or *obtuse*? *Explain* how you found your answer.

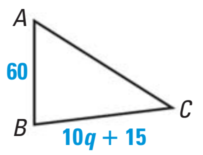
**48.** **TAKS REASONING** Without knowing its side lengths, can you determine the ratio of the perimeter of a square to the length of one of its sides? *Explain.*

**ALGEBRA** **In Exercises 49–51, the ratio of two side lengths for the triangle is given. Solve for the variable.**

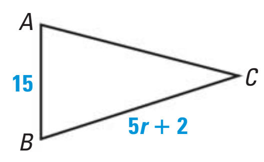
**49.** $AB : BC$ is 3 : 8.




**50.** $AB : BC$ is 3 : 4.




**51.** $AB : BC$ is 5 : 9.




**52.** **TAKS REASONING** What is a value of $x$ that makes $\frac{x}{3} = \frac{4x}{x + 3}$ true?

(A) 3 (B) 4 (C) 9 (D) 12

**53.** **AREA** The area of a rectangle is 4320 square inches. The ratio of the width to the length is 5 : 6. Find the length and the width.

**54.** **COORDINATE GEOMETRY** The points $(-3, 2)$, $(1, 1)$, and $(x, 0)$ are collinear. Use slopes to write a proportion to find the value of $x$.

**55.** **ALGEBRA** Use the proportions $\frac{a + b}{2a - b} = \frac{5}{4}$ and $\frac{b}{a + 9} = \frac{5}{9}$ to find $a$ and $b$.

**56.** **CHALLENGE** Find the ratio of $x$ to $y$ given that $\frac{5}{y} + \frac{7}{x} = 24$ and $\frac{12}{y} + \frac{2}{x} = 24$.

## PROBLEM SOLVING

EXAMPLE 2
on p. 357
for Ex. 57

**57. TILING** The perimeter of a room is 66 feet. The ratio of its length to its width is 6 : 5. You want to tile the floor with 12 inch square tiles. Find the length and width of the room, and the area of the floor. How many tiles will you need? The tiles cost $1.98 each. What is the total cost to tile the floor?

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**58. GEARS** The *gear ratio* of two gears is the ratio of the number of teeth of the larger gear to the number of teeth of the smaller gear. In a set of three gears, the ratio of Gear A to Gear B is equal to the ratio of Gear B to Gear C. Gear A has 36 teeth and Gear C has 16 teeth. How many teeth does Gear B have?

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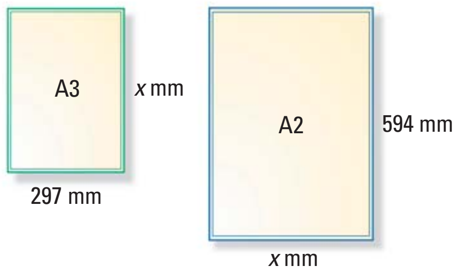
**59. TRAIL MIX** You need to make 36 one-half cup bags of trail mix for a class trip. The recipe calls for peanuts, chocolate chips, and raisins in the extended ratio 5 : 1 : 4. How many cups of each item do you need?

**60. PAPER SIZES** International standard paper sizes are commonly used all over the world. The various sizes all have the same width-to-length ratios. Two sizes of paper are shown, called A3 and A2. The distance labeled $x$ is the geometric mean of 297 mm and 594 mm. Find the value of $x$.




**61. BATTING AVERAGE** The batting average of a baseball player is the ratio of the number of hits to the number of official at-bats. In 2004, Johnny Damon of the Boston Red Sox had 621 official at-bats and a batting average of .304. Use the proportion to find the number of hits made by Johnny Damon.

$$\frac{\text{Number of hits}}{\text{Number of at-bats}} = \frac{\text{Batting average}}{1.000}$$

EXAMPLE 5
on p. 359
for Ex. 62

**62. MULTI-STEP PROBLEM** The population of Red-tailed hawks is increasing in many areas of the United States. One long-term survey of bird populations suggests that the Red-tailed hawk population is increasing nationally by 2.7% each year.

**a.** Write 2.7% as a ratio of hawks in year $n$ to hawks in year $(n - 1)$.

**b.** In 2004, observers in Corpus Christi, TX, spotted 180 migrating Red-tailed hawks. Assuming this population follows the national trend, about how many Red-tailed hawks can they expect to see in 2005?

**c.** Observers in Lipan Point, AZ, spotted 951 migrating Red-tailed hawks in 2004. Assuming this population follows the national trend, about how many Red-tailed hawks can they expect to see in 2006?

**63.** **TAKS REASONING** Some common computer screen resolutions are 1024 : 768, 800 : 600, and 640 : 480. *Explain* why these ratios are equivalent.

**64.** **BIOLOGY** The larvae of the Mother-of-Pearl moth is the fastest moving caterpillar. It can run at a speed of 15 inches per second. When threatened, it can curl itself up and roll away 40 times faster than it can run. How fast can it run in miles per hour? How fast can it roll?

**65.** **CURRENCY EXCHANGE** Emily took 500 U.S. dollars to the bank to exchange for Canadian dollars. The exchange rate on that day was 1.2 Canadian dollars per U.S. dollar. How many Canadian dollars did she get in exchange for the 500 U.S. dollars?

**66.** **MULTIPLE REPRESENTATIONS** Let $x$ and $y$ be two positive numbers whose geometric mean is 6.

**a.** **Making a Table** Make a table of ordered pairs $(x, y)$ such that $\sqrt{xy} = 6$.

**b.** **Drawing a Graph** Use the ordered pairs to make a scatter plot. Connect the points with a smooth curve.

**c.** **Analyzing Data** Is the data linear? Why or why not?

**67.** **ALGEBRA** Use algebra to verify Property 1, the Cross Products Property.

**68.** **ALGEBRA** Show that the geometric mean of two numbers is equal to the arithmetic mean (or average) of the two numbers only when the numbers are equal. (*Hint*: Solve $\sqrt{xy} = \frac{x + y}{2}$ with $x, y \geq 0$.)

**CHALLENGE In Exercises 69–71, use the given information to find the value(s) of *x*. Assume that the given quantities are nonnegative.**

**69.** The geometric mean of the quantities $(\sqrt{x})$ and $(3\sqrt{x})$ is $(x - 6)$.

**70.** The geometric mean of the quantities $(x + 1)$ and $(2x + 3)$ is $(x + 3)$.

**71.** The geometric mean of the quantities $(2x + 1)$ and $(6x + 1)$ is $(4x - 1)$.

## MIXED REVIEW FOR TAKS

TAKS PRACTICE at classzone.com

**REVIEW**
Skills Review Handbook p. 887; TAKS Workbook

**72.** **TAKS PRACTICE** Given the set of data {120, 80, 50, 50, 100, 150, 100, 120, 50, 130, 80, 110}, which statement best interprets the data? *TAKS Obj. 9*

Ⓐ The mean is 100.

Ⓑ The median is 95.

Ⓒ The mode and range are equal.

Ⓓ The median and range are equal.

**REVIEW**
Skills Review Handbook p. 882; TAKS Workbook

**73.** **TAKS PRACTICE** What are the solutions of the quadratic equation $2x^2 + 2x = 12$? *TAKS Obj. 5*

Ⓕ $x = 1$ and $x = -6$

Ⓖ $x = 6$ and $x = -1$

Ⓗ $x = 2$ and $x = -3$

Ⓙ $x = 3$ and $x = -2$

# 6.2 Use Proportions to Solve Geometry Problems

TEKS a.3, G.5.B, G.11.A, G.11.B

**Before** You wrote and solved proportions.

**Now** You will use proportions to solve geometry problems.

**Why?** So you can calculate building dimensions, as in Ex. 22.

**Key Vocabulary**
- **scale drawing**
- **scale**

In Lesson 6.1, you learned to use the Cross Products Property to write equations that are equivalent to a given proportion. Three more ways to do this are given by the properties below.

**REVIEW RECIPROCALS**
For help with reciprocals, see p. 869.

**KEY CONCEPT** *For Your Notebook*

## Additional Properties of Proportions

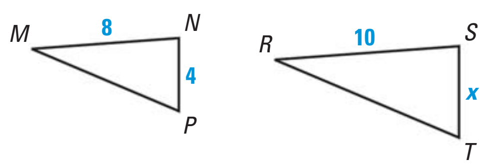
2. **Reciprocal Property** If two ratios are equal, then their reciprocals are also equal. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$.

3. If you interchange the means of a proportion, then you form another true proportion. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$.

4. In a proportion, if you add the value of each ratio's denominator to its numerator, then you form another true proportion. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$.

## EXAMPLE 1 Use properties of proportions

**In the diagram, $\frac{MN}{RS} = \frac{NP}{ST}$. Write four true proportions.**




**Solution**

Because $\frac{MN}{RS} = \frac{NP}{ST}$, then $\frac{8}{10} = \frac{4}{x}$.

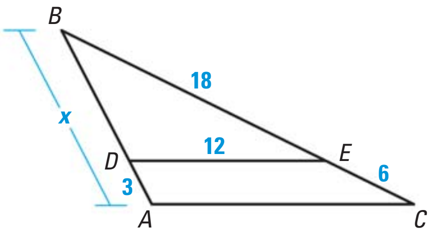
By the Reciprocal Property, the reciprocals are equal, so $\frac{10}{8} = \frac{x}{4}$.

By Property 3, you can interchange the means, so $\frac{8}{4} = \frac{10}{x}$.

By Property 4, you can add the denominators to the numerators, so $\frac{8+10}{10} = \frac{4+x}{x}$, or $\frac{18}{10} = \frac{4+x}{x}$.

## EXAMPLE 2 Use proportions with geometric figures

**ALGEBRA** **In the diagram, $\frac{BD}{DA} = \frac{BE}{EC}$. Find $BA$ and $BD$.**




**Solution**

$$\frac{BD}{DA} = \frac{BE}{EC}$$ Given

$$\frac{BD + DA}{DA} = \frac{BE + EC}{EC}$$ Property of Proportions (Property 4)

$$\frac{x}{3} = \frac{18 + 6}{6}$$ Substitution Property of Equality

$$6x = 3(18 + 6)$$ Cross Products Property

$$x = 12$$ Solve for $x$.

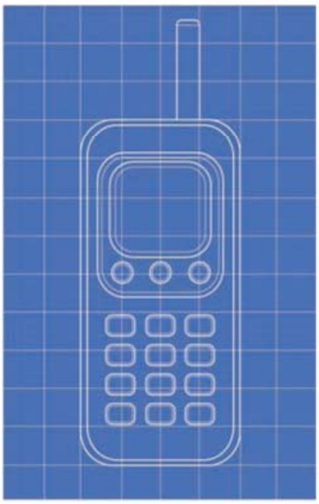
▶ So, $BA = 12$ and $BD = 12 - 3 = 9$.

Animated Geometry at classzone.com

**SCALE DRAWING** A **scale drawing** is a drawing that is the same shape as the object it represents. The **scale** is a ratio that describes how the dimensions in the drawing are related to the actual dimensions of the object.

## EXAMPLE 3 Find the scale of a drawing

**BLUEPRINTS** The blueprint shows a scale drawing of a cell phone. The length of the antenna on the blueprint is 5 centimeters. The actual length of the antenna is 2 centimeters. What is the scale of the blueprint?



**Solution**

To find the scale, write the ratio of a length in the drawing to an actual length, then rewrite the ratio so that the denominator is 1.

$$\frac{\text{length on blueprint}}{\text{length of antenna}} = \frac{5 \text{ cm}}{2 \text{ cm}} = \frac{5 \div 2}{2 \div 2} = \frac{2.5}{1}$$

▶ The scale of the blueprint is 2.5 cm : 1 cm.

## ✓ GUIDED PRACTICE for Examples 1, 2, and 3

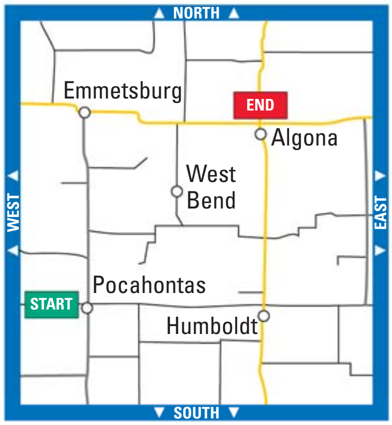
1. In Example 1, find the value of $x$.

2. In Example 2, $\frac{DE}{AC} = \frac{BE}{BC}$. Find $AC$.

3. **WHAT IF?** In Example 3, suppose the length of the antenna on the blueprint is 10 centimeters. Find the new scale of the blueprint.

## EXAMPLE 4 Use a scale drawing

**MAPS** The scale of the map at the right is 1 inch : 26 miles. Find the actual distance from Pocahontas to Algona.




### Solution

Use a ruler. The distance from Pocahontas to Algona on the map is about 1.25 inches. Let $x$ be the actual distance in miles.

$$\frac{1.25 \text{ in.}}{x \text{ mi}} = \frac{1 \text{ in.}}{26 \text{ mi}}$$ ← **distance on map** / ← **actual distance**

$x = 1.25(26)$ **Cross Products Property**

$x = 32.5$ **Simplify.**

▶ The actual distance from Pocahontas to Algona is about 32.5 miles.

## TAKS EXAMPLE 5 TAKS Reasoning: Multi-Step Problem

**SCALE MODEL** You buy a 3-D scale model of the Reunion Tower in Dallas, TX. The actual building is 560 feet tall. Your model is 10 inches tall, and the diameter of the dome on your scale model is about 2.1 inches.

**a.** What is the diameter of the actual dome?

**b.** About how many times as tall as your model is the actual building?

### Solution

**a.** $$\frac{10 \text{ in.}}{560 \text{ ft}} = \frac{2.1 \text{ in.}}{x \text{ ft}}$$ ← **measurement on model** / ← **measurement on actual building**

$10x = 1176$ **Cross Products Property**

$x = 117.6$ **Solve for *x*.**

▶ The diameter of the actual dome is about 118 feet.

**b.** To simplify a ratio with unlike units, multiply by a conversion factor.

$$\frac{560 \text{ ft}}{10 \text{ in.}} = \frac{560 \cancel{\text{ft}}}{10 \cancel{\text{in.}}} \cdot \frac{12 \cancel{\text{in.}}}{1 \cancel{\text{ft}}} = 672$$

▶ The actual building is 672 times as tall as the model.

### ✓ GUIDED PRACTICE for Examples 4 and 5

**4.** Two cities are 96 miles from each other. The cities are 4 inches apart on a map. Find the scale of the map.

**5.** **WHAT IF?** Your friend has a model of the Reunion Tower that is 14 inches tall. What is the diameter of the dome on your friend's model?

# 6.2 EXERCISES

**HOMEWORK KEY**

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 13, and 25

★ = **TAKS PRACTICE AND REASONING** Exs. 18, 24, 38, and 39

## SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A __?__ is a drawing that has the same shape as the object it represents.

2. **WRITING** Suppose the scale of a model of the Eiffel Tower is 1 inch : 20 feet. *Explain* how to determine how many times taller the actual tower is than the model.

**EXAMPLE 1** on p. 364 for Exs. 3–10

**REASONING Copy and complete the statement.**

3. If $\frac{8}{x} = \frac{3}{y}$, then $\frac{8}{3} = \frac{?}{?}$.

4. If $\frac{x}{9} = \frac{y}{20}$, then $\frac{x}{y} = \frac{?}{?}$.

5. If $\frac{x}{6} = \frac{y}{15}$, then $\frac{x+6}{6} = \frac{?}{?}$.

6. If $\frac{14}{3} = \frac{x}{y}$, then $\frac{17}{3} = \frac{?}{?}$.

**REASONING Decide whether the statement is *true* or *false*.**

7. If $\frac{8}{m} = \frac{n}{9}$, then $\frac{8+m}{m} = \frac{n+9}{9}$.

8. If $\frac{5}{7} = \frac{a}{b}$, then $\frac{7}{5} = \frac{a}{b}$.

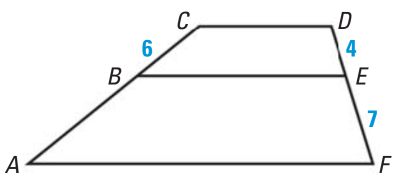
9. If $\frac{d}{2} = \frac{g+10}{11}$, then $\frac{d}{g+10} = \frac{2}{11}$.

10. If $\frac{4+x}{4} = \frac{3+y}{y}$, then $\frac{x}{4} = \frac{3}{y}$.

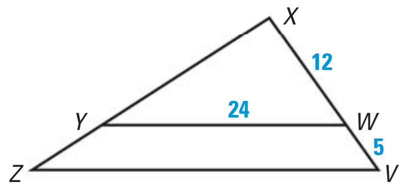
**EXAMPLE 2** on p. 365 for Exs. 11–12

**PROPERTIES OF PROPORTIONS Use the diagram and the given information to find the unknown length.**

11. Given $\frac{CB}{BA} = \frac{DE}{EF}$, find $BA$.




12. Given $\frac{XW}{XV} = \frac{YW}{ZV}$, find $ZV$.




**EXAMPLES 3 and 4** on pp. 365–366 for Exs. 13–14

**SCALE DIAGRAMS In Exercises 13 and 14, use the diagram of the field hockey field in which 1 inch = 50 yards. Use a ruler to approximate the dimension.**

13. Find the actual length of the field.

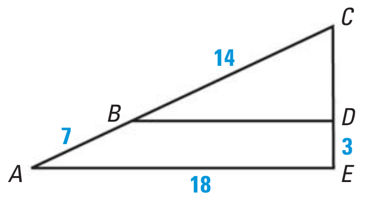
14. Find the actual width of the field.

15. **ERROR ANALYSIS** *Describe* and correct the error made in the reasoning.

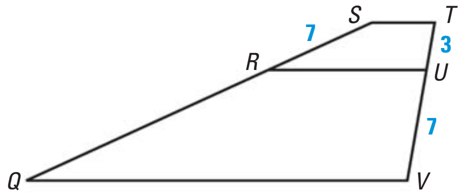
If $\frac{a}{3} = \frac{c}{4}$, then $\frac{a+3}{3} = \frac{c+3}{4}$. ✗

**PROPERTIES OF PROPORTIONS** **Use the diagram and the given information to find the unknown length.**

**16.** Given $\frac{CA}{CB} = \frac{AE}{BD}$, find $BD$.




**17.** Given $\frac{SQ}{SR} = \frac{TV}{TU}$, find $RQ$.




**18.** **TAKS REASONING** If $x$, $y$, $z$, and $q$ are four different numbers, and the proportion $\frac{x}{y} = \frac{z}{q}$ is true, which of the following is false?

Ⓐ $\frac{y}{x} = \frac{q}{z}$ Ⓑ $\frac{x}{z} = \frac{y}{q}$ Ⓒ $\frac{y}{x} = \frac{z}{q}$ Ⓓ $\frac{x+y}{y} = \frac{z+q}{q}$

**CHALLENGE** **Two number patterns are *proportional* if there is a nonzero number $k$ such that $(a_1, b_1, c_1, \ldots) = k(a_2, b_2, c_2, \ldots) = ka_2, kb_2, kc_2, \ldots$.**

**19.** Given the relationship $(8, 16, 20) = k(2, 4, 5)$, find $k$.

**20.** Given that $a_1 = ka_2$, $b_1 = kb_2$, and $c_1 = kc_2$, show that $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

**21.** Given that $a_1 = ka_2$, $b_1 = kb_2$, and $c_1 = kc_2$, show that $\frac{a_1 + b_1 + c_1}{a_2 + b_2 + c_2} = k$.

## PROBLEM SOLVING

**EXAMPLE 5** on p. 366 for Ex. 22

**22.** **ARCHITECTURE** A basket manufacturer has headquarters in an office building that has the same shape as a basket they sell.

**a.** The bottom of the basket is a rectangle with length 15 inches and width 10 inches. The base of the building is a rectangle with length 192 feet. What is the width of the base of the building?

**b.** About how many times as long as the bottom of the basket is the base of the building?

TEXAS @HomeTutor for problem solving help at classzone.com

**Longaberger Company Home Office Newark, Ohio**

**23.** **MAP SCALE** A street on a map is 3 inches long. The actual street is 1 mile long. Find the scale of the map.

TEXAS @HomeTutor for problem solving help at classzone.com

**24.** **TAKS REASONING** A model train engine is 12 centimeters long. The actual engine is 18 meters long. What is the scale of the model?

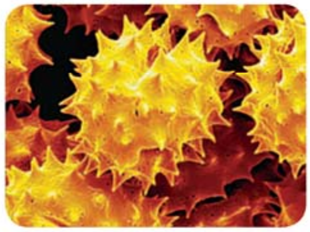
Ⓐ 3 cm : 2 m Ⓑ 1 cm : 1.5 m Ⓒ 1 cm : 3 m Ⓓ 200 cm : 3 m

**MAP READING** **The map of a hiking trail has a scale of 1 inch : 3.2 miles. Use a ruler to approximate the actual distance between the two shelters.**


**25.** Meadow View and Whispering Pines

**26.** Whispering Pines and Blueberry Hill

**27.** **POLLEN** The photograph shows a particle of goldenrod pollen that has been magnified under a microscope. The scale of the photograph is 900 : 1. Use a ruler to estimate the width in millimeters of the particle.



**RAMP DESIGN** **Assume that the wheelchair ramps described each have a slope of $\frac{1}{12}$, which is the maximum slope recommended for a wheelchair ramp.**

**28.** A wheelchair ramp has a 21 foot run. What is its rise?

**29.** A wheelchair ramp rises 4 feet. What is its run?

**30.** **STATISTICS** Researchers asked 4887 people to pick a number between 1 and 10. The results are shown in the table below.

| | | | | | |
|---|---|---|---|---|---|
| **Answer** | 1 | 2 | 3 | 4 | 5 |
| **Percent** | 4.2% | 5.1% | 11.4% | 10.5% | 10.7% |
| **Answer** | 6 | 7 | 8 | 9 | 10 |
| **Percent** | 10.0% | 27.2% | 8.8% | 6.0% | 6.1% |

**a.** Estimate the number of people who picked the number 3.

**b.** You ask a participant what number she picked. Is the participant more likely to answer 6 or 7? *Explain.*

**c.** Conduct this experiment with your classmates. Make a table in which you compare the new percentages with the ones given in the original survey. Why might they be different?

**xy ALGEBRA** **Use algebra to verify the property of proportions.**

**31.** Property 2

**32.** Property 3

**33.** Property 4

**REASONING** **Use algebra to *explain* why the property of proportions is true.**

**34.** If $\frac{a-b}{a+b} = \frac{c-d}{c+d}$, then $\frac{a}{b} = \frac{c}{d}$.

**35.** If $\frac{a+c}{b+d} = \frac{a-c}{b-d}$, then $\frac{a}{b} = \frac{c}{d}$.

**36.** If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, then $\frac{a+c+e}{b+d+f} = \frac{a}{b}$. (*Hint:* Let $\frac{a}{b} = r$.)

**37.** **CHALLENGE** When fruit is dehydrated, water is removed from the fruit. The water content in fresh apricots is about 86%. In dehydrated apricots, the water content is about 75%. Suppose 5 kilograms of raw apricots are dehydrated. How many kilograms of water are removed from the fruit? What is the approximate weight of the dehydrated apricots?

**TAKS PRACTICE** at classzone.com

## MIXED REVIEW FOR TAKS

**REVIEW**
Lesson 3.5;
TAKS Workbook

**38.** **TAKS PRACTICE** What is the $y$-intercept of the function $f(x) = -5(x+3)$? ***TAKS Obj. 3***

Ⓐ −15 Ⓑ −5 Ⓒ −3 Ⓓ 3

**REVIEW**
Skills Review Handbook p. 880;
TAKS Workbook

**39.** **TAKS PRACTICE** Tickets for Heather's school's basketball game cost \$4 for students and \$5.50 for non-students. A total of 535 tickets are sold for \$2455. Which system of equations can be used to find the number of student tickets, $s$, and the number of non-student tickets, $n$, that were sold? ***TAKS Obj. 4***

Ⓕ $s + n = 535$
$4s + 5.5n = 2455$

Ⓖ $s + n = 2455$
$4s + 5.5n = 535$

Ⓗ $s + n = 4$
$535s + 2455n = 5.5$

Ⓙ $4s + 5.5n = 535$
$5.5s + 4n = 2455$

## QUIZ *for Lessons 6.1–6.2*

**Solve the proportion.** *(p. 356)*

**1.** $\frac{10}{y} = \frac{5}{2}$ **2.** $\frac{x}{6} = \frac{9}{3}$ **3.** $\frac{1}{a+3} = \frac{4}{16}$ **4.** $\frac{6}{d-6} = \frac{4}{8}$

**Copy and complete the statement.** *(p. 364)*

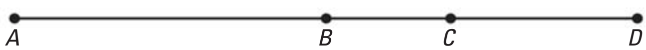
**5.** If $\frac{9}{x} = \frac{5}{2}$, then $\frac{9}{5} = \frac{?}{?}$.

