5 Relationships within Triangles

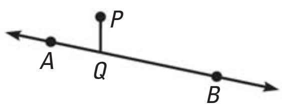
Before

In previous courses and in Chapters 1–4, you learned the following skills, which you'll use in Chapter 5: simplifying expressions, finding distances and slopes, using properties of triangles, and solving equations and inequalities.

Prerequisite Skills

VOCABULARY CHECK

1. Is the *distance from point P to line AB* equal to the length of $\overline{PQ}$? *Explain* why or why not.



SKILLS AND ALGEBRA CHECK

Simplify the expression. All variables are positive. *(Review pp. 139, 870 for 5.1.)*

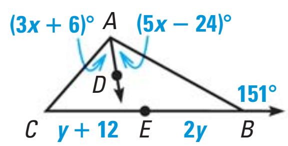
2. $\sqrt{(0-h)^2}$ **3.** $\frac{2m+2n}{2}$ **4.** $|(x+a)-a|$ **5.** $\sqrt{r^2+r^2}$

$\triangle PQR$ has the given vertices. Graph the triangle and classify it by its sides. Then determine if it is a right triangle. *(Review p. 217 for 5.1, 5.4.)*

6. $P(2, 0)$, $Q(6, 6)$, and $R(12, 2)$ **7.** $P(2, 3)$, $Q(4, 7)$, and $R(11, 3)$

Ray AD bisects $\angle BAC$ and point E bisects $\overline{CB}$. Find the measurement. *(Review pp. 15, 24, 217 for 5.2, 5.3, 5.5.)*

8. CE **9.** $m\angle BAC$ **10.** $m\angle ACB$



Solve. *(Review pp. 287, 882 for 5.3, 5.5.)*

11. $x^2 + 24^2 = 26^2$ **12.** $48 + x^2 = 60$ **13.** $43 > x + 35$

TEXAS @HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 5, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 343. You will also use