**6.** If $\frac{x}{15} = \frac{y}{21}$, then $\frac{x}{y} = \frac{?}{?}$.

**7.** If $\frac{x}{8} = \frac{y}{12}$, then $\frac{x+8}{8} = \frac{?}{?}$.

**8.** If $\frac{32}{5} = \frac{x}{y}$, then $\frac{37}{5} = \frac{?}{?}$.

**9.** In the diagram, $AD = 10$, $B$ is the midpoint of $\overline{AD}$, and $AC$ is the geometric mean of $AB$ and $AD$. Find $AC$. *(p. 364)*




 **EXTRA PRACTICE** for Lesson 6.2, p. 906 **ONLINE QUIZ** at classzone.com

Investigating Geometry ACTIVITY *Use before Lesson 6.3*

# 6.3 Similar Polygons

TEKS *a.5, G.2.A, G.3.B, G.9.B*

**MATERIALS** • metric ruler • protractor

**QUESTION** **When a figure is reduced, how are the corresponding angles related? How are the corresponding lengths related?**

**EXPLORE** **Compare measures of lengths and angles in two photos**

**STEP 1** ***Measure segments*** Photo 2 is a reduction of Photo 1. In each photo, find *AB* to the nearest millimeter. Write the ratio of the length of $\overline{AB}$ in Photo 1 to the length of $\overline{AB}$ in Photo 2.

**Photo 1**

**STEP 2** ***Measure angles*** Use a protractor to find the measure of ∠1 in each photo. Write the ratio of $m\angle 1$ in Photo 1 to $m\angle 1$ in Photo 2.

**STEP 3** ***Find measurements*** Copy and complete the table. Use the same units for each measurement. Record your results in a table.

| Measurement | Photo 1 | Photo 2 | $\frac{\text{Photo 1}}{\text{Photo 2}}$ |
|---|---|---|---|
| *AB* | ? | ? | ? |
| *AC* | ? | ? | ? |
| *DE* | ? | ? | ? |
| $m\angle 1$ | ? | ? | ? |
| $m\angle 2$ | ? | ? | ? |

**Photo 2**

**DRAW CONCLUSIONS** **Use your observations to complete these exercises**

1. Make a conjecture about the relationship between corresponding lengths when a figure is reduced.
2. Make a conjecture about the relationship between corresponding angles when a figure is reduced.
3. Suppose the measure of an angle in Photo 2 is 35°. What is the measure of the corresponding angle in Photo 1?
4. Suppose a segment in Photo 2 is 1 centimeters long. What is the measure of the corresponding segment in Photo 1?
5. Suppose a segment in Photo 1 is 5 centimeters long. What is the measure of the corresponding segment in Photo 2?

# 6.3 Use Similar Polygons

TEKS G.11.A, G.11.B, G.11.C, G.11.D

**Before** You used proportions to solve geometry problems.

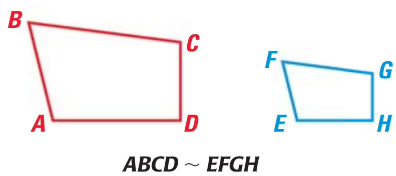
**Now** You will use proportions to identify similar polygons.

**Why?** So you can solve science problems, as in Ex. 34.

**Key Vocabulary**
- similar polygons
- scale factor

Two polygons are **similar polygons** if corresponding angles are congruent and corresponding side lengths are proportional.

In the diagram below, $ABCD$ is similar to $EFGH$. You can write "$ABCD$ is similar to $EFGH$" as $ABCD \sim EFGH$. Notice in the similarity statement that the corresponding vertices are listed in the same order.




**Corresponding angles**

$\angle A \cong \angle E$, $\angle B \cong \angle F$, $\angle C \cong \angle G$, and $\angle D \cong \angle H$

**Ratios of corresponding sides**

$$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE}$$

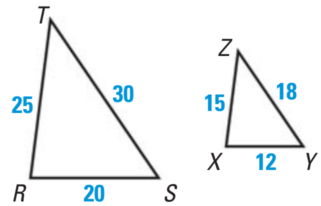
## EXAMPLE 1 Use similarity statements

**In the diagram, $\triangle RST \sim \triangle XYZ$.**

**a.** List all pairs of congruent angles.

**b.** Check that the ratios of corresponding side lengths are equal.

**c.** Write the ratios of the corresponding side lengths in a *statement of proportionality*.




**READ VOCABULARY**
In a *statement of proportionality*, any pair of ratios forms a true proportion.

**Solution**

**a.** $\angle R \cong \angle X$, $\angle S \cong \angle Y$, and $\angle T \cong \angle Z$.

**b.** $\frac{RS}{XY} = \frac{20}{12} = \frac{5}{3}$ $\qquad \frac{ST}{YZ} = \frac{30}{18} = \frac{5}{3}$ $\qquad \frac{TR}{ZX} = \frac{25}{15} = \frac{5}{3}$

**c.** Because the ratios in part (b) are equal, $\frac{RS}{XY} = \frac{ST}{YZ} = \frac{TR}{ZX}$.

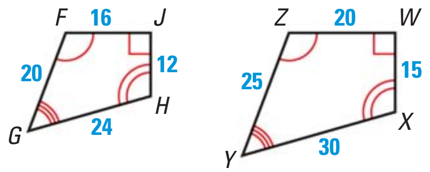
**GUIDED PRACTICE** for Example 1

**1.** Given $\triangle JKL \sim \triangle PQR$, list all pairs of congruent angles. Write the ratios of the corresponding side lengths in a statement of proportionality.

**SCALE FACTOR** If two polygons are similar, then the ratio of the lengths of two corresponding sides is called the **scale factor**. In Example 1, the common ratio of $\frac{5}{3}$ is the scale factor of $\triangle RST$ to $\triangle XYZ$.

## EXAMPLE 2 Find the scale factor

**Determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor of *ZYXW* to *FGHJ*.**




**Solution**

**STEP 1** **Identify** pairs of congruent angles. From the diagram, you can see that $\angle Z \cong \angle F$, $\angle Y \cong \angle G$, and $\angle X \cong \angle H$. Angles $W$ and $J$ are right angles, so $\angle W \cong \angle J$. So, the corresponding angles are congruent.

**STEP 2** **Show** that corresponding side lengths are proportional.

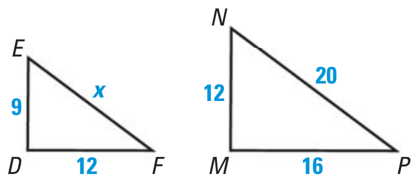
$$\frac{ZY}{FG} = \frac{25}{20} = \frac{5}{4} \qquad \frac{YX}{GH} = \frac{30}{24} = \frac{5}{4} \qquad \frac{XW}{HJ} = \frac{15}{12} = \frac{5}{4} \qquad \frac{WZ}{JF} = \frac{20}{16} = \frac{5}{4}$$

The ratios are equal, so the corresponding side lengths are proportional.

▶ So $ZYXW \sim FGHJ$. The scale factor of $ZYXW$ to $FGHJ$ is $\frac{5}{4}$.

## EXAMPLE 3 Use similar polygons

**ALGEBRA** **In the diagram, $\triangle DEF \sim \triangle MNP$. Find the value of $x$.**




**Solution**

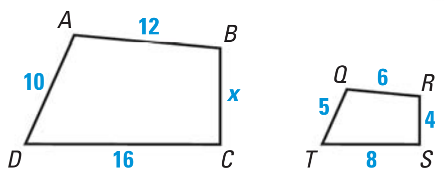
The triangles are similar, so the corresponding side lengths are proportional.

| | |
|---|---|
| $\frac{MN}{DE} = \frac{NP}{EF}$ | **Write proportion.** |
| $\frac{12}{9} = \frac{20}{x}$ | **Substitute.** |
| $12x = 180$ | **Cross Products Property** |
| $x = 15$ | **Solve for $x$.** |

**ANOTHER WAY**
There are several ways to write the proportion. For example, you could write $\frac{DF}{MP} = \frac{EF}{NP}$.

## ✓ GUIDED PRACTICE for Examples 2 and 3

**In the diagram, $ABCD \sim QRST$.**




2. What is the scale factor of $QRST$ to $ABCD$?

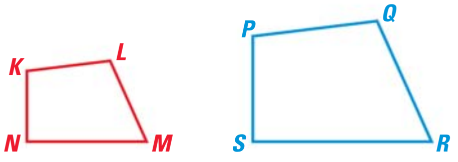
3. Find the value of $x$.

**PERIMETERS** The ratios of lengths in similar polygons is the same as the scale factor. Theorem 6.1 shows this is true for the perimeters of the polygons.

**THEOREM** *For Your Notebook*

**THEOREM 6.1 Perimeters of Similar Polygons**

If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.




If *KLMN* ~ *PQRS*, then $\frac{KL + LM + MN + NK}{PQ + QR + RS + SP} = \frac{KL}{PQ} = \frac{LM}{QR} = \frac{MN}{RS} = \frac{NK}{SP}$.

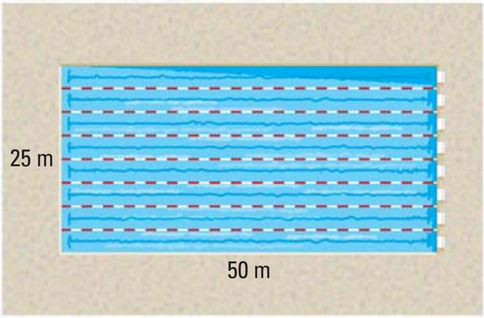
*Proof:* Ex. 38, p. 379

## EXAMPLE 4 Find perimeters of similar figures

**SWIMMING** A town is building a new swimming pool. An Olympic pool is rectangular with length 50 meters and width 25 meters. The new pool will be similar in shape, but only 40 meters long.

**a.** Find the scale factor of the new pool to an Olympic pool.

**b.** Find the perimeter of an Olympic pool and the new pool.




**Solution**

**a.** Because the new pool will be similar to an Olympic pool, the scale factor is the ratio of the lengths, $\frac{40}{50} = \frac{4}{5}$.

**b.** The perimeter of an Olympic pool is $2(50) + 2(25) = 150$ meters. You can use Theorem 6.1 to find the perimeter $x$ of the new pool.

$\frac{x}{150} = \frac{4}{5}$ **Use Theorem 6.1 to write a proportion.**

$x = 120$ **Multiply each side by 150 and simplify.**

▶ The perimeter of the new pool is 120 meters.

**ANOTHER WAY**
Another way to solve Example 4 is to write the scale factor as the decimal 0.8. Then, multiply the perimeter of the Olympic pool by the scale factor to get the perimeter of the new pool:
$0.8(150) = 120$.

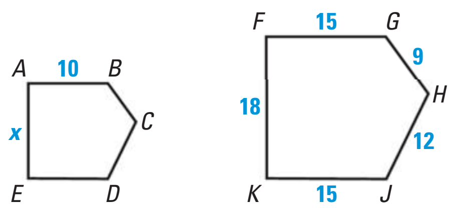
## ✓ GUIDED PRACTICE for Example 4

**In the diagram, *ABCDE* ~ *FGHJK*.**

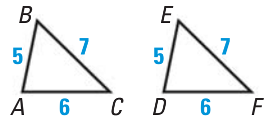
**4.** Find the scale factor of *FGHJK* to *ABCDE*.

**5.** Find the value of *x*.

**6.** Find the perimeter of *ABCDE*.



**SIMILARITY AND CONGRUENCE** Notice that any two congruent figures are also similar. Their scale factor is 1 : 1. In $\triangle ABC$ and $\triangle DEF$, the scale factor is $\frac{5}{5} = 1$. You can write $\triangle ABC \sim \triangle DEF$ and $\triangle ABC \cong \triangle DEF$.




**READ VOCABULARY**
For example, *corresponding lengths* in similar triangles include side lengths, altitudes, medians, midsegments, and so on.

**CORRESPONDING LENGTHS** You know that perimeters of similar polygons are in the same ratio as corresponding side lengths. You can extend this concept to other segments in polygons.

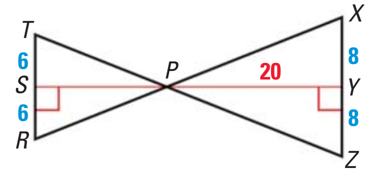
**KEY CONCEPT** *For Your Notebook*

### Corresponding Lengths in Similar Polygons

If two polygons are similar, then the ratio of any two corresponding lengths in the polygons is equal to the scale factor of the similar polygons.

## EXAMPLE 5 Use a scale factor

**In the diagram, $\triangle TPR \sim \triangle XPZ$. Find the length of the altitude $\overline{PS}$.**




**Solution**

First, find the scale factor of $\triangle TPR$ to $\triangle XPZ$.

$$\frac{TR}{XZ} = \frac{6+6}{8+8} = \frac{12}{16} = \frac{3}{4}$$

Because the ratio of the lengths of the altitudes in similar triangles is equal to the scale factor, you can write the following proportion.

$\frac{PS}{PY} = \frac{3}{4}$ **Write proportion.**

$\frac{PS}{20} = \frac{3}{4}$ **Substitute 20 for *PY*.**

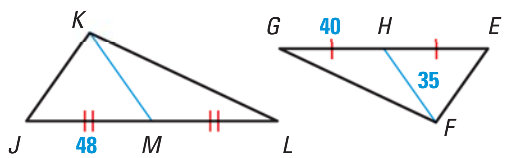
$PS = 15$ **Multiply each side by 20 and simplify.**

▶ The length of the altitude $\overline{PS}$ is 15.

Animated Geometry at classzone.com

✓ **GUIDED PRACTICE** for Example 5

**7.** In the diagram, $\triangle JKL \sim \triangle EFG$. Find the length of the median $\overline{KM}$.



# 6.3 EXERCISES

**HOMEWORK KEY**
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 7, and 31
★ = **TAKS PRACTICE AND REASONING** Exs. 6, 18, 27, 28, 35, 36, 37, 40, 41, and 42
◆ = **MULTIPLE REPRESENTATIONS** Ex. 33

## SKILL PRACTICE

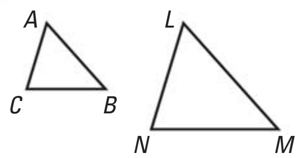
1. **VOCABULARY** Copy and complete: Two polygons are similar if corresponding angles are _?_ and corresponding side lengths are _?_.

2. **WRITING** If two polygons are congruent, must they be similar? If two polygons are similar, must they be congruent? *Explain*.

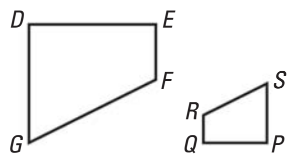
**EXAMPLE 1** on p. 372 for Exs. 3–6

**USING SIMILARITY** **List all pairs of congruent angles for the figures. Then write the ratios of the corresponding sides in a statement of proportionality.**

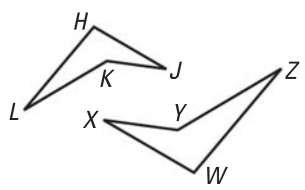
3. $\triangle ABC \sim \triangle LMN$




4. $DEFG \sim PQRS$




5. $HJKL \sim WXYZ$



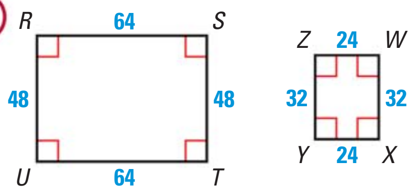

6. ★ **TAKS REASONING** Triangles *ABC* and *DEF* are similar. Which statement is *not* correct?

Ⓐ $\frac{BC}{EF} = \frac{BC}{EF}$ Ⓑ $\frac{AB}{DE} = \frac{CA}{FD}$ Ⓒ $\frac{CA}{FD} = \frac{BC}{EF}$ Ⓓ $\frac{AB}{EF} = \frac{BC}{DE}$

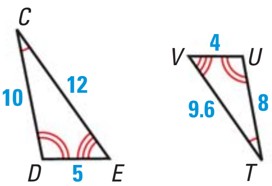
**EXAMPLES 2 and 3** on p. 373 for Exs. 7–10

**DETERMINING SIMILARITY** **Determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor.**

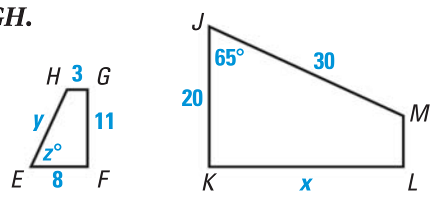
7.




8.




**USING SIMILAR POLYGONS** **In the diagram, $JKLM \sim EFGH$.**




9. Find the scale factor of *JKLM* to *EFGH*.

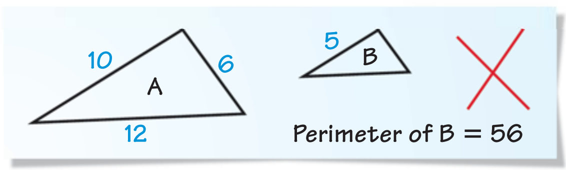
10. Find the values of *x*, *y*, and *z*.

**EXAMPLE 4** on p. 374 for Exs. 11–13

11. Find the perimeter of each polygon.

12. **PERIMETER** Two similar FOR SALE signs have a scale factor of 5 : 3. The large sign's perimeter is 60 inches. Find the small sign's perimeter.

13. **ERROR ANALYSIS** The triangles are similar. *Describe* and correct the error in finding the perimeter of Triangle B.



**REASONING Are the polygons *always*, *sometimes*, or *never* similar?**

**14.** Two isosceles triangles

**15.** Two equilateral triangles

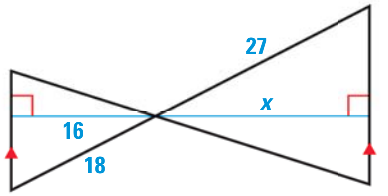
**16.** A right triangle and an isosceles triangle

**17.** A scalene triangle and an isosceles triangle

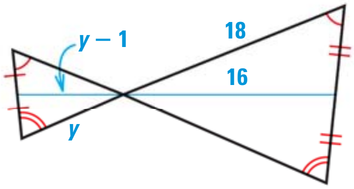
**18.** **TAKS REASONING** The scale factor of Figure A to Figure B is $1 : x$. What is the scale factor of Figure B to Figure A? *Explain* your reasoning.

**SIMILAR TRIANGLES Identify the type of special segment shown in blue, and find the value of the variable.**

**19.**




**20.**




**EXAMPLE 5** on p. 375 for Exs. 21–22

**USING SCALE FACTOR Triangles *NPQ* and *RST* are similar. The side lengths of $\triangle NPQ$ are 6 inches, 8 inches, and 10 inches, and the length of an altitude is 4.8 inches. The shortest side of $\triangle RST$ is 8 inches long.**

**21.** Find the lengths of the other two sides of $\triangle RST$.

**22.** Find the length of the corresponding altitude in $\triangle RST$.

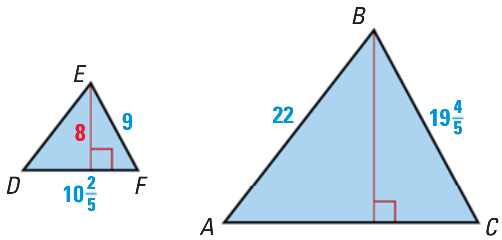
**USING SIMILAR TRIANGLES In the diagram, $\triangle ABC \sim \triangle DEF$.**

**23.** Find the scale factor of $\triangle ABC$ to $\triangle DEF$.

**24.** Find the unknown side lengths in both triangles.

**25.** Find the length of the altitude shown in $\triangle ABC$.

**26.** Find and compare the areas of both triangles.

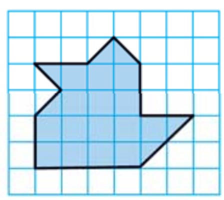



**27.** **TAKS REASONING** Suppose you are told that $\triangle PQR \sim \triangle XYZ$ and that the extended ratio of the angle measures in $\triangle PQR$ is $x : x + 30 : 3x$. Do you need to know anything about $\triangle XYZ$ to be able to write its extended ratio of angle measures? *Explain* your reasoning.

**28.** **TAKS REASONING** The lengths of the legs of right triangle *ABC* are 3 feet and 4 feet. The shortest side of $\triangle UVW$ is 4.5 feet and $\triangle UVW \sim \triangle ABC$. How long is the hypotenuse of $\triangle UVW$?

Ⓐ 1.5 ft　Ⓑ 5 ft　Ⓒ 6 ft　Ⓓ 7.5 ft

**29.** **CHALLENGE** Copy the figure at the right and divide it into two similar figures.

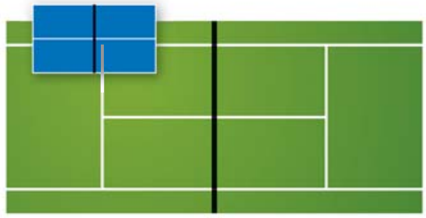


**30.** **REASONING** Is similarity reflexive? symmetric? transitive? Give examples to support your answers.

## PROBLEM SOLVING

EXAMPLE 2 on p. 373 for Exs. 31–32

31. **TENNIS** In table tennis, the table is a rectangle 9 feet long and 5 feet wide. A tennis court is a rectangle 78 feet long and 36 feet wide. Are the two surfaces similar? *Explain*. If so, find the scale factor of the tennis court to the table.



TEXAS @HomeTutor for problem solving help at classzone.com

32. **DIGITAL PROJECTOR** You are preparing a computer presentation to be digitally projected onto the wall of your classroom. Your computer screen is 13.25 inches wide and 10.6 inches high. The projected image on the wall is 53 inches wide and 42.4 inches high. Are the two shapes similar? If so, find the scale factor of the computer screen to the projected image.

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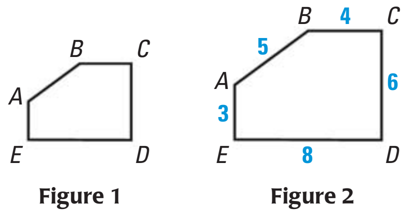
33. **MULTIPLE REPRESENTATIONS** Use the similar figures shown. The scale factor of Figure 1 to Figure 2 is 7 : 10.

a. **Making a Table** Copy and complete the table.

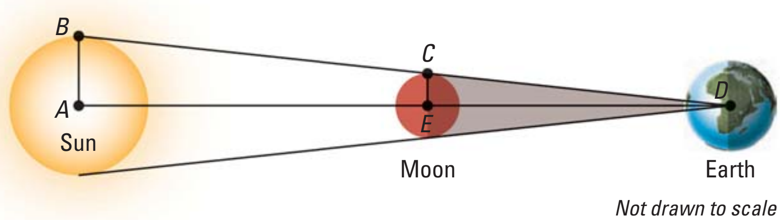
| | *AB* | *BC* | *CD* | *DE* | *EA* |
|---|---|---|---|---|---|
| **Figure 1** | 3.5 | ? | ? | ? | ? |
| **Figure 2** | 5.0 | 4.0 | 6.0 | 8.0 | 3.0 |




b. **Drawing a Graph** Graph the data in the table. Let $x$ represent the length of a side in Figure 1 and let $y$ represent the length of the corresponding side in Figure 2. Is the relationship linear?

c. **Writing an Equation** Write an equation that relates $x$ and $y$. What is its slope? How is the slope related to the scale factor?

34. **MULTI-STEP PROBLEM** During a total eclipse of the sun, the moon is directly in line with the sun and blocks the sun's rays. The distance *ED* between Earth and the moon is 240,000 miles, the distance *DA* between Earth and the sun is 93,000,000 miles, and the radius *AB* of the sun is 432,500 miles.




a. Copy the diagram and label the known distances.

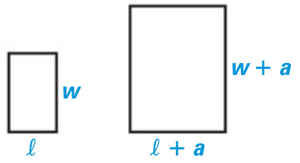
b. In the diagram, $\triangle BDA \sim \triangle CDE$. Use this fact to explain a total eclipse of the sun.

c. Estimate the radius *CE* of the moon.



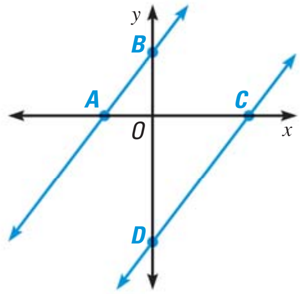
= WORKED-OUT SOLUTIONS on p. WS1 = TAKS PRACTICE AND REASONING = MULTIPLE REPRESENTATIONS

35. **TAKS REASONING** A rectangular image is enlarged on each side by the same amount. The angles remain unchanged. Can the larger image be similar to the original? *Explain* your reasoning, and give an example to support your answer.




36. **TAKS REASONING** How are the areas of similar rectangles related to the scale factor? Use examples to *justify* your reasoning.

37. **TAKS REASONING** The equations of two lines in the coordinate plane are $y = \frac{4}{3}x + 4$ and $y = \frac{4}{3}x - 8$.




**a.** *Explain* why the two lines are parallel.

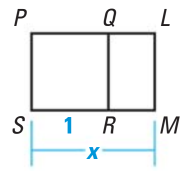
**b.** Show that $\angle BOA \cong \angle DOC$, $\angle OBA \cong \angle ODC$, and $\angle BAO \cong \angle DCO$.

**c.** Find the coordinates of points $A$, $B$, $C$, and $D$. Find the lengths of the sides of $\triangle AOB$ and $\triangle COD$.

**d.** Show that $\triangle AOB \sim \triangle COD$.

38. **PROVING THEOREM 6.1** Prove the Perimeters of Similar Polygons Theorem for similar rectangles. Include a diagram in your proof.

39. **CHALLENGE** In the diagram, $PQRS$ is a square, and $PLMS \sim LMRQ$. Find the exact value of $x$. This value is called the *golden ratio*. Golden rectangles have their length and width in this ratio. Show that the similar rectangles in the diagram are golden rectangles.

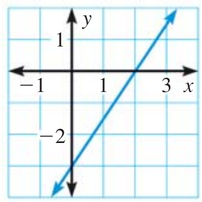



## MIXED REVIEW FOR TAKS

**TAKS PRACTICE** at classzone.com

**REVIEW**
Lesson 3.4;
TAKS Workbook

40. **TAKS PRACTICE** What is the rate of change of the graph? *TAKS Obj. 3*




(A) $\frac{3}{2}$  (B) $\frac{2}{3}$

(C) $-\frac{3}{2}$  (D) $-\frac{2}{3}$

**REVIEW**
Skills Review Handbook p. 894;
TAKS Workbook

41. **TAKS PRACTICE** Jesse buys a used set of drill bits at a garage sale. One of the bits is missing. Originally, the set contained seven bits whose diameter measures were in consecutive increments of $\frac{1}{32}$ inch. The remaining bits have the following diameter measures: $\frac{1}{16}$ inch, $\frac{3}{32}$ inch, $\frac{1}{8}$ inch, $\frac{3}{16}$ inch, $\frac{7}{32}$ inch, and $\frac{1}{4}$ inch. Which size bit is missing? *TAKS Obj. 10*

(F) $\frac{1}{32}$ in.  (G) $\frac{5}{32}$ in.  (H) $\frac{5}{16}$ in.  (J) $\frac{3}{8}$ in.

**REVIEW**
Skills Review Handbook p. 884;
TAKS Workbook

42. **TAKS PRACTICE** The function $y = 6x \mp 17$ has a domain of {0, 4, 7}. What is its range? *TAKS Obj. 2*

(A) {17, 7, 42}  V (B) {11, −7, 42}  (C) {−11, 6, 24}  (D) {−17, 7, 25}

# MIXED REVIEW FOR TEKS

TAKS PRACTICE
classzone.com

## Lessons 6.1–6.3

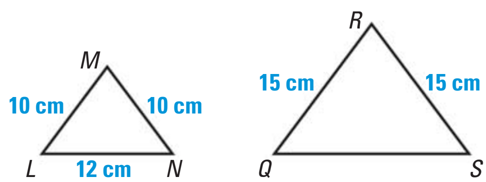
1. **LINE SEGMENTS** In the diagram, $AB:BC$ is $3:8$. What is $AC$? *TEKS G.11.B*


Ⓐ 1136 units Ⓑ 1278 units
Ⓒ 1562 units Ⓓ 3408 units

2. **SCALE MODEL** The Flatiron Building in New York City is 285 feet high and 190 feet wide along its Broadway front. A scale model of the building is 60 inches high. How wide is the model along the corresponding front? *TEKS G.11.B*

Ⓕ 1.5 ft. Ⓖ 4.75 ft.
Ⓗ 40 in. Ⓙ 90 in.

3. **PERIMETER** In the diagram, $\triangle LMN$ and $\triangle QRS$ are similar. What is the perimeter of $\triangle QRS$? *TEKS G.11.B*




Ⓐ $14\frac{2}{3}$ cm

Ⓑ 18 cm

Ⓒ 33 cm

Ⓓ 48 cm

4. **EXCHANGE RATES** Kelly is going on a trip to Mexico. She takes 150 U.S. dollars with her. In Mexico, she exchanges her U.S. dollars for Mexican pesos. During her stay, Kelly spends 640 pesos. How many Mexican pesos does she have left? *TEKS a.6*


Ⓕ 314.5 Mexican pesos

Ⓖ 799.5 Mexican pesos

Ⓗ 954.5 Mexican pesos

Ⓙ 1,594.5 Mexican pesos

5. **PEACHES** In the United States, 1504 million pounds of peaches were consumed in 2002, when the U.S. population was 290 million. The per capita consumption of peaches is the ratio of the total amount of peaches consumed to the population. What was the approximate per capita consumption of peaches in the U.S. in 2002? *TEKS a.6*

Ⓐ 3.5 pounds per person

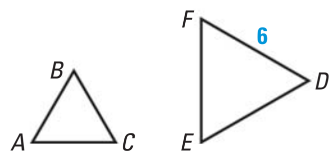
Ⓑ 5.2 pounds per person

Ⓒ 6.9 pounds per person

Ⓓ 10.4 pounds per person

### GRIDDED ANSWER

6. **SIDE LENGTH** In the diagram, $\triangle ABC$ and $\triangle DEF$ are similar. The scale factor of $\triangle ABC$ to $\triangle DEF$ is $2:3$. Find $AC$. *TEKS G.11.B*



# 6.4 Prove Triangles Similar by AA

TEKS G.1.A, G.2.A, G.3.E, G.11.C

**Before** You used the AAS Congruence Theorem.

**Now** You will use the AA Similarity Postulate.

**Why?** So you can use similar triangles to understand aerial photography, as in Ex. 34.

**Key Vocabulary**
- **similar polygons,** *p. 372*

## ACTIVITY ANGLES AND SIMILAR TRIANGLES

**QUESTION** What can you conclude about two triangles if you know two pairs of corresponding angles are congruent?

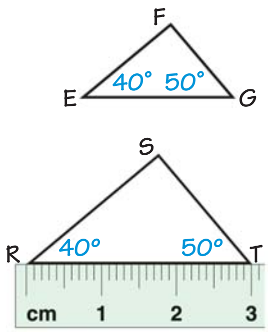
**Materials:**
- protractor
- metric ruler

**STEP 1** **Draw** $\triangle EFG$ so that $m\angle E = 40°$ and $m\angle G = 50°$.

**STEP 2** **Draw** $\triangle RST$ so that $m\angle R = 40°$ and $m\angle T = 50°$, and $\triangle RST$ is not congruent to $\triangle EFG$.

**STEP 3** **Calculate** $m\angle F$ and $m\angle S$ using the Triangle Sum Theorem. Use a protractor to check that your results are true.

**STEP 4** **Measure** and record the side lengths of both triangles. Use a metric ruler.




**DRAW CONCLUSIONS**

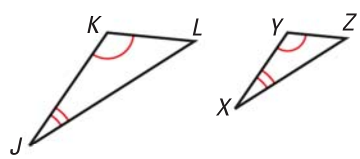
1. Are the triangles similar? Explain your reasoning.
2. Repeat the steps above using different angle measures. Make a conjecture about two triangles with two pairs of congruent corresponding angles.

**TRIANGLE SIMILARITY** The Activity suggests that two triangles are similar if two pairs of corresponding angles are congruent. In other words, you do not need to know the measures of the sides or the third pair of angles.

## POSTULATE *For Your Notebook*

**POSTULATE 22 Angle-Angle (AA) Similarity Postulate**

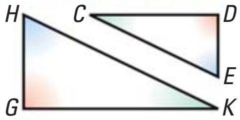
If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.




$\triangle JKL \sim \triangle XYZ$

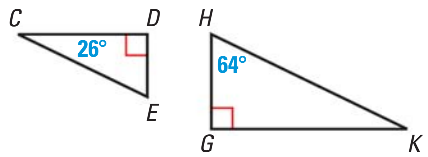
## EXAMPLE 1 Use the AA Similarity Postulate

**DRAW DIAGRAMS**




Use colored pencils to show congruent angles. This will help you write similarity statements.

**Determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.**




### Solution

Because they are both right angles, $\angle D$ and $\angle G$ are congruent.

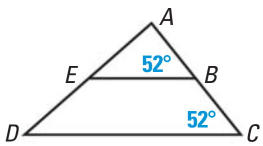
By the Triangle Sum Theorem, $26° + 90° + m\angle E = 180°$, so $m\angle E = 64°$. Therefore, $\angle E$ and $\angle H$ are congruent.

▶ So, $\triangle CDE \sim \triangle KGH$ by the AA Similarity Postulate.

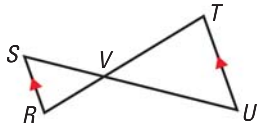
## EXAMPLE 2 Show that triangles are similar

**Show that the two triangles are similar.**

**a.** $\triangle ABE$ and $\triangle ACD$




**b.** $\triangle SVR$ and $\triangle UVT$



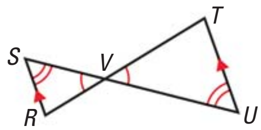

### Solution

**a.** You may find it helpful to redraw the triangles separately.

Because $m\angle ABE$ and $m\angle C$ both equal 52°, $\angle ABE \cong \angle C$. By the Reflexive Property, $\angle A \cong \angle A$.

▶ So, $\triangle ABE \sim \triangle ACD$ by the AA Similarity Postulate.

**b.** You know $\angle SVR \cong \angle UVT$ by the Vertical Angles Congruence Theorem. The diagram shows $\overline{RS} \parallel \overline{UT}$ so $\angle S \cong \angle U$ by the Alternate Interior Angles Theorem.

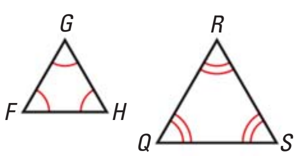



▶ So, $\triangle SVR \sim \triangle UVT$ by the AA Similarity Postulate.

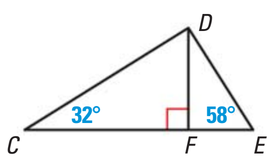
## ✓ GUIDED PRACTICE for Examples 1 and 2

**Show that the triangles are similar. Write a similarity statement.**

**1.** $\triangle FGH$ and $\triangle RQS$




**2.** $\triangle CDF$ and $\triangle DEF$




**3. REASONING** Suppose in Example 2, part (b), $\overline{SR} \nparallel \overline{TU}$. Could the triangles still be similar? *Explain.*

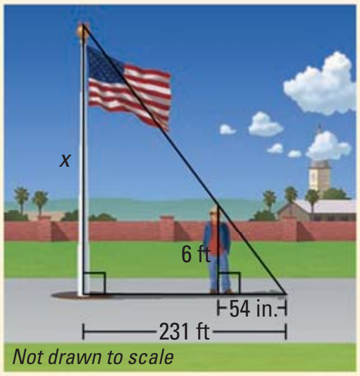
**INDIRECT MEASUREMENT** In Lesson 4.6, you learned a way to use congruent triangles to find measurements indirectly. Another useful way to find measurements indirectly is by using similar triangles.

## EXAMPLE 3 TAKS PRACTICE: Multiple Choice

A flagpole in Laredo, Texas, casts a shadow that is 231 feet long. At the same time, a man standing nearby who is six feet tall casts a shadow that is 54 inches long. How tall is the flagpole to the nearest foot?

Ⓐ 26 feet Ⓑ 173 feet

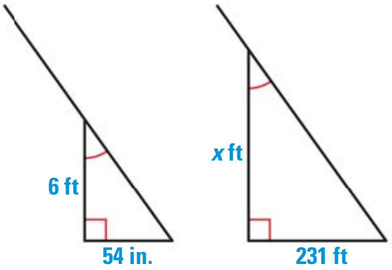
Ⓒ 257 feet Ⓓ 308 feet




**ELIMINATE CHOICES**
Notice that the man's height is greater than his shadow's length. So the flagpole must be taller than its shadow's length. Eliminate choices A and B.

### Solution

The flagpole and the man form sides of two right triangles with the ground, as shown below. The sun hits the flagpole and the man at the same angle. You have two pairs of congruent angles, so the triangles are similar by the AA Similarity Postulate.




You can use a proportion to find the height $x$. Write 6 feet as 72 inches so that you can form two ratios of feet to inches.

$$\frac{x \text{ ft}}{72 \text{ in.}} = \frac{231 \text{ ft}}{54 \text{ in.}}$$ **Write proportion of side lengths.**

$$54x = 72(231)$$ **Cross Products Property**

$$x = 308$$ **Solve for $x$.**

▶ The flagpole is 308 feet tall. The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

## ✓ GUIDED PRACTICE for Example 3

4. **WHAT IF?** A child who is 58 inches tall is standing next to the woman in Example 3. How long is the child's shadow?

5. You are standing in your backyard, and you measure the lengths of the shadows cast by both you and a tree. Write a proportion showing how you could find the height of the tree.

# 6.4 EXERCISES

**HOMEWORK KEY**
- ○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 13, and 33
- ★ = **TAKS PRACTICE AND REASONING** Exs. 16, 18, 19, 20, 33, 38, 41, and 42

## SKILL PRACTICE

**1. VOCABULARY** Copy and complete: If two angles of one triangle are congruent to two angles of another triangle, then the triangles are __?__.

**2. WRITING** Can you assume that corresponding sides and corresponding angles of any two similar triangles are congruent? *Explain.*

**EXAMPLE 1** on p. 382 for Exs. 3–11

**REASONING** **Use the diagram to complete the statement.**

**3.** $\triangle ABC \sim$ __?__

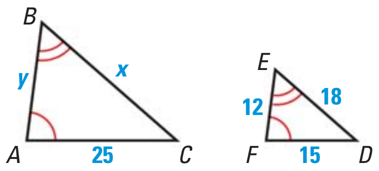
**4.** $\frac{BA}{?} = \frac{AC}{?} = \frac{CB}{?}$

**5.** $\frac{25}{?} = \frac{?}{12}$

**6.** $\frac{?}{25} = \frac{18}{?}$

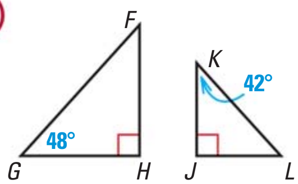
**7.** $y =$ __?__

**8.** $x =$ __?__

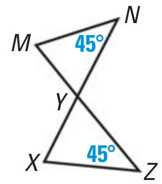



**AA SIMILARITY POSTULATE** **In Exercises 9–14, determine whether the triangles are similar. If they are, write a similarity statement.**

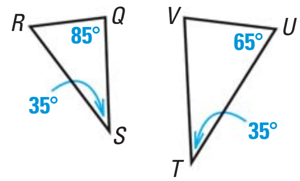
**9.**




**10.**

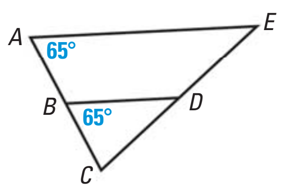



**11.**

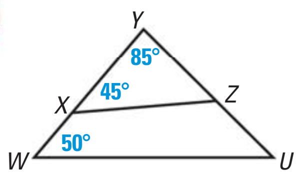



**EXAMPLE 2** on p. 382 for Exs. 12–16

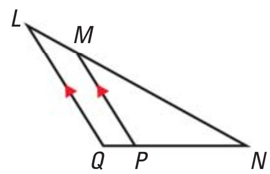
**12.**



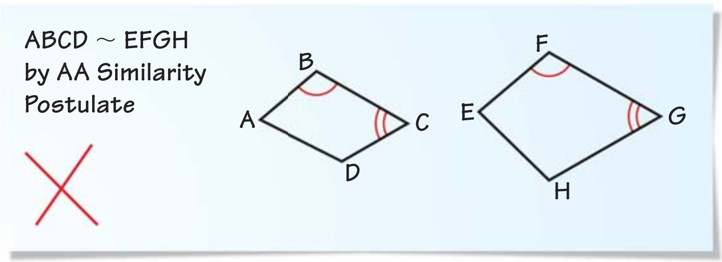

**13.**




**14.**



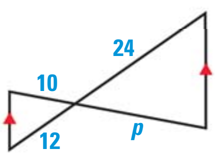

**15. ERROR ANALYSIS** *Explain* why the student's similarity statement is incorrect.



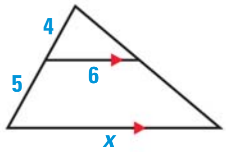

**16.** ★ **TAKS REASONING** What is the value of $p$?

Ⓐ 5 Ⓑ 20

Ⓒ 28.8 Ⓓ Cannot be determined



**17. ERROR ANALYSIS** A student uses the proportion $\frac{4}{6} = \frac{5}{x}$ to find the value of $x$ in the figure. *Explain* why this proportion is incorrect and write a correct proportion.

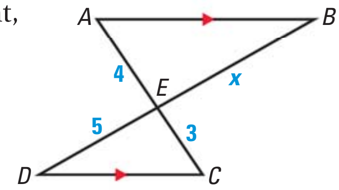



**TAKS REASONING** **In Exercises 18 and 19, make a sketch that can be used to show that the statement is false.**

**18.** If two pairs of sides of two triangles are congruent, then the triangles are similar.

**19.** If the ratios of two pairs of sides of two triangles are proportional, then the triangles are similar.

**20. TAKS REASONING** In the figure at the right, find the length of $\overline{BD}$.




Ⓐ $\frac{35}{3}$ Ⓑ $\frac{37}{5}$

Ⓒ $\frac{20}{3}$ Ⓓ $\frac{12}{5}$

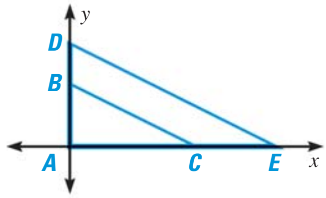
**xy ALGEBRA** **Find coordinates for point *E* so that $\triangle ABC \sim \triangle ADE$.**

**21.** $A(0, 0)$, $B(0, 4)$, $C(8, 0)$, $D(0, 5)$, $E(x, y)$

**22.** $A(0, 0)$, $B(0, 3)$, $C(4, 0)$, $D(0, 7)$, $E(x, y)$

**23.** $A(0, 0)$, $B(0, 1)$, $C(6, 0)$, $D(0, 4)$, $E(x, y)$

**24.** $A(0, 0)$, $B(0, 6)$, $C(3, 0)$, $D(0, 9)$, $E(x, y)$



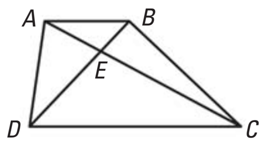

**25. MULTI-STEP PROBLEM** In the diagram, $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$, $AE = 6$, $AB = 8$, $CE = 15$, and $DE = 10$.

**a.** Copy the diagram and mark all given information.

**b.** List two pairs of congruent angles in the diagram.

**c.** Name a pair of similar triangles and write a similarity statement.

**d.** Find $BE$ and $DC$.




**REASONING** **In Exercises 26–29, is it possible for $\triangle JKL$ and $\triangle XYZ$ to be similar? *Explain* why or why not.**

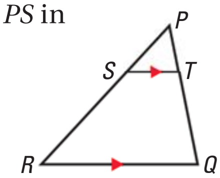
**26.** $m\angle J = 71°$, $m\angle K = 52°$, $m\angle X = 71°$, and $m\angle Z = 57°$

**27.** $\triangle JKL$ is a right triangle and $m\angle X + m\angle Y = 150°$.

**28.** $m\angle J = 87°$ and $m\angle Y = 94°$

**29.** $m\angle J + m\angle K = 85°$ and $m\angle Y + m\angle Z = 80°$

**30. CHALLENGE** If $PT = x$, $PQ = 3x$, and $SR = \frac{8}{3}x$, find $PS$ in terms of $x$. *Explain* your reasoning.

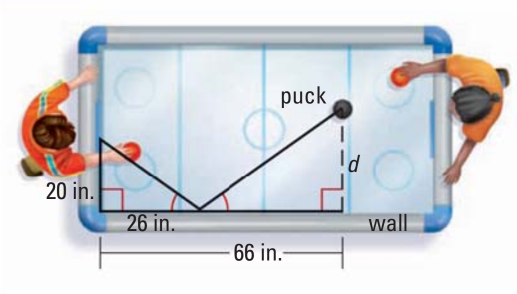


## PROBLEM SOLVING

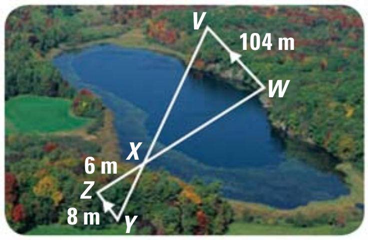
**EXAMPLE 3** on p. 383 for Exs. 31–32

31. **AIR HOCKEY** An air hockey player returns the puck to his opponent by bouncing the puck off the wall of the table as shown. From physics, the angles that the path of the puck makes with the wall are congruent. What is the distance $d$ between the puck and the wall when the opponent returns it?

TEXAS @HomeTutor for problem solving help at classzone.com




32. **LAKES** You can measure the width of the lake using a surveying technique, as shown in the diagram.
    a. What postulate or theorem can you use to show that the triangles are similar?
    b. Find the width of the lake, $WX$.
    c. If $XY = 10$ meters, find $VX$.

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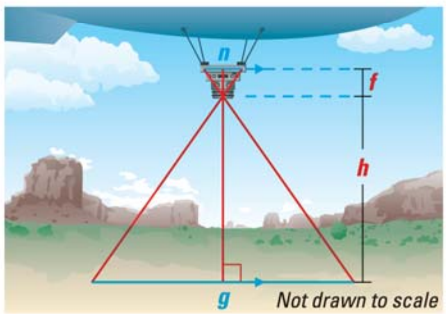
33. **TAKS REASONING** *Explain* why all equilateral triangles are similar. Include sketches in your answer.

34. **AERIAL PHOTOGRAPHY** Low-level aerial photos can be taken using a remote-controlled camera suspended from a blimp. You want to take an aerial photo that covers a ground distance $g$ of 50 meters. Use the proportion $\frac{f}{h} = \frac{n}{g}$ to estimate the altitude $h$ that the blimp should fly at to take the photo. In the proportion, use $f = 8$ centimeters and $n = 3$ centimeters. These two variables are determined by the type of camera used.



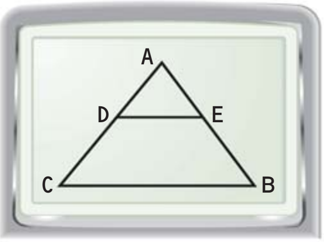

35. **PROOF** Use the given information to draw a sketch. Then write a proof.

**GIVEN ▶** $\triangle STU \sim \triangle PQR$
Point $V$ lies on $\overline{TU}$ so that $\overline{SV}$ bisects $\angle TSU$.
Point $N$ lies on $\overline{QR}$ so that $\overline{PN}$ bisects $\angle QPR$.

**PROVE ▶** $\frac{SV}{PN} = \frac{ST}{PQ}$

36. **PROOF** Prove that if an acute angle in one right triangle is congruent to an acute angle in another right triangle, then the triangles are similar.

**37. TECHNOLOGY** Use a graphing calculator or computer.




**a.** Draw $\triangle ABC$. Draw $\overline{DE}$ through two sides of the triangle, parallel to the third side.

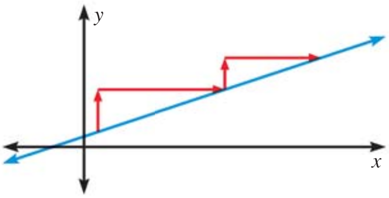
**b.** Measure $\angle ADE$ and $\angle ACB$. Measure $\angle AED$ and $\angle ABC$. What do you notice?

**c.** What does a postulate in this lesson tell you about $\triangle ADE$ and $\triangle ACB$?

**d.** Measure all the sides. Show that corresponding side lengths are proportional.

**e.** Move vertex $A$ to form new triangles. How do your measurements in parts (b) and (d) change? Are the new triangles still similar? *Explain*.

**38. TAKS REASONING** *Explain* how you could use similar triangles to show that any two points on a line can be used to calculate its slope.




**39. CORRESPONDING LENGTHS** Without using the Corresponding Lengths Property on page 375, prove that the ratio of two corresponding angle bisectors in similar triangles is equal to the scale factor.

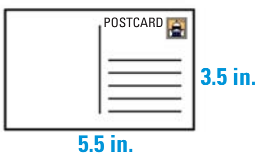
**40. CHALLENGE** Prove that if the lengths of two sides of a triangle are $a$ and $b$ respectively, then the lengths of the corresponding altitudes to those sides are in the ratio $\frac{b}{a}$.

## MIXED REVIEW FOR TAKS

**TAKS PRACTICE** at classzone.com

**REVIEW**
Lesson 6.2;
TAKS Workbook

**41. TAKS PRACTICE** Which of the following dimensions are *not* proportional to the dimensions of the postcard? *TAKS Obj. 6*




Ⓐ 11 in. by 7 in. Ⓑ 6.6 in. by 4.2 in.

Ⓒ 5 in. by 3 in. Ⓓ 4.4 in. by 2.8 in.

**REVIEW**
Skills Review Handbook p. 887;
TAKS Workbook

**42. TAKS PRACTICE** Janice's scores for six games in a bowling tournament are shown below. Which calculation gives Janice the highest final score? *TAKS Obj. 9*

120, 142, 158, 138, 142, 156

Ⓕ Mean Ⓖ Median Ⓗ Mode Ⓙ Range

# 6.5 Prove Triangles Similar by SSS and SAS

TEKS a.3, G.1.A, G.3.E, G.11.C

**Before** You used the AA Similarity Postulate to prove triangles similar.

**Now** You will use the SSS and SAS Similarity Theorems.

**Why?** So you can show that triangles are similar, as in Ex. 28.

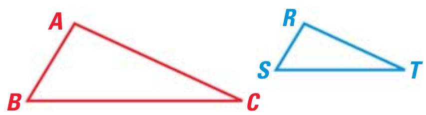
**Key Vocabulary**
- **ratio,** *p. 356*
- **proportion,** *p. 358*
- **similar polygons,** *p. 372*

In addition to using congruent corresponding angles to show that two triangles are similar, you can use proportional corresponding side lengths.

**THEOREM** *For Your Notebook*

**THEOREM 6.2 Side-Side-Side (SSS) Similarity Theorem**

If the corresponding side lengths of two triangles are proportional, then the triangles are similar.

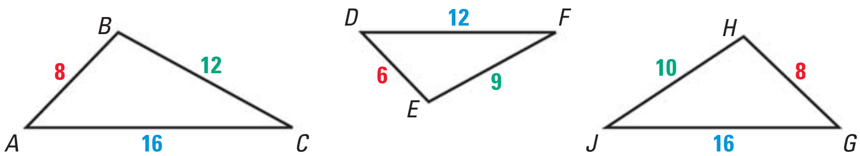



If $\frac{AB}{RS} = \frac{BC}{ST} = \frac{CA}{TR}$, then $\triangle ABC \sim \triangle RST$.

*Proof:* p. 389

## EXAMPLE 1 Use the SSS Similarity Theorem

**Is either $\triangle DEF$ or $\triangle GHJ$ similar to $\triangle ABC$?**




**Solution**

**APPLY THEOREMS**
When using the SSS Similarity Theorem, compare the shortest sides, the longest sides, and then the remaining sides.

Compare $\triangle ABC$ and $\triangle DEF$ by finding ratios of corresponding side lengths.

**Shortest sides** $\frac{AB}{DE} = \frac{8}{6} = \frac{4}{3}$

**Longest sides** $\frac{CA}{FD} = \frac{16}{12} = \frac{4}{3}$

**Remaining sides** $\frac{BC}{EF} = \frac{12}{9} = \frac{4}{3}$

▶ All of the ratios are equal, so $\triangle ABC \sim \triangle DEF$.

Compare $\triangle ABC$ and $\triangle GHJ$ by finding ratios of corresponding side lengths.

**Shortest sides** $\frac{AB}{GH} = \frac{8}{8} = 1$

**Longest sides** $\frac{CA}{JG} = \frac{16}{16} = 1$

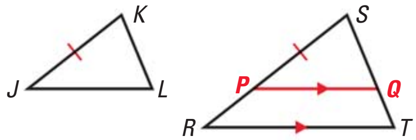
**Remaining sides** $\frac{BC}{HJ} = \frac{12}{10} = \frac{6}{5}$

▶ The ratios are not all equal, so $\triangle ABC$ and $\triangle GHJ$ are not similar.

## PROOF SSS Similarity Theorem

**GIVEN** ▶ $\frac{RS}{JK} = \frac{ST}{KL} = \frac{TR}{LJ}$

**PROVE** ▶ $\triangle RST \sim \triangle JKL$




**USE AN AUXILIARY LINE**

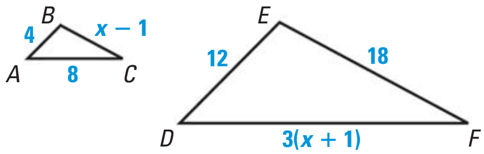
The Parallel Postulate allows you to draw an auxiliary line $\overleftrightarrow{PQ}$ in $\triangle RST$. There is only one line through point $P$ parallel to $\overleftrightarrow{RT}$, so you are able to draw it.

Locate $P$ on $\overline{RS}$ so that $PS = JK$. Draw $\overline{PQ}$ so that $\overline{PQ} \parallel \overline{RT}$. Then $\triangle RST \sim \triangle PSQ$ by the AA Similarity Postulate, and $\frac{RS}{PS} = \frac{ST}{SQ} = \frac{TR}{QP}$.

You can use the given proportion and the fact that $PS = JK$ to deduce that $SQ = KL$ and $QP = LJ$. By the SSS Congruence Postulate, it follows that $\triangle PSQ \cong \triangle JKL$. Finally, use the definition of congruent triangles and the AA Similarity Postulate to conclude that $\triangle RST \sim \triangle JKL$.

## EXAMPLE 2 Use the SSS Similarity Theorem

**xy ALGEBRA** **Find the value of $x$ that makes $\triangle ABC \sim \triangle DEF$.**




**Solution**

**STEP 1** **Find** the value of $x$ that makes corresponding side lengths proportional.

**CHOOSE A METHOD**

You can use either $\frac{AB}{DE} = \frac{BC}{EF}$ or $\frac{AB}{DE} = \frac{AC}{DF}$ in Step 1.

| | |
|---|---|
| $\frac{4}{12} = \frac{x-1}{18}$ | **Write proportion.** |
| $4 \cdot 18 = 12(x - 1)$ | **Cross Products Property** |
| $72 = 12x - 12$ | **Simplify.** |
| $7 = x$ | **Solve for $x$.** |

**STEP 2** **Check** that the side lengths are proportional when $x = 7$.

$BC = x - 1 = 6$ $\qquad$ $DF = 3(x + 1) = 24$

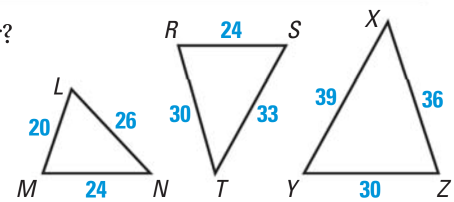
$\frac{AB}{DE} \stackrel{?}{=} \frac{BC}{EF} \rightarrow \frac{4}{12} = \frac{6}{18}$ ✓ $\qquad$ $\frac{AB}{DE} \stackrel{?}{=} \frac{AC}{DF} \rightarrow \frac{4}{12} = \frac{8}{24}$ ✓

▶ When $x = 7$, the triangles are similar by the SSS Similarity Theorem.

## ✓ GUIDED PRACTICE for Examples 1 and 2

1. Which of the three triangles are similar? Write a similarity statement.

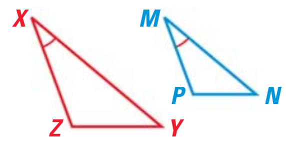
2. The shortest side of a triangle similar to $\triangle RST$ is 12 units long. Find the other side lengths of the triangle.



## THEOREM — *For Your Notebook*

**THEOREM 6.3 Side-Angle-Side (SAS) Similarity Theorem**

If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.

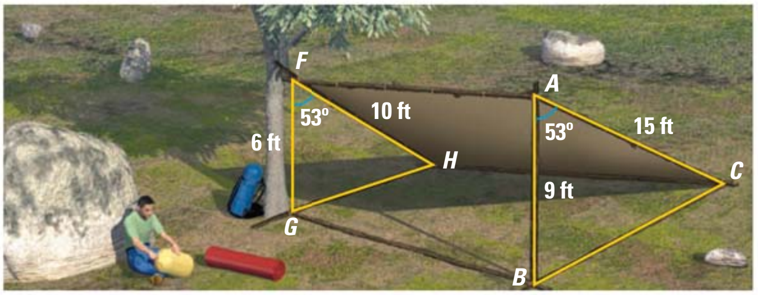



If $\angle X \cong \angle M$ and $\frac{ZX}{PM} = \frac{XY}{MN}$, then $\triangle XYZ \sim \triangle MNP$.

*Proof:* Ex. 37, p. 395

## EXAMPLE 3 Use the SAS Similarity Theorem

**LEAN-TO SHELTER** You are building a lean-to shelter starting from a tree branch, as shown. Can you construct the right end so it is similar to the left end using the angle measure and lengths shown?




**Solution**

Both $m\angle A$ and $m\angle F$ equal 53°, so $\angle A \cong \angle F$. Next, compare the ratios of the lengths of the sides that include $\angle A$ and $\angle F$.

**Shorter sides** $\frac{AB}{FG} = \frac{9}{6} = \frac{3}{2}$  **Longer sides** $\frac{AC}{FH} = \frac{15}{10} = \frac{3}{2}$

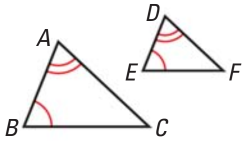
The lengths of the sides that include $\angle A$ and $\angle F$ are proportional.

▶ So, by the SAS Similarity Theorem, $\triangle ABC \sim \triangle FGH$. Yes, you can make the right end similar to the left end of the shelter.

## CONCEPT SUMMARY — *For Your Notebook*

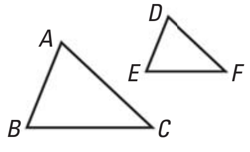
**Triangle Similarity Postulate and Theorems**

**AA Similarity Postulate**



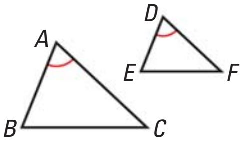

If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.

**SSS Similarity Theorem**




If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

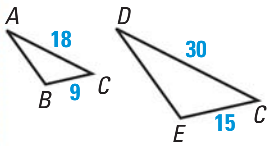
**SAS Similarity Theorem**



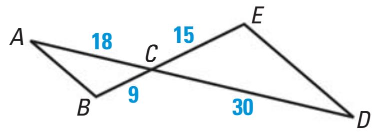

If $\angle A \cong \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

## EXAMPLE 4 Choose a method

**VISUAL REASONING**
To identify corresponding parts, redraw the triangles so that the corresponding parts have the same orientation.




**Tell what method you would use to show that the triangles are similar.**




### Solution

Find the ratios of the lengths of the corresponding sides.

**Shorter sides** $\frac{BC}{EC} = \frac{9}{15} = \frac{3}{5}$ **Longer sides** $\frac{CA}{CD} = \frac{18}{30} = \frac{3}{5}$

The corresponding side lengths are proportional. The included angles $\angle ACB$ and $\angle DCE$ are congruent because they are vertical angles. So, $\triangle ACB \sim \triangle DCE$ by the SAS Similarity Theorem.

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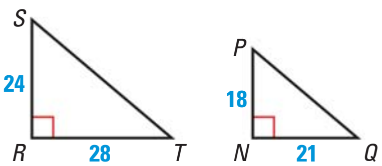
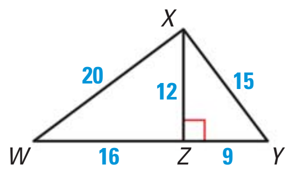
## ✓ GUIDED PRACTICE for Examples 3 and 4

***Explain*** **how to show that the indicated triangles are similar.**

**3.** $\triangle SRT \sim \triangle PNQ$




**4.** $\triangle XZW \sim \triangle YZX$




# 6.5 EXERCISES

**HOMEWORK KEY**
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 7, and 31
★ = **TAKS PRACTICE AND REASONING** Exs. 14, 32, 34, 39, and 40

## SKILL PRACTICE

**1. VOCABULARY** You plan to prove that $\triangle ACB$ is similar to $\triangle PXQ$ by the SSS Similarity Theorem. Copy and complete the proportion that is needed to use this theorem: $\frac{AC}{?} = \frac{?}{XQ} = \frac{AB}{?}$.

**2. WRITING** If you know two triangles are similar by the SAS Similarity Theorem, what additional piece(s) of information would you need to know to show that the triangles are congruent?

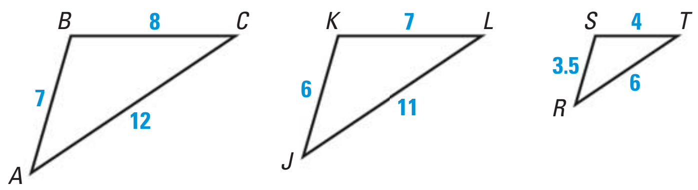
**EXAMPLES 1 and 2**
on pp. 388–389 for Exs. 3–6

**SSS SIMILARITY THEOREM** **Verify that $\triangle ABC \sim \triangle DEF$. Find the scale factor of $\triangle ABC$ to $\triangle DEF$.**

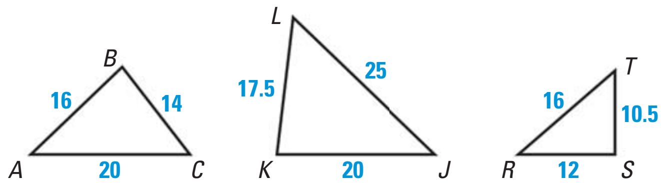
**3.** $\triangle ABC$: $BC = 18$, $AB = 15$, $AC = 12$
$\triangle DEF$: $EF = 12$, $DE = 10$, $DF = 8$

**4.** $\triangle ABC$: $AB = 10$, $BC = 16$, $CA = 20$
$\triangle DEF$: $DE = 25$, $EF = 40$, $FD = 50$

5. **SSS SIMILARITY THEOREM** Is either $\triangle JKL$ or $\triangle RST$ similar to $\triangle ABC$?



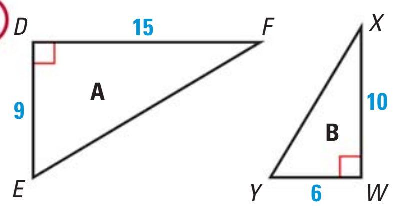

6. **SSS SIMILARITY THEOREM** Is either $\triangle JKL$ or $\triangle RST$ similar to $\triangle ABC$?



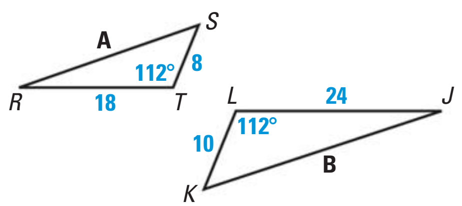

**EXAMPLE 3**
on p. 390
for Exs. 7–9

**SAS SIMILARITY THEOREM** **Determine whether the two triangles are similar. If they are similar, write a similarity statement and find the scale factor of Triangle B to Triangle A.**

7.




8.

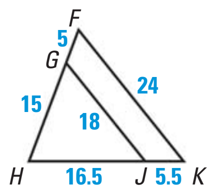



9. **ALGEBRA** Find the value of $n$ that makes $\triangle PQR \sim \triangle XYZ$ when $PQ = 4$, $QR = 5$, $XY = 4(n + 1)$, $YZ = 7n - 1$, and $\angle Q \cong \angle Y$. Include a sketch.

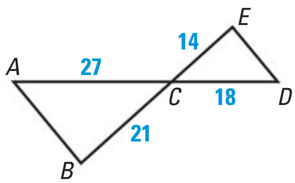
**EXAMPLE 4**
on p. 391
for Exs. 10–12

**SHOWING SIMILARITY** **Show that the triangles are similar and write a similarity statement. *Explain* your reasoning.**

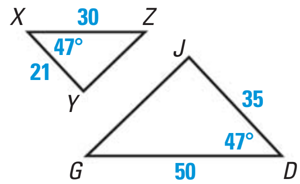
10.



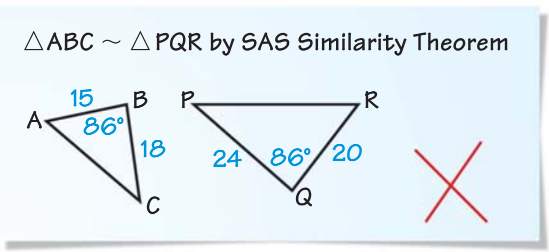

11.



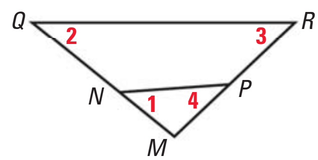

12.




13. **ERROR ANALYSIS** *Describe* and correct the student's error in writing the similarity statement.




14. **TAKS REASONING** In the diagram, $\frac{MN}{MR} = \frac{MP}{MQ}$. Which of the statements must be true?




Ⓐ $\angle 1 \cong \angle 2$ Ⓑ $\overline{QR} \parallel \overline{NP}$

Ⓒ $\angle 1 \cong \angle 4$ Ⓓ $\triangle MNP \sim \triangle MRQ$



○ = WORKED-OUT SOLUTIONS on p. WS1 = TAKS PRACTICE AND REASONING

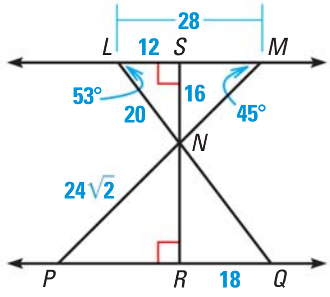
**DRAWING TRIANGLES** **Sketch the triangles using the given description. *Explain* whether the two triangles can be similar.**

**15.** In $\triangle XYZ$, $m\angle X = 66°$ and $m\angle Y = 34°$. In $\triangle LMN$, $m\angle M = 34°$ and $m\angle N = 80°$.

**16.** In $\triangle RST$, $RS = 20$, $ST = 32$, and $m\angle S = 16°$. In $\triangle FGH$, $GH = 30$, $HF = 48$, and $m\angle H = 24°$.

**17.** The side lengths of $\triangle ABC$ are 24, $8x$, and 54, and the side lengths of $\triangle DEF$ are 15, 25, and $7x$.

**FINDING MEASURES** **In Exercises 18–23, use the diagram to copy and complete the statements.**




**18.** $m\angle NQP = \underline{\ ?\ }$

**19.** $m\angle QPN = \underline{\ ?\ }$

**20.** $m\angle PNQ = \underline{\ ?\ }$

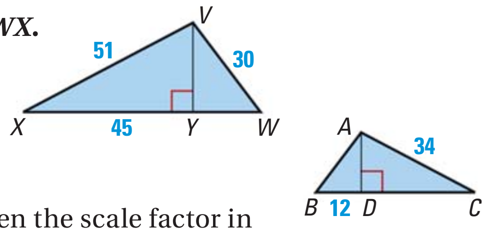
**21.** $RN = \underline{\ ?\ }$

**22.** $PQ = \underline{\ ?\ }$

**23.** $NM = \underline{\ ?\ }$

**24.** **SIMILAR TRIANGLES** In the diagram at the right, name the three pairs of triangles that are similar.

**CHALLENGE** **In the figure at the right, $\triangle ABC \sim \triangle VWX$.**




**25.** Find the scale factor of $\triangle VWX$ to $\triangle ABC$.

**26.** Find the ratio of the area of $\triangle VWX$ to the area of $\triangle ABC$.

**27.** Make a conjecture about the relationship between the scale factor in Exercise 25 and the ratio in Exercise 26. *Justify* your conjecture.

## PROBLEM SOLVING

**28.** **RACECAR NET** Which postulate or theorem could you use to show that the three triangles that make up the racecar window net are similar? *Explain*.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**EXAMPLE 1** on p. 388 for Ex. 29

**29.** **STAINED GLASS** Certain sections of stained glass are sold in triangular *beveled* pieces. Which of the three beveled pieces, if any, are similar?


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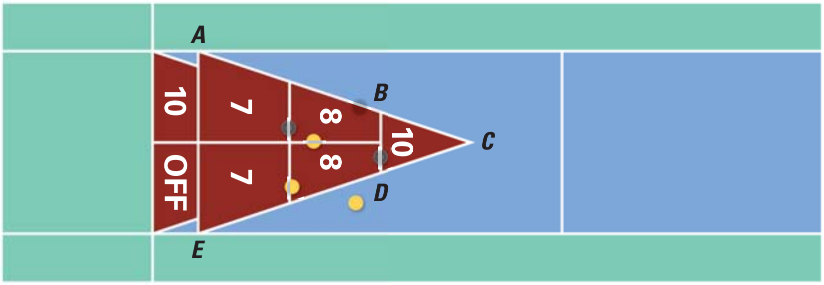
**SHUFFLEBOARD** In the portion of the shuffleboard court shown, $\frac{BC}{AC} = \frac{BD}{AE}$.



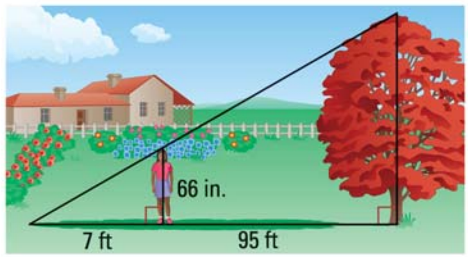

**30.** What additional piece of information do you need in order to show that $\triangle BCD \sim \triangle ACE$ using the SSS Similarity Theorem?

**31.** What additional piece of information do you need in order to show that $\triangle BCD \sim \triangle ACE$ using the SAS Similarity Theorem?

**32.** **TAKS REASONING** Use a diagram to show why there is no Side-Side-Angle Similarity Postulate.

**EXAMPLE 4** on p. 391 for Ex. 33

**33.** **MULTI-STEP PROBLEM** Ruby is standing in her back yard and she decides to estimate the height of a tree. She stands so that the tip of her shadow coincides with the tip of the tree's shadow, as shown. Ruby is 66 inches tall. The distance from the tree to Ruby is 95 feet and the distance between the tip of the shadows and Ruby is 7 feet.




**a.** What postulate or theorem can you use to show that the triangles in the diagram are similar?

**b.** About how tall is the tree, to the nearest foot?

**c.** **What If?** Curtis is 75 inches tall. At a different time of day, he stands so that the tip of his shadow and the tip of the tree's shadow coincide, as described above. His shadow is 6 feet long. How far is Curtis from the tree?

*Animated* **Geometry** at classzone.com

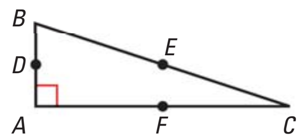
**34.** **TAKS REASONING** Suppose you are given two right triangles with one pair of corresponding legs and the pair of corresponding hypotenuses having the same length ratios.

**a.** The lengths of the given pair of corresponding legs are 6 and 18, and the lengths of the hypotenuses are 10 and 30. Use the Pythagorean Theorem to solve for the lengths of the other pair of corresponding legs. Draw a diagram.

**b.** Write the ratio of the lengths of the second pair of corresponding legs.

**c.** Are these triangles similar? Does this suggest a Hypotenuse-Leg Similarity Theorem for right triangles?

**35.** **PROOF** Given that $\triangle ABC$ is a right triangle and $D$, $E$, and $F$ are midpoints, prove that $m\angle DEF = 90°$.

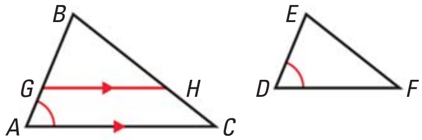



**36.** **WRITING** Can two triangles have all pairs of corresponding angles in proportion? *Explain.*

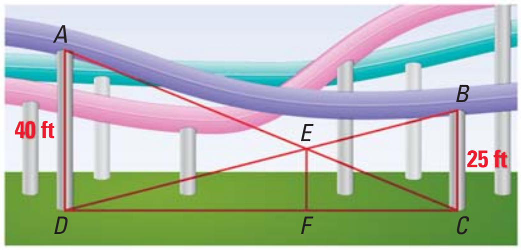
**37.** **PROVING THEOREM 6.3** Write a paragraph proof of the SAS Similarity Theorem.

**GIVEN** ▶ $\angle A \cong \angle D, \dfrac{AB}{DE} = \dfrac{AC}{DF}$

**PROVE** ▶ $\triangle ABC \sim \triangle DEF$




**38.** **CHALLENGE** A portion of a water slide in an amusement park is shown. Find the length of $\overline{EF}$. (*Note:* The posts form right angles with the ground.)




**TAKS PRACTICE** at classzone.com

## MIXED REVIEW FOR TAKS

**REVIEW**
Lesson 6.2;
TAKS Workbook

**39.** **TAKS PRACTICE** The blueprint dimensions for a rectangular parking lot are proportional to the actual dimensions of the parking lot. On the blueprint, the parking lot is 48 centimeters long and 22 centimeters wide. The actual length of the parking lot is 42 meters. What is the actual width of the parking lot? *TAKS Obj. 7*

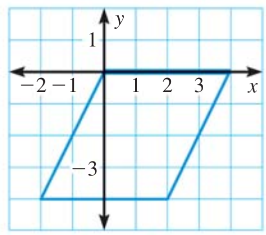
Ⓐ 5.19 m  Ⓑ 16 m  Ⓒ 19.25 m  Ⓓ 25.14 m

**REVIEW**
Lesson 3.5;
TAKS Workbook

**40.** **TAKS PRACTICE** Which of the following functions describes a line that would include an edge of the parallelogram shown in the diagram? *TAKS Obj. 4*

Ⓕ $y = x$  Ⓖ $x = -4$

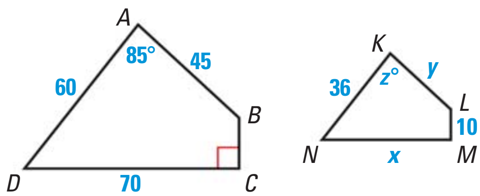
Ⓗ $y = 2x$  Ⓙ $y = 2x + 8$




## QUIZ *for Lessons 6.3–6.5*

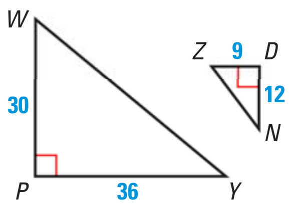
**In the diagram, *ABCD* ~ *KLMN*.** *(p. 372)*

1. Find the scale factor of *ABCD* to *KLMN*.
2. Find the values of *x*, *y*, and *z*.
3. Find the perimeter of each polygon.

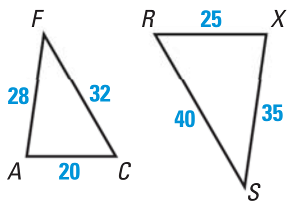



**Determine whether the triangles are similar. If they are similar, write a similarity statement.** *(pp. 381, 388)*

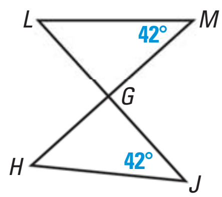
**4.**




**5.**




**6.**



Investigating Geometry **ACTIVITY** *Use before Lesson 6.6*

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classzone.com
Keystrokes

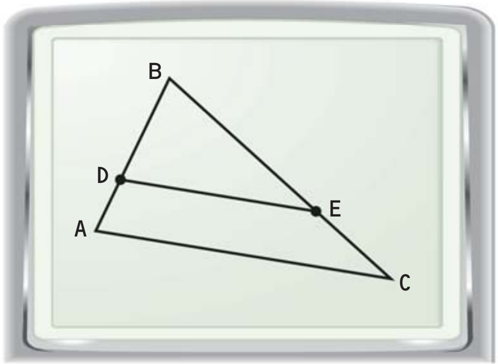
# 6.6 Investigate Proportionality

TEKS *a.5, G.2.A, G.3.B, G.9.B*

**MATERIALS** • graphing calculator or computer

**QUESTION** **How can you use geometry drawing software to compare segment lengths in triangles?**

## EXPLORE 1 Construct a line parallel to a triangle's third side



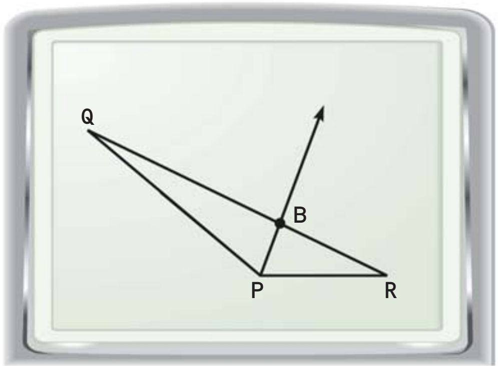

**STEP 1** ***Draw a triangle*** Draw a triangle. Label the vertices $A$, $B$, and $C$. Draw a point on $\overline{AB}$. Label the point $D$.

**STEP 2** ***Draw a parallel line*** Draw a line through $D$ that is parallel to $\overline{AC}$. Label the intersection of the line and $\overline{BC}$ as point $E$.

**STEP 3** ***Measure segments*** Measure $\overline{BD}$, $\overline{DA}$, $\overline{BE}$, and $\overline{EC}$. Calculate the ratios $\frac{BD}{DA}$ and $\frac{BE}{EC}$.

**STEP 4** ***Compare ratios*** Move one or more of the triangle's vertices to change its shape. *Compare* the ratios from Step 3 as the shape changes. Save as "EXPLORE1."

## EXPLORE 2 Construct an angle bisector of a triangle




**STEP 1** ***Draw a triangle*** Draw a triangle. Label the vertices $P$, $Q$, and $R$. Draw the angle bisector of $\angle QPR$. Label the intersection of the angle bisector and $\overline{QR}$ as point $B$.

**STEP 2** ***Measure segments*** Measure $\overline{BR}$, $\overline{RP}$, $\overline{BQ}$, and $\overline{QP}$. Calculate the ratios $\frac{BR}{BQ}$ and $\frac{RP}{QP}$.

**STEP 3** ***Compare ratios*** Move one or more of the triangle's vertices to change its shape. *Compare* the ratios from Step 3. Save as "EXPLORE2."

## DRAW CONCLUSIONS Use your observations to complete these exercises

**1.** Make a conjecture about the ratios of the lengths of the segments formed when two sides of a triangle are cut by a line parallel to the triangle's third side.

**2.** Make a conjecture about how the ratio of the lengths of two sides of a triangle is related to the ratio of the lengths of the segments formed when an angle bisector is drawn to the third side.

# 6.6 Use Proportionality Theorems

TEKS G.1.A, G.2.A, G.5.A, G.11.C

**Before** You used proportions with similar triangles.

**Now** You will use proportions with a triangle or parallel lines.

**Why?** So you can use perspective drawings, as in Ex. 28.

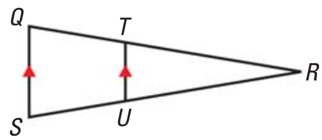
**Key Vocabulary**
- **corresponding angles,** *p. 147*
- **ratio,** *p. 356*
- **proportion,** *p. 358*

The Midsegment Theorem, which you learned on page 295, is a special case of the Triangle Proportionality Theorem and its converse.

## THEOREMS *For Your Notebook*

### THEOREM 6.4 Triangle Proportionality Theorem

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

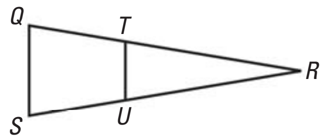



If $\overline{TU} \parallel \overline{QS}$, then $\frac{RT}{TQ} = \frac{RU}{US}$.

*Proof:* Ex. 22, p. 402

### THEOREM 6.5 Converse of the Triangle Proportionality Theorem

If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

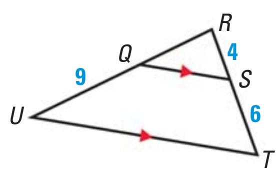



If $\frac{RT}{TQ} = \frac{RU}{US}$, then $\overline{TU} \parallel \overline{QS}$.

*Proof:* Ex. 26, p. 402

## EXAMPLE 1 Find the length of a segment

**In the diagram, $\overline{QS} \parallel \overline{UT}$, $RS = 4$, $ST = 6$, and $QU = 9$. What is the length of $\overline{RQ}$?**




**Solution**

$\frac{RQ}{QU} = \frac{RS}{ST}$ Triangle Proportionality Theorem

$\frac{RQ}{9} = \frac{4}{6}$ Substitute.

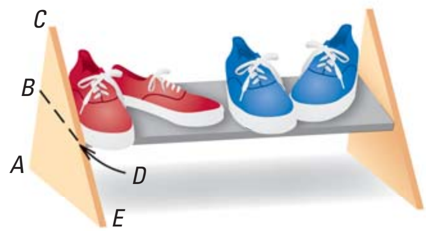
$RQ = 6$ Multiply each side by 9 and simplify.

**REASONING** Theorems 6.4 and 6.5 also tell you that if the lines are *not* parallel, then the proportion is *not* true, and vice-versa.

So if $\overline{TU} \nparallel \overline{QS}$, then $\frac{RT}{TQ} \neq \frac{RU}{US}$. Also, if $\frac{RT}{TQ} \neq \frac{RU}{US}$, then $\overline{TU} \nparallel \overline{QS}$.

## EXAMPLE 2 Solve a real-world problem

**SHOERACK** On the shoerack shown, $AB = 33$ cm, $BC = 27$ cm, $CD = 44$ cm, and $DE = 25$ cm. *Explain* why the gray shelf is not parallel to the floor.




**Solution**

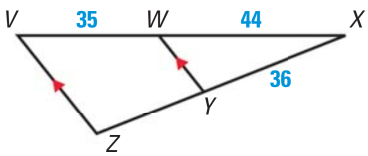
Find and simplify the ratios of lengths determined by the shoerack.

$$\frac{CD}{DE} = \frac{44}{25} \qquad \frac{CB}{BA} = \frac{27}{33} = \frac{9}{11}$$

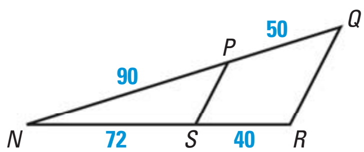
▶ Because $\frac{44}{25} \neq \frac{9}{11}$, $\overline{BD}$ is not parallel to $\overline{AE}$. So, the shelf is not parallel to the floor.

## ✓ GUIDED PRACTICE for Examples 1 and 2

1. Find the length of $\overline{YZ}$.



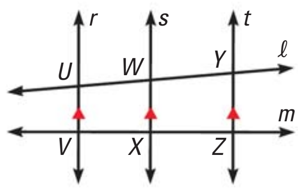

2. Determine whether $\overline{PS} \parallel \overline{QR}$.




## THEOREMS *For Your Notebook*

### THEOREM 6.6

If three parallel lines intersect two transversals, then they divide the transversals proportionally.

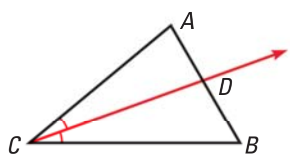



$$\frac{UW}{WY} = \frac{VX}{XZ}$$

*Proof:* Ex. 23, p. 402

### THEOREM 6.7

If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.

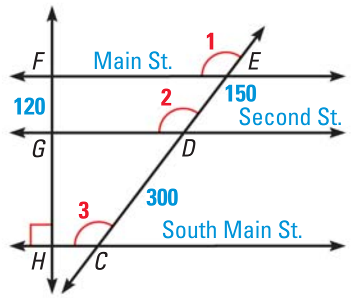



$$\frac{AD}{DB} = \frac{CA}{CB}$$

*Proof:* Ex. 27, p. 403

## EXAMPLE 3 Use Theorem 6.6

**CITY TRAVEL** In the diagram, $\angle 1$, $\angle 2$, and $\angle 3$ are all congruent and $GF = 120$ yards, $DE = 150$ yards, and $CD = 300$ yards. Find the distance $HF$ between Main Street and South Main Street.




**ANOTHER WAY**

For alternative methods for solving the problem in Example 3, turn to page 404 for the **Problem Solving Workshop**.

### Solution

Corresponding angles are congruent, so $\overleftrightarrow{FE}$, $\overleftrightarrow{GD}$, and $\overleftrightarrow{HC}$ are parallel. Use Theorem 6.6.

$\frac{HG}{GF} = \frac{CD}{DE}$ **Parallel lines divide transversals proportionally.**

$\frac{HG + GF}{GF} = \frac{CD + DE}{DE}$ **Property of proportions (Property 4)**

$\frac{HF}{120} = \frac{300 + 150}{150}$ **Substitute.**

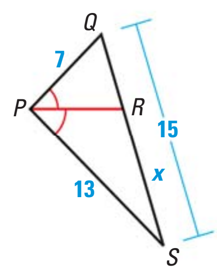
$\frac{HF}{120} = \frac{450}{150}$ **Simplify.**

$HF = 360$ **Multiply each side by 120 and simplify.**

▶ The distance between Main Street and South Main Street is 360 yards.

## EXAMPLE 4 Use Theorem 6.7

**In the diagram, $\angle QPR \cong \angle RPS$. Use the given side lengths to find the length of $\overline{RS}$.**




### Solution

Because $\overrightarrow{PR}$ is an angle bisector of $\angle QPS$, you can apply Theorem 6.7. Let $RS = x$. Then $RQ = 15 - x$.

$\frac{RQ}{RS} = \frac{PQ}{PS}$ **Angle bisector divides opposite side proportionally.**

$\frac{15 - x}{x} = \frac{7}{13}$ **Substitute.**

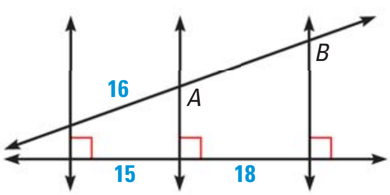
$7x = 195 - 13x$ **Cross Products Property**

$x = 9.75$ **Solve for *x*.**

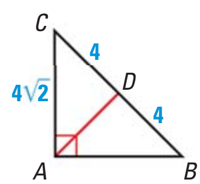
## ✓ GUIDED PRACTICE for Examples 3 and 4

**Find the length of $\overline{AB}$.**

**3.**




**4.**



# 6.6 EXERCISES

**HOMEWORK KEY**
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 9, and 21
★ = **TAKS PRACTICE AND REASONING** Exs. 8, 13, 25, 28, 30, and 31

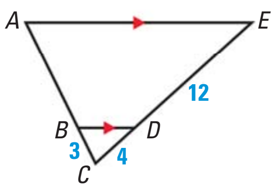
## SKILL PRACTICE

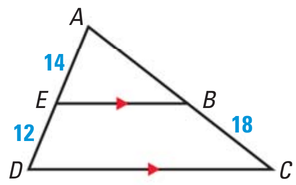
1. **VOCABULARY** State the Triangle Proportionality Theorem. Draw a diagram.

2. **WRITING** *Compare* the Midsegment Theorem (see page 295) and the Triangle Proportionality Theorem. How are they related?

**EXAMPLE 1** on p. 397 for Exs. 3–4

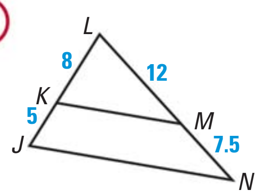
**FINDING THE LENGTH OF A SEGMENT** **Find the length of $\overline{AB}$.**

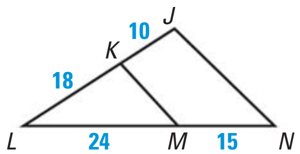
3. 

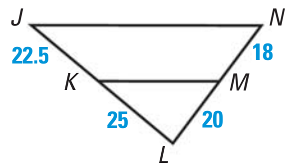

4. 


**EXAMPLE 2** on p. 398 for Exs. 5–7

**REASONING** **Use the given information to determine whether $\overline{KM} \parallel \overline{JN}$. *Explain* your reasoning.**

5. 


6. 

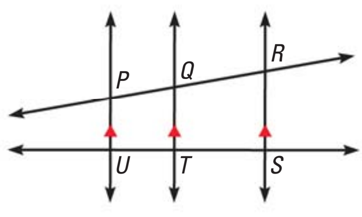

7. 


**EXAMPLE 3** on p. 399 for Ex. 8

8. ★ **TAKS REASONING** For the figure at the right, which statement is *not* necessarily true?

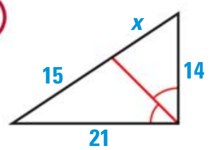
Ⓐ $\dfrac{PQ}{QR} = \dfrac{UT}{TS}$ Ⓑ $\dfrac{TS}{UT} = \dfrac{QR}{PQ}$

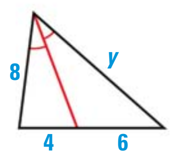
Ⓒ $\dfrac{QR}{RS} = \dfrac{TS}{RS}$ Ⓓ $\dfrac{PQ}{PR} = \dfrac{UT}{US}$

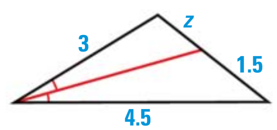



**EXAMPLE 4** on p. 399 for Exs. 9–12

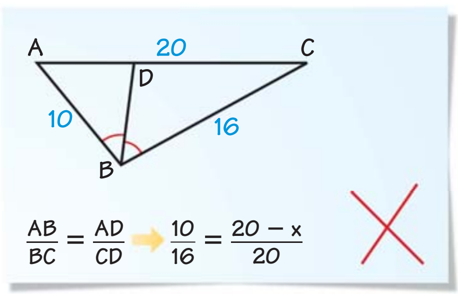
**ALGEBRA** **Find the value of the variable.**

9. 


10. 


11. 

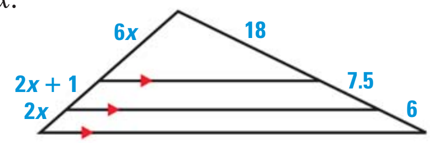

12. **ERROR ANALYSIS** A student begins to solve for the length of $\overline{AD}$ as shown. *Describe* and correct the student's error.



**13.** **TAKS REASONING** Find the value of $x$.

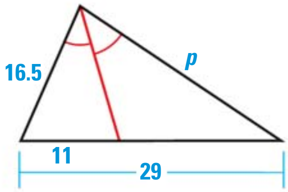
Ⓐ $\frac{1}{2}$ Ⓑ 1

Ⓒ 2 Ⓓ 3

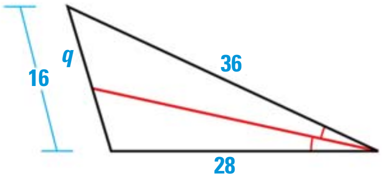



**ALGEBRA** **Find the value of the variable.**

**14.**

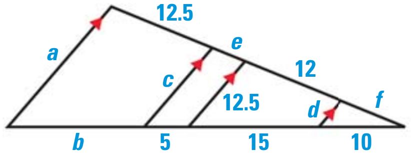



**15.**

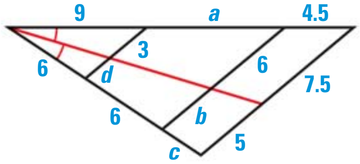



**FINDING SEGMENT LENGTHS** **Use the diagram to find the value of each variable.**

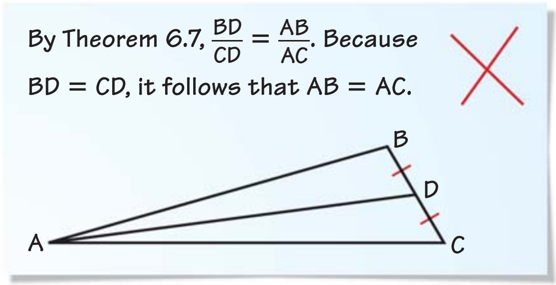
**16.**




**17.**




**18.** **ERROR ANALYSIS** A student claims that $AB = AC$ using the method shown. *Describe* and correct the student's error.

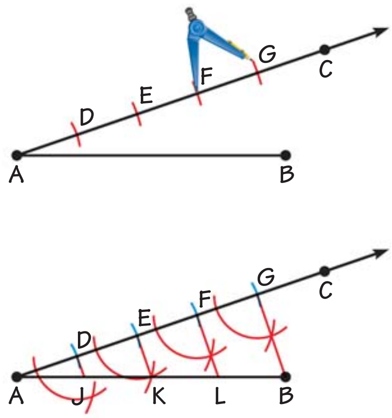



**19.** **CONSTRUCTION** Follow the instructions for constructing a line segment that is divided into four equal parts.

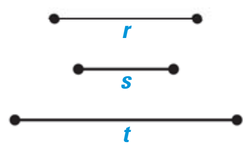
**a.** Draw a line segment that is about 3 inches long, and label its endpoints $A$ and $B$. Choose any point $C$ not on $\overline{AB}$. Draw $\overrightarrow{AC}$.

**b.** Using any length, place the compass point at $A$ and make an arc intersecting $\overrightarrow{AC}$ at $D$. Using the same compass setting, make additional arcs on $\overrightarrow{AC}$. Label the points $E$, $F$, and $G$ so that $AD = DE = EF = FG$.

**c.** Draw $\overline{GB}$. Construct a line parallel to $\overline{GB}$ through $D$. Continue constructing parallel lines and label the points as shown. *Explain* why $AJ = JK = KL = LB$.

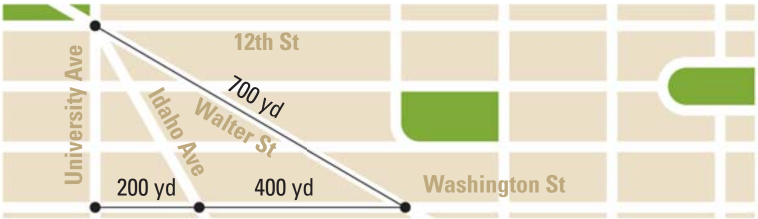



**20.** **CHALLENGE** Given segments with lengths $r$, $s$, and $t$, construct a segment of length $x$, such that $\frac{r}{s} = \frac{t}{x}$.



## PROBLEM SOLVING

21. **CITY MAP** On the map below, Idaho Avenue bisects the angle between University Avenue and Walter Street. To the nearest yard, what is the distance along University Avenue from 12th Street to Washington Street?

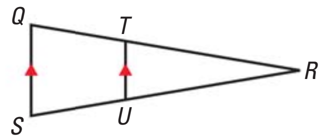



**TEXAS @HomeTutor** for problem solving help at classzone.com

22. **PROVING THEOREM 6.4** Prove the Triangle Proportionality Theorem.

**GIVEN** ▶ $\overline{QS} \parallel \overline{TU}$

**PROVE** ▶ $\frac{QT}{TR} = \frac{SU}{UR}$

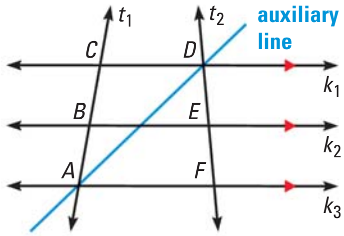



**TEXAS @HomeTutor** for problem solving help at classzone.com

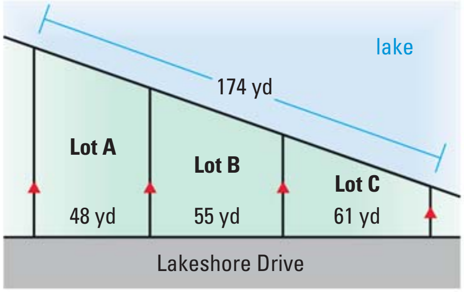
23. **PROVING THEOREM 6.6** Use the diagram with the auxiliary line drawn to write a paragraph proof of Theorem 6.6.

**GIVEN** ▶ $k_1 \parallel k_2$, $k_2 \parallel k_3$

**PROVE** ▶ $\frac{CB}{BA} = \frac{DE}{EF}$




24. **MULTI-STEP PROBLEM** The real estate term *lake frontage* refers to the distance along the edge of a piece of property that touches a lake.




a. Find the lake frontage (to the nearest tenth of a yard) for each lot shown.

b. In general, the more lake frontage a lot has, the higher its selling price. Which of the lots should be listed for the highest price?

c. Suppose that lot prices are in the same ratio as lake frontages. If the least expensive lot is \$100,000, what are the prices of the other lots? *Explain* your reasoning.

25. **TAKS REASONING** Sketch an isosceles triangle. Draw a ray that bisects the angle opposite the base. This ray divides the base into two segments. By Theorem 6.7, the ratio of the legs is proportional to the ratio of these two segments. *Explain* why this ratio is 1 : 1 for an isosceles triangle.

26. **PLAN FOR PROOF** Use the diagram given for the proof of Theorem 6.4 in Exercise 22 to write a plan for proving Theorem 6.5, the Triangle Proportionality Converse.



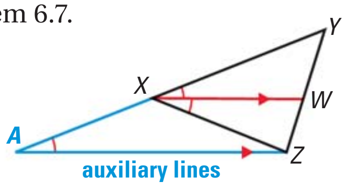
○ = **WORKED-OUT SOLUTIONS** on p. WS1

= **TAKS PRACTICE AND REASONING**

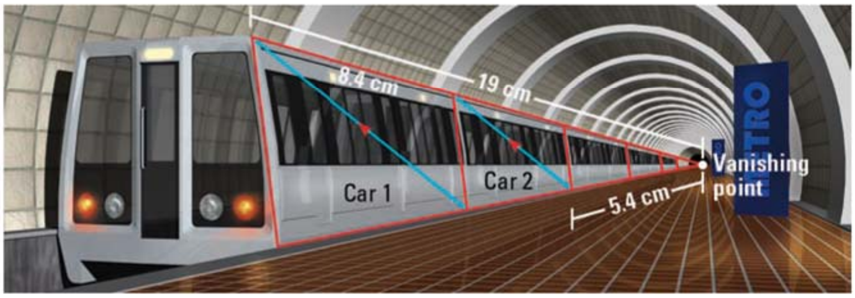
27. **PROVING THEOREM 6.7** Use the diagram with the auxiliary lines drawn to write a paragraph proof of Theorem 6.7.

**GIVEN** ▶ $\angle YXW \cong \angle WXZ$

**PROVE** ▶ $\frac{YW}{WZ} = \frac{XY}{XZ}$




28. **TAKS REASONING** In *perspective drawing*, lines that are parallel in real life must meet at a vanishing point on the horizon. To make the train cars in the drawing appear equal in length, they are drawn so that the lines connecting the opposite corners of each car are parallel.

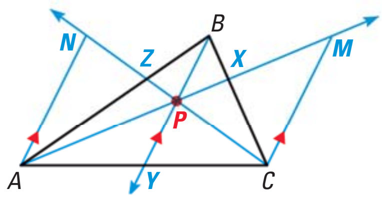



**a.** Use the dimensions given and the red parallel lines to find the length of the bottom edge of the drawing of Car 2.

**b.** What other set of parallel lines exist in the figure? *Explain* how these can be used to form a set of similar triangles.

**c.** Find the length of the top edge of the drawing of Car 2.

29. **CHALLENGE** Prove *Ceva's Theorem:* If $P$ is any point inside $\triangle ABC$, then $\frac{AY}{YC} \cdot \frac{CX}{XB} \cdot \frac{BZ}{ZA} = 1$. (*Hint:* Draw lines parallel to $\overline{BY}$ through $A$ and $C$. Apply Theorem 6.4 to $\triangle ACM$. Show that $\triangle APN \sim \triangle MPC$, $\triangle CXM \sim \triangle BXP$, and $\triangle BZP \sim \triangle AZN$.)




## MIXED REVIEW FOR TAKS

**TAKS PRACTICE** at classzone.com

**REVIEW**
Skills Review Handbook p. 882; TAKS Workbook

30. **TAKS PRACTICE** What are the roots of the quadratic equation $x^2 + x - 20 = 0$? *TAKS Obj. 5*

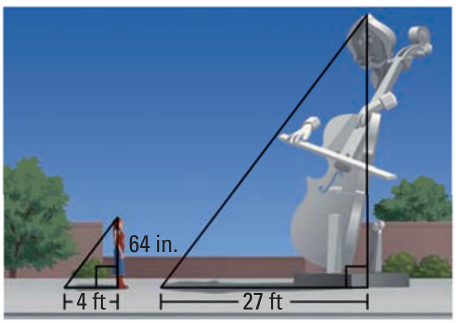
(A) 4 and 5 (B) 4 and $-5$ (C) $-4$ and $-5$ (D) $-4$ and 5

**REVIEW**
Lesson 6.2; TAKS Workbook

31. **TAKS PRACTICE** Maria is standing next to a statue, as shown in the diagram. The statue casts a shadow that is 27 feet long. At the same time, Maria's shadow is 4 feet long. Maria is 64 inches tall. How tall is the statue? *TAKS Obj. 8*

(F) 16 ft (G) 22 ft

(H) 36 ft (J) 44 ft



PROBLEM SOLVING WORKSHOP
LESSON 6.6

TEKS G.3.B, G.5.C, G.9.B

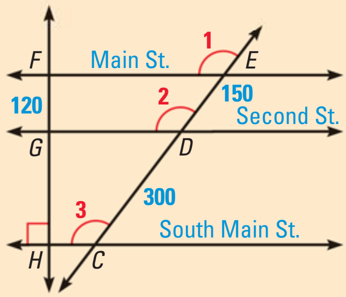
# *Using* ALTERNATIVE METHODS

## *Another Way to Solve Example 3, page 399*

**MULTIPLE REPRESENTATIONS** In Lesson 6.6, you used proportionality theorems to find lengths of segments formed when transversals intersect two or more parallel lines. Now, you will learn two different ways to solve Example 3 on page 399.

### PROBLEM

**CITY TRAVEL** In the diagram, $\angle 1$, $\angle 2$, and $\angle 3$ are all congruent and $GF = 120$ yards, $DE = 150$ yards, and $CD = 300$ yards. Find the distance $HF$ between Main Street and South Main Street.




### METHOD 1

**Applying a Ratio** One alternative approach is to look for ratios in the diagram.

***STEP 1*** **Read** the problem. Because Main Street, Second Street, and South Main Street are all parallel, the lengths of the segments of the cross streets will be in proportion, so they have the same ratio.

***STEP 2*** **Apply** a ratio. Notice that on $\overleftrightarrow{CE}$, the distance $CD$ between South Main Street and Second Street is twice the distance $DE$ between Second Street and Main Street. So the same will be true for the distances $HG$ and $GF$.

$HG = 2 \cdot GF$ **Write equation.**

$= 2 \cdot 120$ **Substitute.**

$= 240$ **Simplify.**

***STEP 3*** **Calculate** the distance. Line $HF$ is perpendicular to both Main Street and South Main Street, so the distance between Main Street and South Main Street is this perpendicular distance, $HF$.

$HF = HG + GF$ **Segment Addition Postulate**

$= 120 + 240$ **Substitute.**

$= 360$ **Simplify.**

***STEP 4*** **Check** page 399 to verify your answer, and confirm that it is the same.

**METHOD 2** **Writing a Proportion** Another alternative approach is to use a graphic organizer to set up a proportion.

*STEP 1* **Make** a table to compare the distances.

| | $\overleftrightarrow{CE}$ | $\overleftrightarrow{HF}$ |
|---|---|---|
| **Total distance** | 300 + 150, or 450 | $x$ |
| **Partial distance** | 150 | 120 |

*STEP 2* **Write** and solve a proportion.

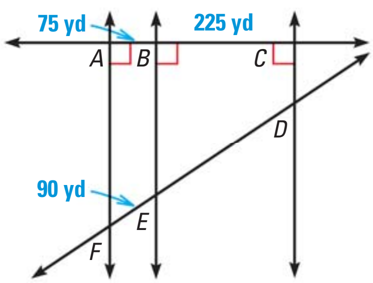
$$\frac{450}{150} = \frac{x}{120}$$ **Write proportion.**

$$360 = x$$ **Multiply each side by 12 and simplify.**

▶ The distance is 360 yards.

## PRACTICE

1. **MAPS** Use the information on the map.




a. Find $DE$.

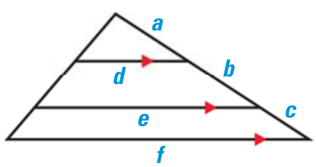
b. **What If?** Suppose there is an alley one fourth of the way from $\overline{BE}$ to $\overline{CD}$ and parallel to $\overline{BE}$. What is the distance from $E$ to the alley along $\overleftrightarrow{FD}$?

2. **REASONING** Given the diagram below, *explain* why the three given proportions are true.

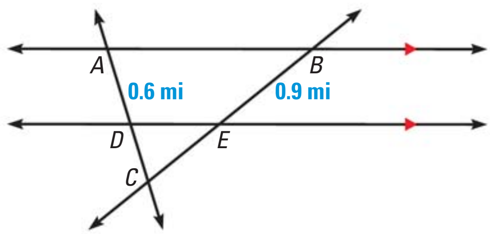
$$\frac{a}{a+b} = \frac{d}{e}$$

$$\frac{a}{a+b+c} = \frac{d}{f}$$

$$\frac{a+b}{a+b+c} = \frac{d}{f}$$

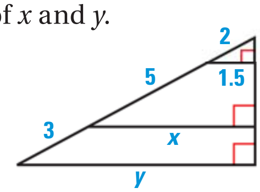



3. **WALKING** Two people leave points $A$ and $B$ at the same time. They intend to meet at point $C$ at the same time. The person who leaves point $A$ walks at a speed of 3 miles per hour. How fast must the person who leaves point $B$ walk?




4. **ERROR ANALYSIS** A student who attempted to solve the problem in Exercise 3 claims that you need to know the length of $\overline{AC}$ to solve the problem. *Describe* and correct the error that the student made.

5. **xy ALGEBRA** Use the diagram to find the values of $x$ and $y$.



# Extension

*Use after Lesson 6.6*

# Fractals

TEKS *G.2.A, G.3.B, G.5.C, G.9.B*

**GOAL** Explore the properties of fractals.

**Key Vocabulary**
- fractal
- self-similarity
- iteration

**HISTORY NOTE**

Computers made it easier to study mathematical iteration by reducing the time needed to perform calculations. Using fractals, mathematicians have been able to create better models of coastlines, clouds, and other natural objects.

A **fractal** is an object that is *self-similar*. An object is **self-similar** if one part of the object can be enlarged to look like the whole object. In nature, fractals can be found in ferns and branches of a river. Scientists use fractals to map out clouds in order to predict rain.

Many fractals are formed by a repetition of a sequence of the steps called **iteration**. The first stage of drawing a fractal is considered Stage 0. Helge van Koch (1870–1924) described a fractal known as the *Koch snowflake*, shown in Example 1.

A Mandelbrot fractal

## EXAMPLE 1 Draw a fractal

**Use the directions below to draw a Koch snowflake.**

Starting with an equilateral triangle, at each stage each side is divided into thirds and a new equilateral triangle is formed using the middle third as the triangle side length.

**Solution**

*STAGE 0* **Draw** an equilateral triangle with a side length of one unit.

*STAGE 1* **Replace** the middle third of each side with an equilateral triangle.

*STAGE 2* **Repeat** Stage 1 with the six smaller equilateral triangles.

*STAGE 3* **Repeat** Stage 1 with the eighteen smaller equilateral triangles.

**MEASUREMENT** Benoit Mandelbrot (b. 1924) was the first mathematician to formalize the idea of fractals when he observed methods used to measure the lengths of coastlines. Coastlines cannot be measured as straight lines because of the inlets and rocks. Mandelbrot used fractals to model coastlines.

## EXAMPLE 2 Find lengths in a fractal

**Make a table to study the lengths of the sides of a Koch snowflake at different stages.**

| Stage number | Edge length | Number of edges | Perimeter |
|---|---|---|---|
| 0 | 1 | 3 | 3 |
| 1 | $\frac{1}{3}$ | $3 \cdot 4 = 12$ | 4 |
| 2 | $\frac{1}{9}$ | $12 \cdot 4 = 48$ | $\frac{48}{9} = 5\frac{1}{3}$ |
| 3 | $\frac{1}{27}$ | $48 \cdot 4 = 192$ | $\frac{192}{27} = 7\frac{1}{9}$ |
| $n$ | $\frac{1}{3^n}$ | $3 \cdot 4^n$ | $\frac{4^n}{3^{n-1}}$ |

AnimatedGeometry at classzone.com

## PRACTICE

**EXAMPLES 1 and 2** for Exs. 1–3

1. **PERIMETER** Find the ratio of the edge length of the triangle in Stage 0 of a Koch snowflake to the edge length of the triangle in Stage 1. How is the perimeter of the triangle in Stage 0 related to the perimeter of the triangle in Stage 1? *Explain.*

2. **MULTI-STEP PROBLEM** Use the *Cantor set*, which is a fractal whose iteration consists of dividing a segment into thirds and erasing the middle third.
   **a.** Draw Stage 0 through Stage 5 of the Cantor set. Stage 0 has a length of one unit.
   **b.** Make a table showing the stage number, number of segments, segment length, and total length of the Cantor set.
   **c.** What is the total length of the Cantor set at Stage 10? Stage 20? Stage $n$?

3. **EXTENDED RESPONSE** A *Sierpinski carpet* starts with a square with side length one unit. At each stage, divide the square into nine equal squares with the middle square shaded a different color.
   **a.** Draw Stage 0 through Stage 3 of a Sierpinski Carpet.
   **b.** *Explain* why the carpet is said to be *self-similar* by comparing the upper left hand square to the whole square.
   **c.** Make a table to find the total area of the colored squares at Stage 3.

Investigating Geometry **ACTIVITY** *Use before Lesson 6.7*

# 6.7 Dilations

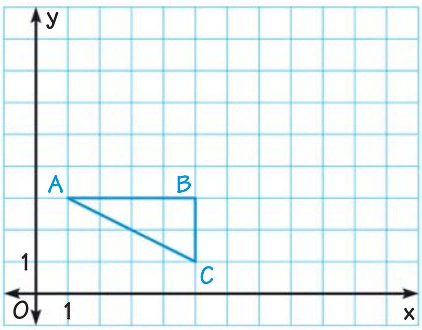
TEKS *a.5, G.5.C, G.7.A, G.11.A*

**MATERIALS** • graph paper • straightedge • compass • ruler

**QUESTION** How can you construct a similar figure?

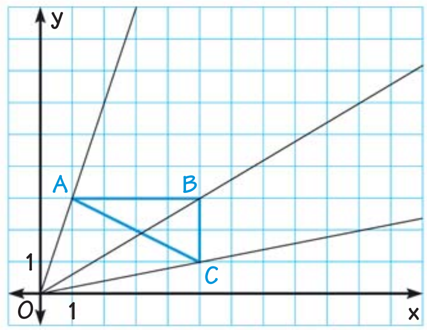
**EXPLORE** Construct a similar triangle

**STEP 1**



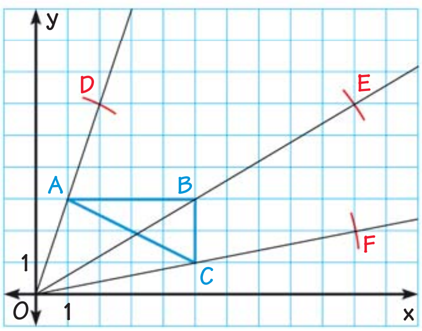

***Draw a triangle*** Plot the points $A(1, 3)$, $B(5, 3)$, and $C(5, 1)$ in a coordinate plane. Draw $\triangle ABC$.

**STEP 2**



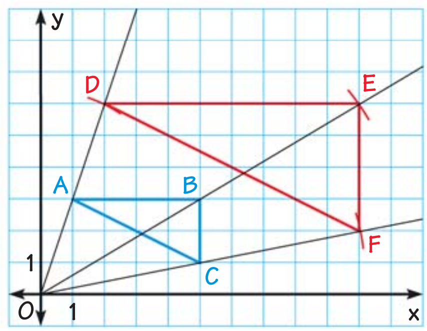

***Draw rays*** Using the origin as an endpoint $O$, draw $\overrightarrow{OA}$, $\overrightarrow{OB}$, and $\overrightarrow{OC}$.

**STEP 3**




***Draw equal segments*** Use a compass to mark a point $D$ on $\overrightarrow{OA}$ so $OA = AD$. Mark a point $E$ on $\overrightarrow{OB}$ so $OB = BE$. Mark a point $F$ on $\overrightarrow{OC}$ so $OC = CF$.

**STEP 4**




***Draw the image*** Connect points $D$, $E$, and $F$ to form a right triangle.

**DRAW CONCLUSIONS** Use your observations to complete these exercises

1. Measure $\overline{AB}$, $\overline{BC}$, $\overline{DE}$, and $\overline{EF}$. Calculate the ratios $\frac{DE}{AB}$ and $\frac{EF}{BC}$. Using this information, show that the two triangles are similar.
2. Repeat the steps in the Explore to construct $\triangle GHJ$ so that $3 \cdot OA = AG$, $3 \cdot OB = BH$, and $3 \cdot OC = CJ$.

# 6.7 Perform Similarity Transformations

TEKS G.5.C, G.7.A, G.7.C, G.11.A

**Before** You performed congruence transformations.

**Now** You will perform dilations.

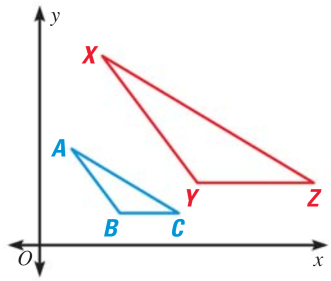
**Why?** So you can solve problems in art, as in Ex. 26.

**Key Vocabulary**
- dilation
- center of dilation
- scale factor of a dilation
- reduction
- enlargement
- transformation, *p. 272*

A **dilation** is a transformation that stretches or shrinks a figure to create a similar figure. A dilation is a type of *similarity transformation*.

In a dilation, a figure is enlarged or reduced with respect to a fixed point called the **center of dilation**.

The **scale factor of a dilation** is the ratio of a side length of the image to the corresponding side length of the original figure. In the figure shown, $\triangle XYZ$ is the image of $\triangle ABC$. The center of dilation is (0, 0) and the scale factor is $\frac{XY}{AB}$.




## KEY CONCEPT — *For Your Notebook*

### Coordinate Notation for a Dilation

You can describe a dilation with respect to the origin with the notation $(x, y) \rightarrow (kx, ky)$, where $k$ is the scale factor.

If $0 < k < 1$, the dilation is a **reduction**. If $k > 1$, the dilation is an **enlargement**.

## EXAMPLE 1 Draw a dilation with a scale factor greater than 1

**READ DIAGRAMS**
All of the dilations in this lesson are in the coordinate plane and each center of dilation is the origin.

**Draw a dilation of quadrilateral $ABCD$ with vertices $A(2, 1)$, $B(4, 1)$, $C(4, -1)$, and $D(1, -1)$. Use a scale factor of 2.**

**Solution**

First draw $ABCD$. Find the dilation of each vertex by multiplying its coordinates by 2. Then draw the dilation.

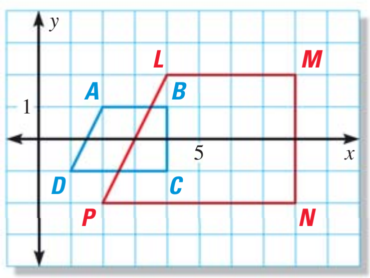
$$(x, y) \rightarrow (2x, 2y)$$

$A(2, 1) \rightarrow L(4, 2)$

$B(4, 1) \rightarrow M(8, 2)$

$C(4, -1) \rightarrow N(8, -2)$

$D(1, -1) \rightarrow P(2, -2)$



## EXAMPLE 2 Verify that a figure is similar to its dilation

**A triangle has the vertices $A(4, -4)$, $B(8, 2)$, and $C(8, -4)$. The image of $\triangle ABC$ after a dilation with a scale factor of $\frac{1}{2}$ is $\triangle DEF$.**

**a.** Sketch $\triangle ABC$ and $\triangle DEF$.

**b.** Verify that $\triangle ABC$ and $\triangle DEF$ are similar.

### Solution

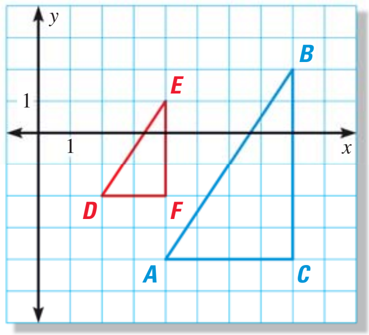
**a.** The scale factor is less than one, so the dilation is a reduction.

$$(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$$

$A(4, -4) \rightarrow D(2, -2)$

$B(8, 2) \rightarrow E(4, 1)$

$C(8, -4) \rightarrow F(4, -2)$




**b.** Because $\angle C$ and $\angle F$ are both right angles, $\angle C \cong \angle F$. Show that the lengths of the sides that include $\angle C$ and $\angle F$ are proportional. Find the horizontal and vertical lengths from the coordinate plane.

$$\frac{AC}{DF} \stackrel{?}{=} \frac{BC}{EF} \rightarrow \frac{4}{2} = \frac{6}{3} \checkmark$$

So, the lengths of the sides that include $\angle C$ and $\angle F$ are proportional.

▶ Therefore, $\triangle ABC \sim \triangle DEF$ by the SAS Similarity Theorem.

## ✓ GUIDED PRACTICE for Examples 1 and 2

**Find the coordinates of $L$, $M$, and $N$ so that $\triangle LMN$ is a dilation of $\triangle PQR$ with a scale factor of $k$. Sketch $\triangle PQR$ and $\triangle LMN$.**

1. $P(-2, -1)$, $Q(-1, 0)$, $R(0, -1)$; $k = 4$

2. $P(5, -5)$, $Q(10, -5)$, $R(10, 5)$; $k = 0.4$

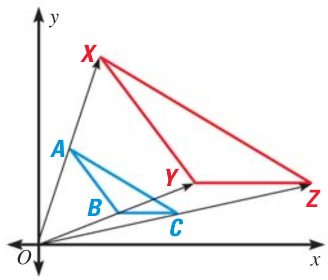
## EXAMPLE 3 Find a scale factor

**PHOTO STICKERS** You are making your own photo stickers. Your photo is 4 inches by 4 inches. The image on the stickers is 1.1 inches by 1.1 inches. What is the scale factor of the reduction?

### Solution

The scale factor is the ratio of a side length of the sticker image to a side length of the original photo, or $\frac{1.1 \text{ in.}}{4 \text{ in.}}$. In simplest form, the scale factor is $\frac{11}{40}$.

**READING DIAGRAMS** Generally, for a center of dilation at the origin, a point of the figure and its image lie on the same ray from the origin. However, if a point of the figure *is* the origin, its image is also the origin.

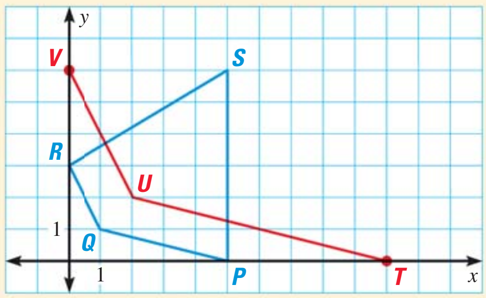



TAKS

## EXAMPLE 4 TAKS PRACTICE: Multiple Choice

**Quadrilateral *PQRS* is dilated to create a quadrilateral *TUVW* that is similar to *PQRS*. What are the coordinates of *W*?**

Ⓐ (10, −12) Ⓑ (8, 8) Ⓒ (10, 12) Ⓓ (12, 14)




**Solution**

**ELIMINATE CHOICES**
You can eliminate choice A, because you can tell by looking at the graph that *W* is in Quadrant I.

Determine if *TUVW* is a dilation of *PQRS* by checking whether the same scale factor can be used to obtain *T*, *U*, and *V* from *P*, *Q*, and *R*.

$(x, y) \to (kx, ky)$

$P(5, 0) \to T(10, 0)$ $\quad k = 2$

$Q(1, 1) \to U(2, 2)$ $\quad k = 2$

$R(0, 3) \to V(0, 6)$ $\quad k = 2$

Because $k$ is the same in each case, the image is a dilation with a scale factor of 2. So, you can use the scale factor to find the image *W* of point *S*.

$S(5, 6) \to W(2 \cdot 5, 2 \cdot 6) = W(10, 12)$

▶ The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

***CHECK*** by drawing the rays from the origin through each point and its image.

## ✓ GUIDED PRACTICE for Examples 3 and 4

3. **WHAT IF?** In Example 3, what is the scale factor of the reduction if your photo is 5.5 inches by 5.5 inches?

4. Suppose a figure containing the origin is dilated. *Explain* why the corresponding point in the image of the figure is also the origin.

# 6.7 EXERCISES

**HOMEWORK KEY**

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 11, and 27

★ = **TAKS PRACTICE AND REASONING** Exs. 13, 21, 22, 28, 30, 31, and 35

## SKILL PRACTICE

1. **VOCABULARY** Copy and complete: In a dilation, the image is _?_ to the original figure.

2. **WRITING** *Explain* how to find the scale factor of a dilation. How do you know whether a dilation is an enlargement or a reduction?

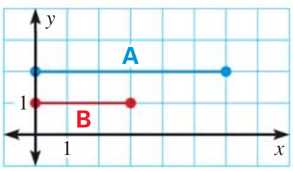
**EXAMPLES 1 and 2** on pp. 409–410 for Exs. 3–8

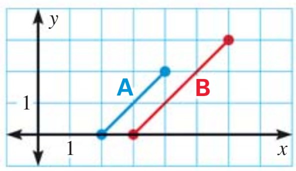
**DRAWING DILATIONS** **Draw a dilation of the polygon with the given vertices using the given scale factor *k*.**

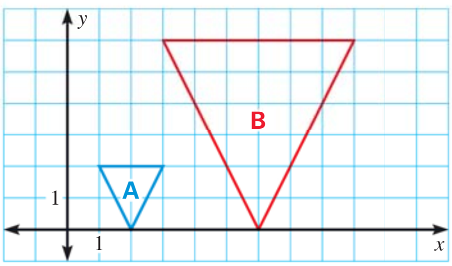
3. $A(-2, 1), B(-4, 1), C(-2, 4); k = 2$
4. $A(-5, 5), B(-5, -10), C(10, 0); k = \frac{3}{5}$
5. $A(1, 1), B(6, 1), C(6, 3); k = 1.5$
6. $A(2, 8), B(8, 8), C(16, 4); k = 0.25$
7. $A(-8, 0), B(0, 8), C(4, 0), D(0, -4); k = \frac{3}{8}$
8. $A(0, 0), B(0, 3), C(2, 4), D(2, -1); k = \frac{13}{2}$

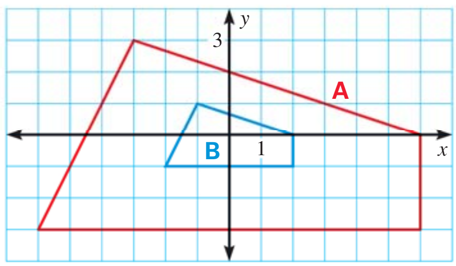
**EXAMPLE 3** on p. 410 for Exs. 9–12

**IDENTIFYING DILATIONS** **Determine whether the dilation from Figure A to Figure B is a *reduction* or an *enlargement*. Then find its scale factor.**

9.


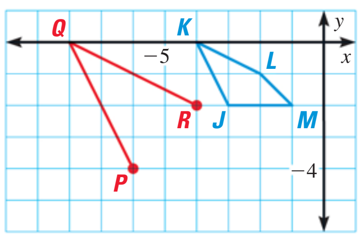

10.



11.



12.



**EXAMPLE 4** on p. 411 for Ex. 13

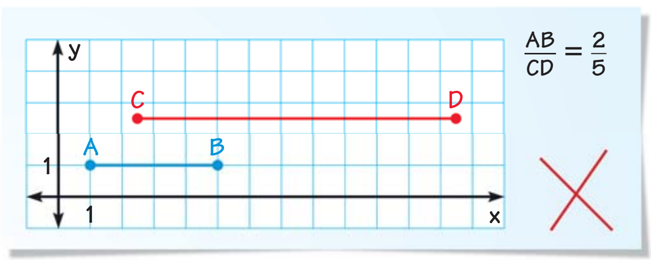
13. ★ **TAKS REASONING** You want to create a quadrilateral *PQRS* that is similar to quadrilateral *JKLM*. What are the coordinates of *S*?



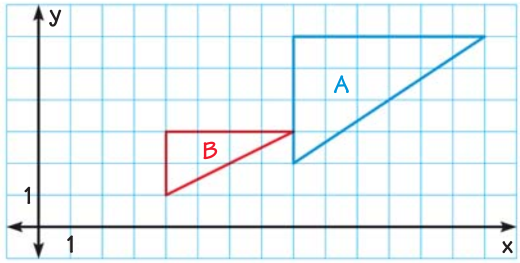

Ⓐ (2, 4) Ⓑ (4, −2)

Ⓒ (−2, −4) Ⓓ (−4, −2)

14. **ERROR ANALYSIS** A student found the scale factor of the dilation from $\overline{AB}$ to $\overline{CD}$ to be $\frac{2}{5}$. *Describe* and correct the student's error.

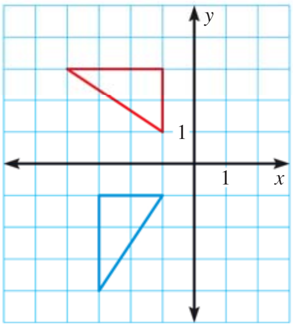


15. **ERROR ANALYSIS** A student says that the figure shown represents a dilation. What is wrong with this statement?

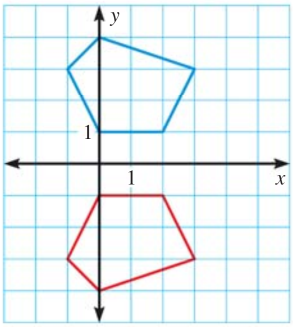



**IDENTIFYING TRANSFORMATIONS** **Determine whether the transformation shown is a *translation*, *reflection*, *rotation*, or *dilation*.**

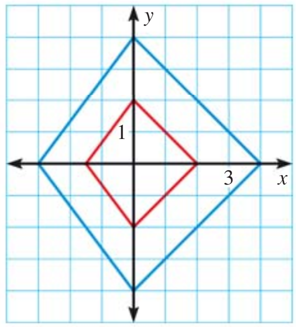
16.




17.

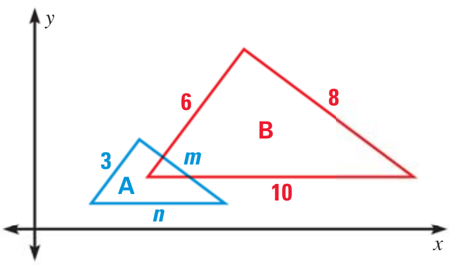



18.

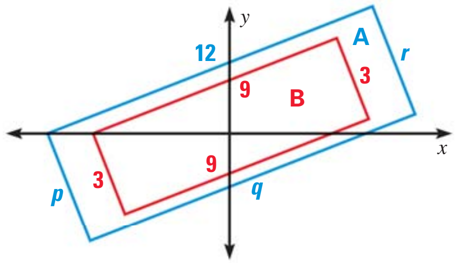



**FINDING SCALE FACTORS** **Find the scale factor of the dilation of Figure A to Figure B. Then give the unknown lengths of Figure A.**

19.



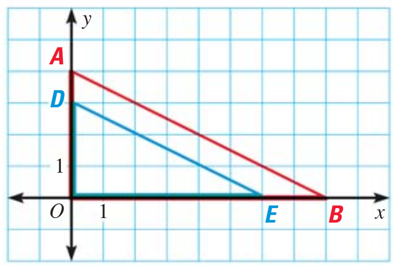

20.




21. **TAKS REASONING** In the diagram shown, $\triangle ABO$ is a dilation of $\triangle DEO$. The length of a median of $\triangle ABO$ is what percent of the length of the corresponding median of $\triangle DEO$?

Ⓐ 50% Ⓑ 75%

Ⓒ $133\frac{1}{3}\%$ Ⓓ 200%




22. **TAKS REASONING** Suppose you dilate a figure using a scale factor of 2. Then, you dilate the image using a scale factor of $\frac{1}{2}$. *Describe* the size and shape of this new image.

**CHALLENGE** ***Describe*** **the two transformations, the first followed by the second, that combined will transform $\triangle ABC$ into $\triangle DEF$.**

23. $A(-3, 3)$, $B(-3, 1)$, $C(0, 1)$
$D(6, 6)$, $E(6, 2)$, $F(0, 2)$

24. $A(6, 0)$, $B(9, 6)$, $C(12, 6)$
$D(0, 3)$, $E(1, 5)$, $F(2, 5)$

## PROBLEM SOLVING

EXAMPLE 3 on p. 410 for Exs. 25–27

25. **BILLBOARD ADVERTISEMENT** A billboard advertising agency requires each advertisement to be drawn so that it fits in a 12-inch by 6-inch rectangle. The agency uses a scale factor of 24 to enlarge the advertisement to create the billboard. What are the dimensions of a billboard, in feet?

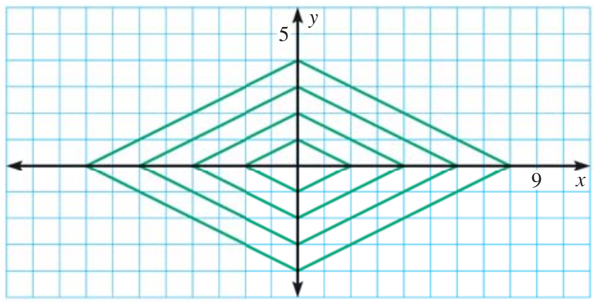
TEXAS @HomeTutor for problem solving help at classzone.com

26. **POTTERY** Your pottery is used on a poster for a student art show. You want to make postcards using the same image. On the poster, the image is 8 inches in width and 6 inches in height. If the image on the postcard can be 5 inches wide, what scale should you use for the image on the postcard?


TEXAS @HomeTutor for problem solving help at classzone.com

27. **SHADOWS** You and your friend are walking at night. You point a flashlight at your friend, and your friend's shadow is cast on the building behind him. The shadow is an enlargement, and is 15 feet tall. Your friend is 6 feet tall. What is the scale factor of the enlargement?

28. **TAKS REASONING** *Describe* how you can use dilations to create the figure shown below.




Animated Geometry at classzone.com

29. **MULTI-STEP PROBLEM** $\triangle ABC$ has vertices $A(3, -3)$, $B(3, 6)$, and $C(15, 6)$.

    a. Draw a dilation of $\triangle ABC$ using a scale factor of $\frac{2}{3}$.

    b. Find the ratio of the perimeter of the image to the perimeter of the original figure. How does this ratio compare to the scale factor?

    c. Find the ratio of the area of the image to the area of the original figure. How does this ratio compare to the scale factor?

30. **TAKS REASONING** Look at the coordinate notation for a dilation on page 409. Suppose the definition of dilation allowed $k < 0$.

    a. *Describe* the dilation if $-1 < k < 0$.

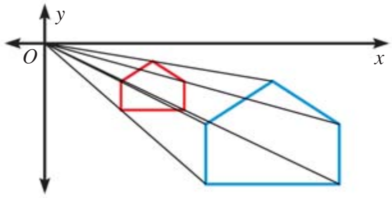
    b. *Describe* the dilation if $k < -1$.

    c. Use a rotation to describe a dilation with $k = -1$.



○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = TAKS PRACTICE AND REASONING

31. **TAKS REASONING** *Explain* how you can use dilations to make a perspective drawing with the center of dilation as a vanishing point. Draw a diagram.




32. **MIDPOINTS** Let $\overline{XY}$ be a dilation of $\overline{PQ}$ with scale factor $k$. Show that the image of the midpoint of $\overline{PQ}$ is the midpoint of $\overline{XY}$.

33. **REASONING** In Exercise 32, show that $\overline{XY} \parallel \overline{PQ}$.

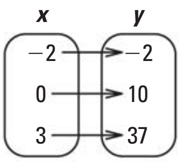
34. **CHALLENGE** A rectangle has vertices $A(0, 0)$, $B(0, 6)$, $C(9, 6)$, and $D(9, 0)$. *Explain* how to dilate the rectangle to produce an image whose area is twice the area of the original rectangle. Make a conjecture about how to dilate any polygon to produce an image whose area is $n$ times the area of the original polygon.

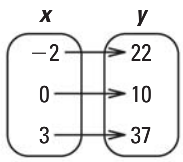
## MIXED REVIEW FOR TAKS

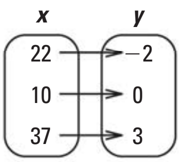
**TAKS PRACTICE** at classzone.com

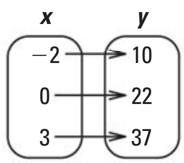
**REVIEW**
Skills Review Handbook p. 884
TAKS Workbook

35. **TAKS PRACTICE** Which mapping best represents the function $y = 3x^2 + 10$ when the replacement set for $x$ is $\{-2, 0, 3\}$? *TAKS Obj. 1*

(A)


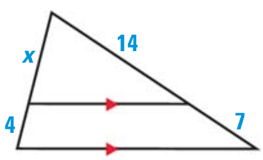

(B)


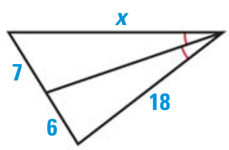

(C)


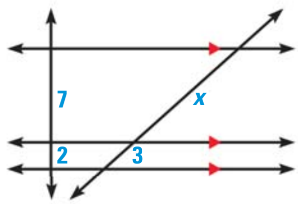

(D)



## QUIZ for Lessons 6.6–6.7

**Find the value of $x$.** *(p. 397)*

1.



2.



3.



**Draw a dilation of $\triangle ABC$ with the given vertices and scale factor $k$.** *(p. 409)*

4. $A(-5, 5)$, $B(-5, -10)$, $C(10, 0)$; $k = 0.4$

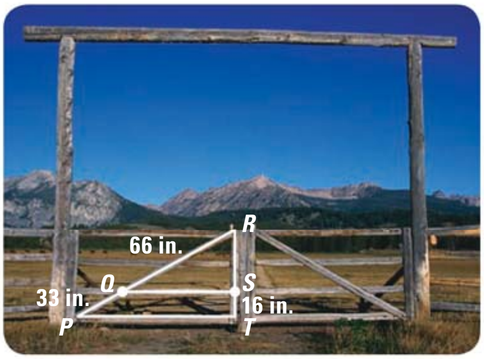
5. $A(-2, 1)$, $B(-4, 1)$, $C(-2, 4)$; $k = 2.5$

# MIXED REVIEW FOR TEKS

TAKS PRACTICE
classzone.com

## Lessons 6.4–6.7

1. **GATE** In the photo of the gate below, $\overline{QS}$ is parallel to $\overline{PT}$. What is $RT$? *TEKS G.11.B*

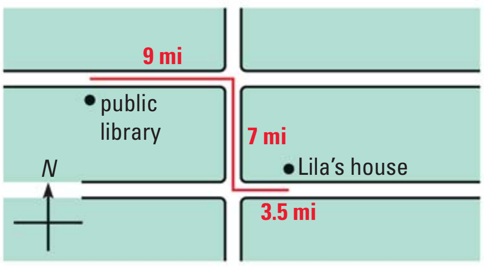



Ⓐ 24 in. Ⓑ 32 in.
Ⓒ 48 in. Ⓓ 64 in.

2. **AREA** Rectangle $ABCD$ has vertices $A(2, 2)$, $B(4, 2)$, $C(4, -4)$, and $D(2, -4)$. Suppose rectangle $ABCD$ is dilated using a scale factor of $\frac{5}{4}$. What is the ratio of the area of the image to the area of the original figure? *TEKS G.11.A*

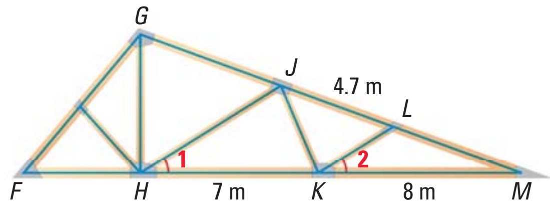
Ⓕ $\frac{5}{8}$ Ⓖ $\frac{5}{4}$
Ⓗ $\frac{25}{16}$ Ⓙ $\frac{5}{2}$

3. **DRIVING DISTANCE** Lila leaves the public library to go home and drives due east 9 miles, due south 7 miles, and due east again 3.5 miles. What is the distance between the library and Lila's house? *TEKS G.11.B*



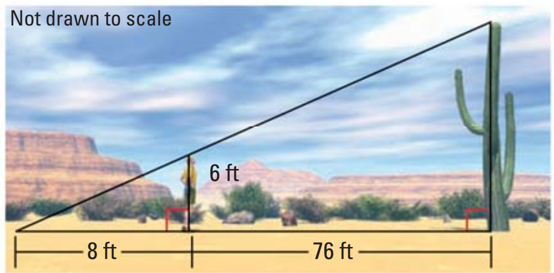

Ⓐ 11.8 miles Ⓑ 12.2 miles
Ⓒ 12.5 miles Ⓓ 14.3 miles

4. **ROOF TRUSS** In the diagram of the roof truss, $HK = 7$ meters, $KM = 8$ meters, $JL = 4.7$ meters, and $\angle 1 \cong \angle 2$. What is $LM$? *TEKS G.11.B*




Ⓕ 2.2 m Ⓖ 4.8 m
Ⓗ 5.4 m Ⓙ 6.7 m

5. **CACTUS** The Cardon cactus, found in the Sonoran Desert in Mexico, is the world's tallest type of cactus. Marco stands 76 feet from a Cardon cactus so that the tip of his shadow coincides with the tip of the cactus' shadow, as shown. Marco is 6 feet tall and his shadow is 8 feet long. How tall is the cactus? *TEKS G.11.B*




Ⓐ 56 ft Ⓑ 57 ft
Ⓒ 63 ft Ⓓ 65 ft

**GRIDDED ANSWER** ⓪ ① ● ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨

6. **GREETING CARDS** Naomi is designing a catalog for a greeting card company. The catalog features a 2.8 inch by 2 inch photograph of each card. The actual dimensions of a greeting card are 7 inches by 5 inches. What is the scale factor of the reduction? Write your answer as a decimal. *TEKS G.11.A*

# 6 CHAPTER SUMMARY

## BIG IDEAS — *For Your Notebook*

**Big Idea 1** — TEKS G.11.B

### Using Ratios and Proportions to Solve Geometry Problems

You can use properties of proportions to solve a variety of algebraic and geometric problems.


For example, in the diagram above, suppose you know that $\frac{AB}{BC} = \frac{ED}{DC}$. Then you can write any of the following relationships.

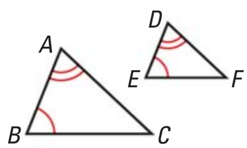
$$\frac{5}{x} = \frac{6}{18} \qquad 5 \cdot 18 = 6x \qquad \frac{x}{5} = \frac{18}{6} \qquad \frac{5}{6} = \frac{x}{18} \qquad \frac{5+x}{x} = \frac{6+18}{18}$$

**Big Idea 2** — TEKS G.11.C

### Showing that Triangles are Similar

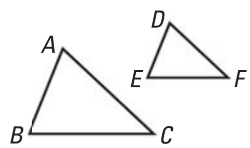
You learned three ways to prove two triangles are similar.

**AA Similarity Postulate**



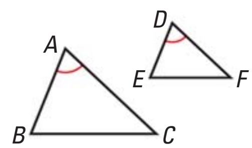

If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.

**SSS Similarity Theorem**




If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

**SAS Similarity Theorem**

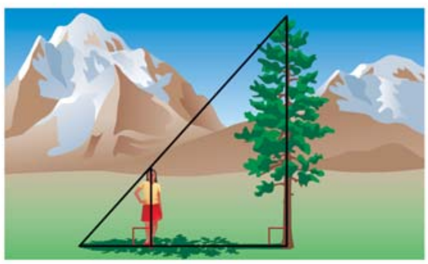



If $\angle A \cong \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$, then $\triangle ABC \sim \triangle DEF$.

**Big Idea 3** — TEKS G.11.C

### Using Indirect Measurement and Similarity

You can use triangle similarity theorems to apply indirect measurement in order to find lengths that would be inconvenient or impossible to measure directly.



Consider the diagram shown. Because the two triangles formed by the person and the tree are similar by the AA Similarity Postulate, you can write the following proportion to find the height of the tree.

$$\frac{\text{height of person}}{\text{length of person's shadow}} = \frac{\text{height of tree}}{\text{length of tree's shadow}}$$

You also learned about dilations, a type of similarity transformation. In a dilation, a figure is either enlarged or reduced in size.

# 6 CHAPTER REVIEW

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- Multi-Language Glossary
- Vocabulary practice

## REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- ratio, *p. 356*
- proportion, *p. 358* means, extremes
- geometric mean, *p. 359*
- scale drawing, *p. 365*
- scale, *p. 365*
- similar polygons, *p. 372*
- scale factor of two similar polygons, *p. 373*
- dilation, *p. 409*
- center of dilation, *p. 409*
- scale factor of a dilation, *p. 409*
- reduction, *p. 409*
- enlargement, *p. 409*

### VOCABULARY EXERCISES

**Copy and complete the statement.**

1. A _?_ is a transformation in which the original figure and its image are similar.
2. If $\triangle PQR \sim \triangle XYZ$, then $\frac{PQ}{XY} = \frac{?}{YZ} = \frac{?}{?}$.
3. **WRITING** *Describe* the relationship between a ratio and a proportion. Give an example of each.

## REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 6.

### 6.1 Ratios, Proportions, and the Geometric Mean *pp. 356–363*

**EXAMPLE**

**The measures of the angles in $\triangle ABC$ are in the extended ratio of 3 : 4 : 5. Find the measures of the angles.**

Use the extended ratio of 3 : 4 : 5 to label the angle measures as $3x°$, $4x°$, and $5x°$.

$3x° + 4x° + 5x° = 180°$ — **Triangle Sum Theorem**

$12x = 180$ — **Combine like terms.**

$x = 15$ — **Divide each side by 12.**

So, the angle measures are $3(15°) = 45°$, $4(15°) = 60°$, and $5(15°) = 75°$.

### EXERCISES

**EXAMPLES 1, 3, and 6** on pp. 356–359 for Exs. 4–6

4. The length of a rectangle is 20 meters and the width is 15 meters. Find the ratio of the width to the length of the rectangle. Then simplify the ratio.
5. The measures of the angles in $\triangle UVW$ are in the extended ratio of 1 : 1 : 2. Find the measures of the angles.
6. Find the geometric mean of 8 and 12.

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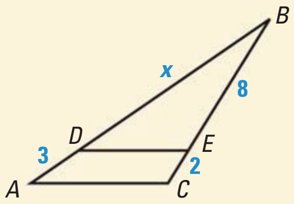
Chapter Review Practice

## 6.2 Use Proportions to Solve Geometry Problems
*pp. 364–370*

**EXAMPLE**

**In the diagram, $\frac{BA}{DA} = \frac{BC}{EC}$. Find $BD$.**



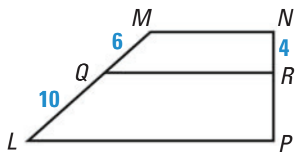

| | |
|---|---|
| $\frac{x+3}{3} = \frac{8+2}{2}$ | Substitution Property of Equality |
| $2x + 6 = 30$ | Cross Products Property |
| $x = 12$ | Solve for $x$. |

### EXERCISES

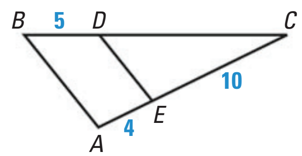
**EXAMPLE 2**
on p. 365
for Exs. 7–8

**Use the diagram and the given information to find the unknown length.**

**7.** Given $\frac{RN}{RP} = \frac{QM}{QL}$, find $RP$.



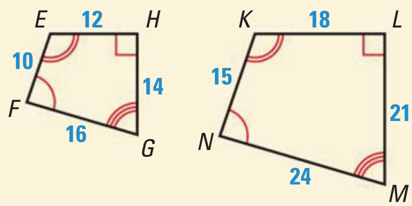

**8.** Given $\frac{CD}{DB} = \frac{CE}{EA}$, find $CD$.




## 6.3 Use Similar Polygons
*pp. 372–379*

**EXAMPLE**

**In the diagram, $EHGF \sim KLMN$. Find the scale factor.**



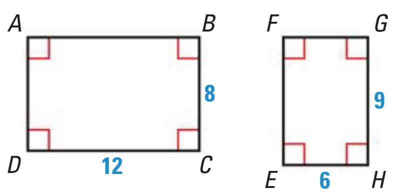

From the diagram, you can see that $\overline{EH}$ and $\overline{KL}$ correspond. So, the scale factor of $EHGF$ to $KLMN$ is $\frac{EH}{KL} = \frac{12}{18} = \frac{2}{3}$.

### EXERCISES

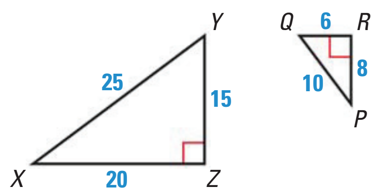
**EXAMPLES 2 and 4**
on pp. 373–374
for Exs. 9–11

**In Exercises 9 and 10, determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor.**

**9.**




**10.**




**11. POSTERS** Two similar posters have a scale factor of 4 : 5. The large poster's perimeter is 85 inches. Find the small poster's perimeter.

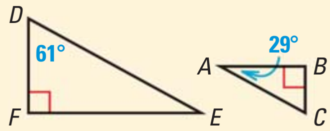
# 6 CHAPTER REVIEW

## 6.4 Prove Triangles Similar by AA

pp. 381–387

**EXAMPLE**

**Determine whether the triangles are similar. If they are, write a similarity statement. Explain your reasoning.**



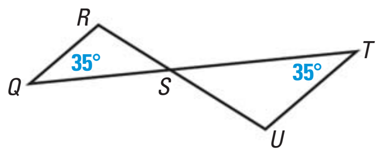

Because they are right angles, $\angle F \cong \angle B$. By the Triangle Sum Theorem, $61° + 90° + m\angle E = 180°$, so $m\angle E = 29°$ and $\angle E \cong \angle A$. Then, two angles of $\triangle DFE$ are congruent to two angles of $\triangle CBA$. So, $\triangle DFE \sim \triangle CBA$.

### EXERCISES

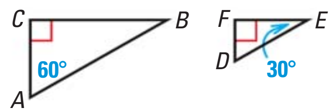
**EXAMPLES 2 and 3** on pp. 382–383 for Exs. 12–14

**Use the AA Similarity Postulate to show that the triangles are similar.**

**12.**




**13.**



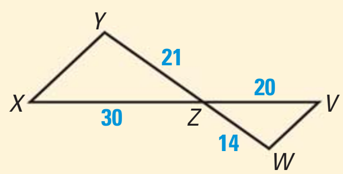

**14. CELL TOWER** A cellular telephone tower casts a shadow that is 72 feet long, while a tree nearby that is 27 feet tall casts a shadow that is 6 feet long. How tall is the tower?

## 6.5 Prove Triangles Similar by SSS and SAS

pp. 388–395

**EXAMPLE**

**Show that the triangles are similar.**




Notice that the lengths of two pairs of corresponding sides are proportional.

$$\frac{WZ}{YZ} = \frac{14}{21} = \frac{2}{3} \qquad \frac{VZ}{XZ} = \frac{20}{30} = \frac{2}{3}$$

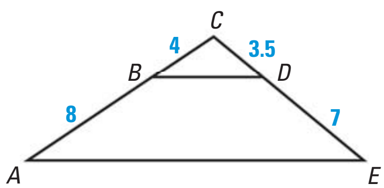
The included angles for these sides, $\angle XZY$ and $\angle VZW$, are vertical angles, so $\angle XZY \cong \angle VZW$. Then $\triangle XYZ \sim \triangle VWZ$ by the SAS Similarity Theorem.

### EXERCISES

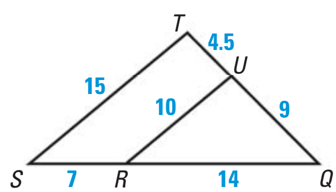
**EXAMPLE 4** on p. 391 for Exs. 15–16

**Use the SSS Similarity Theorem or SAS Similarity Theorem to show that the triangles are similar.**

**15.**




**16.**



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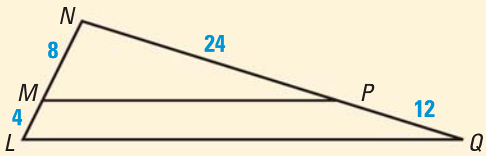
Chapter Review Practice

## 6.6 Use Proportionality Theorems

pp. 397–403

**EXAMPLE**

**Determine whether $\overline{MP} \parallel \overline{LQ}$.**




Begin by finding and simplifying ratios of lengths determined by $\overline{MP}$.

$$\frac{NM}{ML} = \frac{8}{4} = \frac{2}{1} \qquad \frac{NP}{PQ} = \frac{24}{12} = \frac{2}{1}$$

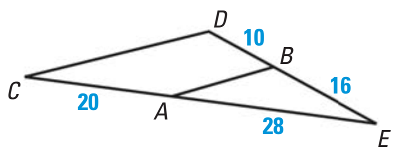
Because $\frac{NM}{ML} = \frac{NP}{PQ}$, $\overline{MP}$ is parallel to $\overline{LQ}$ by Theorem 6.5, the Triangle Proportionality Converse.

### EXERCISES

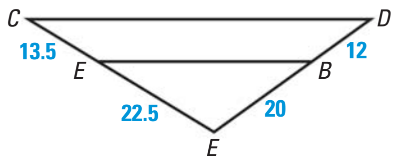
**EXAMPLE 2**
on p. 398
for Exs. 17–18

**Use the given information to determine whether $\overline{AB} \parallel \overline{CD}$.**

**17.**




**18.**



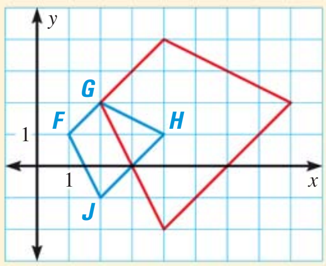

## 6.7 Perform Similarity Transformations

pp. 409–415

**EXAMPLE**

**Draw a dilation of quadrilateral *FGHJ* with vertices $F(1, 1)$, $G(2, 2)$, $H(4, 1)$, and $J(2, -1)$. Use a scale factor of 2.**

First draw *FGHJ*. Find the dilation of each vertex by multiplying its coordinates by 2. Then draw the dilation.




$(x, y) \rightarrow (2x, 2y)$

$F(1, 1) \rightarrow (2, 2)$

$G(2, 2) \rightarrow (4, 4)$

$H(4, 1) \rightarrow (8, 2)$

$J(2, -1) \rightarrow (4, -2)$

### EXERCISES

**EXAMPLES 1 and 2**
on pp. 409–410
for Exs. 19–21

**Draw a dilation of the polygon with the given vertices using the given scale factor *k*.**

**19.** $T(0, 8)$, $U(6, 0)$, $V(0, 0)$; $k = \frac{3}{2}$

**20.** $A(6, 0)$, $B(3, 9)$, $C(0, 0)$, $D(3, 1)$; $k = 4$

**21.** $P(8, 2)$, $Q(4, 0)$, $R(3, 1)$, $S(6, 4)$; $k = 0.5$

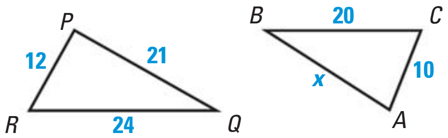
# 6 CHAPTER TEST

**Solve the proportion.**

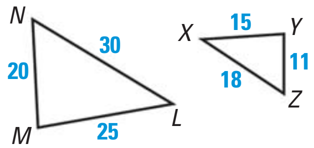
1. $\frac{6}{x} = \frac{9}{24}$ 2. $\frac{5}{4} = \frac{y-5}{12}$ 3. $\frac{3-2b}{4} = \frac{3}{2}$ 4. $\frac{7}{2a+8} = \frac{1}{a-1}$

**In Exercises 5–7, use the diagram where $\triangle PQR \sim \triangle ABC$.**

5. List all pairs of congruent angles.
6. Write the ratios of the corresponding sides in a statement of proportionality.
7. Find the value of $x$.




**Determine whether the triangles are similar. If so, write a similarity statement and the postulate or theorem that justifies your answer.**

8.
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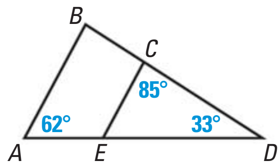


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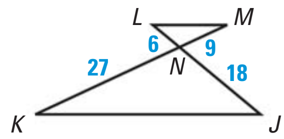
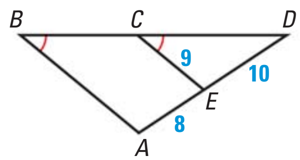
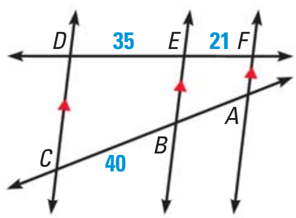



**In Exercises 11–13, find the length of $\overline{AB}$.**

11.



12.
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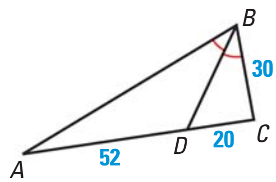
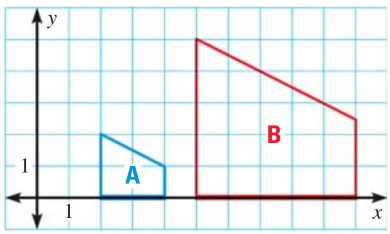
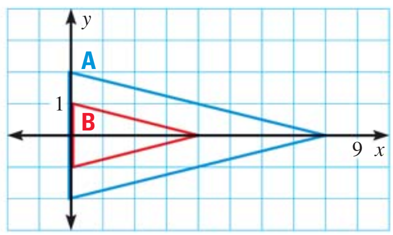
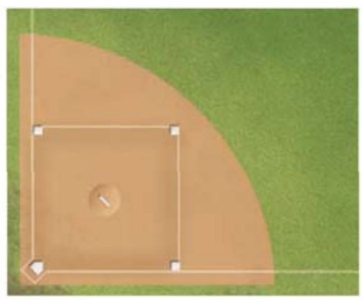



**Determine whether the dilation from Figure A to Figure B is a *reduction* or an *enlargement*. Then find its scale factor.**

14.



15.



16. **SCALE MODEL** You are making a scale model of your school's baseball diamond as part of an art project. The distance between two consecutive bases is 90 feet. If you use a scale factor of $\frac{1}{180}$ to build your model, what will be the distance around the bases on your model?



# 6 ALGEBRA REVIEW

Animated Algebra classzone.com

## SOLVE QUADRATIC EQUATIONS AND SIMPLIFY RADICALS

A radical expression is *simplified* when the radicand has no perfect square factor except 1, there is no fraction in the radicand, and there is no radical in a denominator.

### EXAMPLE 1 Solve quadratic equations by finding square roots

**Solve the equation $4x^2 - 3 = 109$.**

| | |
|---|---|
| $4x^2 - 3 = 109$ | Write original equation. |
| $4x^2 = 112$ | Add 3 to each side. |
| $x^2 = 28$ | Divide each side by 4. |
| $x = \pm\sqrt{28}$ | $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, so $\sqrt{28} = \pm\sqrt{4} \cdot \sqrt{7}$. |
| $x = \pm 2\sqrt{7}$ | Simplify. |

### EXAMPLE 2 Simplify quotients with radicals

**Simplify the expression.**

**a.** $\sqrt{\frac{10}{8}}$ **b.** $\sqrt{\frac{1}{5}}$

**Solution**

**a.**

| | |
|---|---|
| $\sqrt{\frac{10}{8}} = \sqrt{\frac{5}{4}}$ | Simplify fraction. |
| $= \frac{\sqrt{5}}{\sqrt{4}}$ | $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$. |
| $= \frac{\sqrt{5}}{2}$ | Simplify. |

**b.**

| | |
|---|---|
| $\sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}}$ | $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$. $\sqrt{1} = 1$. |
| $= \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$ | Multiply numerator and denominator by $\sqrt{5}$. |
| $= \frac{\sqrt{5}}{5}$ | Multiply fractions. $\sqrt{a} \cdot \sqrt{a} = a$. |

## EXERCISES

EXAMPLE 1 for Exs. 1–9

**Solve the equation or write *no solution*.**

**1.** $x^2 + 8 = 108$ **2.** $2x^2 - 1 = 49$ **3.** $x^2 - 9 = 8$

**4.** $5x^2 + 11 = 1$ **5.** $2(x^2 - 7) = 6$ **6.** $9 = 21 + 3x^2$

**7.** $3x^2 - 17 = 43$ **8.** $56 - x^2 = 20$ **9.** $-3(-x^2 + 5) = 39$

EXAMPLE 2 for Exs. 10–17

**Simplify the expression.**

**10.** $\sqrt{\frac{7}{81}}$ **11.** $\sqrt{\frac{3}{5}}$ **12.** $\sqrt{\frac{24}{27}}$ **13.** $\frac{3\sqrt{7}}{\sqrt{12}}$

**14.** $\sqrt{\frac{75}{64}}$ **15.** $\frac{\sqrt{2}}{\sqrt{200}}$ **16.** $\frac{9}{\sqrt{27}}$ **17.** $\sqrt{\frac{21}{42}}$

# 6 TAKS PREPARATION

TEXAS
**TAKS Obj. 2**
**TEKS A.2.A, A.2.B, A.2.C**

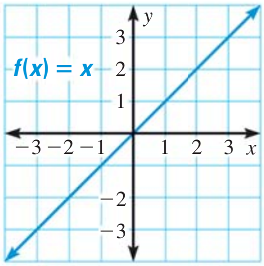
## REVIEWING GRAPHS OF FUNCTIONS PROBLEMS

Recall from Algebra that a *function* is a rule that establishes a relationship between an input and an output. For each input, there is exactly one output. All possible input values make up the *domain* of the function, and all possible output values make up the *range* of the function.

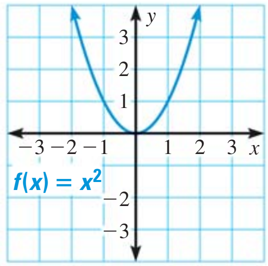
The *graph of a function* is the set of all ordered pairs $(x, f(x))$ such that $x$ is in the domain of the function.

Four functions and their graphs are shown below.

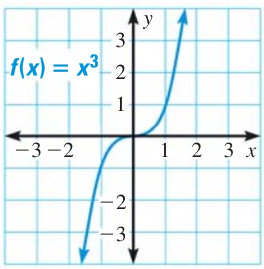
**READ SYMBOLS**
When the domain or range of a function has no upper limit, the upper limit is positive infinity ($\infty$). If they have no lower limit, that limit is negative infinity ($-\infty$).



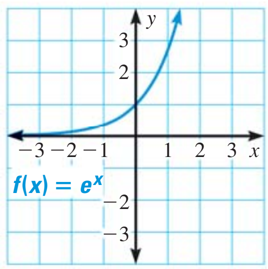

**Linear function**
Domain: $-\infty < x < \infty$
Range: $-\infty < y < \infty$




**Quadratic function**
Domain: $-\infty < x < \infty$
Range: $0 \le y < \infty$



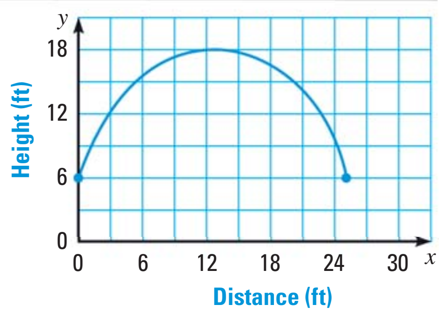

**Cubic function**
Domain: $-\infty < x < \infty$
Range: $-\infty < y < \infty$




**Exponential function**
Domain: $-\infty < x < \infty$
Range: $0 < y < \infty$

### EXAMPLE

The graph shows the path of a baseball thrown during a game of catch. Find the domain and range of the function.



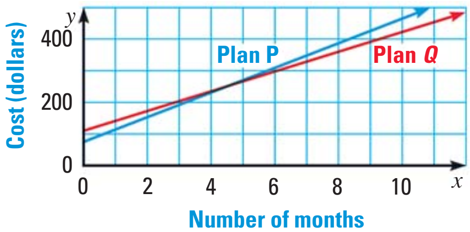

**Solution**

The ball travels a distance of 25 feet, so the domain is $0 \le x \le 25$. The ball was thrown from a height of 6 feet, reached a maximum height of 18 feet, and was caught at a height of 6 feet. So, the range is $6 \le y \le 18$.

# GRAPHS OF FUNCTIONS PROBLEMS ON TAKS

Below are examples of interpreting graphs of functions in multiple choice format. Try solving the problems before looking at the solutions. (Cover the solutions with a piece of paper.) Then check your solutions against the ones given.

**1.** The graph shows the total cost for two phone plans. Which statement is true?




**A** Plan P costs less per month than Plan Q.

**B** Plan P costs less per year than Plan Q.

**C** Plan P costs less until the fifth month.

**D** Plan P costs less after the fifth month.

### Solution

The slope represents the cost per month. Because the graph for Plan P has a steeper slope than the graph for Plan Q, Plan P costs more per month than Plan Q. So, the correct answer is not A.

The total cost of Plan P after 12 months, or 1 year, is about \$580. The total cost of Plan Q after 1 year is about \$460. So, the correct answer is not B.

Plan P costs less than Plan Q for the first 4 months, the plans cost the same during the fifth month, and Plan P costs more than Plan Q after the fifth month. So, the correct answer is C.

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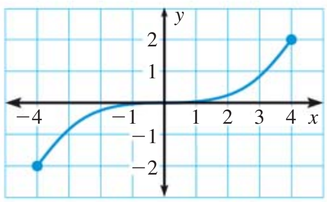
**2.** What is the range of the function shown?




**F** $-4 < x < 4$

**G** $-4 \le x \le 4$

**H** $-2 < y < 2$

**J** $-2 \le y \le 2$

### Solution

Because the graph does not extend below $y = -2$ or above $y = 2$, the range of the function is $-2 \le y \le 2$.

So, the correct answer is H.

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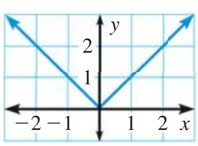
**3.** What type of function is shown in the graph?

**A** Linear function

**B** Quadratic function

**C** Exponential function

**D** Absolute value function




### Solution

The two halves of the graph are linear, but the range does not include values less than zero. So this is an absolute value function.

So, the correct answer is D.

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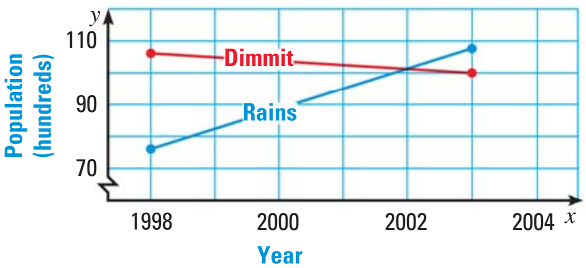
# 6 TAKS PRACTICE

## PRACTICE FOR TAKS OBJECTIVE 2

**1.** The graph shows the approximate populations (in hundreds of people) of Dimmit County and Rains County from 1998 to 2003. According to the graph, which statement is true?



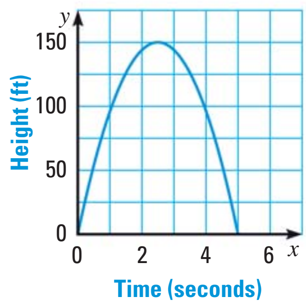

**A** The population of Rains was less than the population of Dimmit after 2002.

**B** The population of Dimmit was less than the population of Rains before 2002.

**C** The population of Rains was less than the population of Dimmit during 2002.

**D** The population of Dimmit was less than the population of Rains after 2002.

**2.** The graph shows the path of a model rocket after it is launched. What is the range of the function?



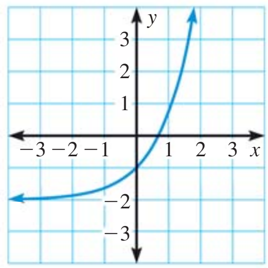

**F** $0 < y < 150$

**G** $0 \le y \le 150$

**H** $0 < x < 5$

**J** $0 \le x \le 5$

**3.** Which type of function is shown in the graph?



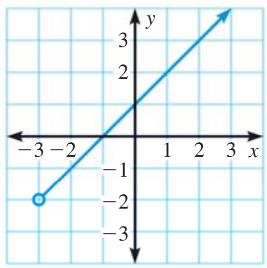
**A** Exponential function

**B** Cubic function

**C** Linear function

**D** Quadratic function

**4.** Which inequality best describes the range of the function shown in the graph?



**F** $y < -2$

**G** $x < -3$

**H** $y > -2$

**J** $x > -3$

## MIXED TAKS PRACTICE

**5.** Which is an equation for the line through the points (4, −2) and (−2, 1)? ***TAKS Obj. 3***

**A** $y = -0.5x$

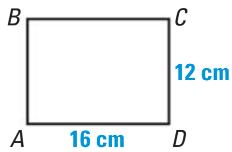
**B** $y = 0.5x - 4$

**C** $y = -2x + 6$

**D** $y = 2x - 10$

# MIXED TAKS PRACTICE

**6.** Which dimensions produce a rectangle similar to rectangle $ABCD$? *TAKS Obj. 8*




**F** 8 cm by 6 cm

**G** 12 cm by 8 cm

**H** 18 cm by 15 cm

**J** 18 cm by 12 cm

**7.** What are the roots of the quadratic equation $x^2 + 3x - 28 = 0$? *TAKS Obj. 5*

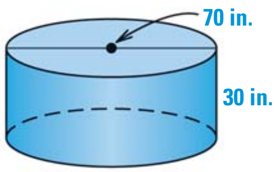
**A** 4 and 7

**B** 4 and $-7$

**C** $-4$ and 7

**D** $-4$ and $-7$

**8.** One gallon equals 231 cubic inches. About how many gallons does the cylindrical tank shown below hold? *TAKS Obj. 8*




**F** 428 gallons

**G** 500 gallons

**H** 857 gallons

**J** 1999 gallons

**9.** Which expression is equivalent to $4(x + 3) - 2x(5x + 1)$? *TAKS Obj. 2*

**A** $3 + 2x - 10x^2$

**B** $12 + 6x - 10x^2$

**C** $12 + 2x - 10x^2$

**D** $3 - 8x$

**10.** At a book sale, all new books are the same price and all used books are the same price. Jim buys 1 new book and 3 used books for \$15.75. Tim spends \$19 on 2 new books and 1 used book. Which system of equations can be used to find the cost of a new book, $x$, and the cost of a used book, $y$? *TAKS Obj. 4*

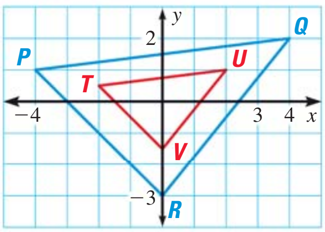
**F** $x + 3y = 15.75$
$2x + y = 19$

**G** $x + 2y = 15.75$
$3x + y = 19$

**H** $x + 3y = 19$
$2x + y = 15.75$

**J** $x + 2y = 19$
$3x + y = 15.75$

**11.** The graph below shows $\triangle PQR$ and its image $\triangle TUV$ after a dilation. Which statement is true? *TAKS Obj. 6*




**A** The measure of $\angle U$ is twice the measure of $\angle Q$.

**B** The measure of $\angle U$ is one-half the measure of $\angle Q$.

**C** All of the corresponding sides are proportional with a scale factor of 2.

**D** All of the corresponding sides are proportional with a scale factor of $\frac{1}{2}$.

**12.** **GRIDDED ANSWER** In a survey of 200 students, 44% chose summer as their favorite season. Of the remaining students, 25% chose winter. How many students chose winter as their favorite season? *TAKS Obj. 9*

*Record your answer and fill in the bubbles on your answer document. Be sure to use the correct place value.*

# CUMULATIVE REVIEW Chapters 1–6

**Find $m\angle 2$ if $\angle 1$ and $\angle 2$ are (a) complementary angles and (b) supplementary angles.** *(p. 24)*

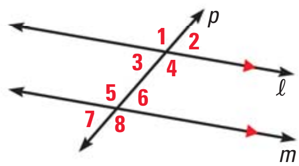
1. $m\angle 1 = 57°$
2. $m\angle 1 = 23°$
3. $m\angle 1 = 88°$
4. $m\angle 1 = 46°$

**Solve the equation and write a reason for each step.** *(p. 105)*

5. $3x - 19 = 47$
6. $30 - 4(x - 3) = -x + 18$
7. $-5(x + 2) = 25$

**State the postulate or theorem that justifies the statement.** *(pp. 147, 154)*

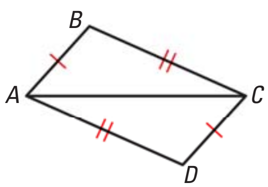
8. $\angle 1 \cong \angle 8$
9. $\angle 3 \cong \angle 6$
10. $m\angle 3 + m\angle 5 = 180°$
11. $\angle 3 \cong \angle 7$
12. $\angle 2 \cong \angle 3$
13. $m\angle 7 + m\angle 8 = 180°$

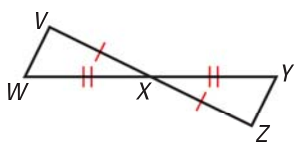


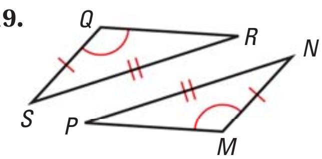

**The variable expressions represent the angle measures of a triangle. Find the measure of each angle. Then classify the triangle by its angles.** *(p. 217)*

14. $m\angle A = x°$
    $m\angle B = 3x°$
    $m\angle C = 4x°$
15. $m\angle A = 2x°$
    $m\angle B = 2x°$
    $m\angle C = (x - 15)°$
16. $m\angle A = (3x - 15)°$
    $m\angle B = (x + 5)°$
    $m\angle C = (x - 20)°$

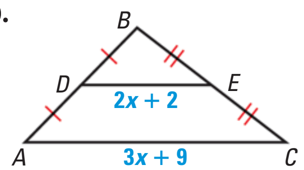
**Determine whether the triangles are congruent. If so, write a congruence statement and state the postulate or theorem you used.** *(pp. 234, 240, 249)*

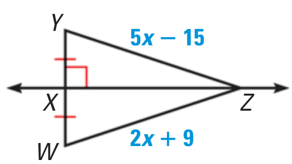
17. 

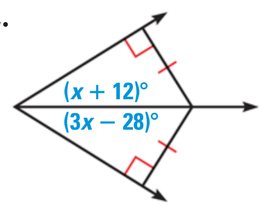

18. 


19. 

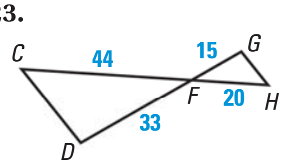

**Find the value of $x$.** *(pp. 295, 303, 310)*

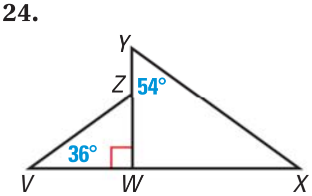
20. 


21. 


22. 


**Determine whether the triangles are similar. If they are, write a similarity statement and state the postulate or theorem you used.** *(pp. 381, 388)*

23. 


24. 


25.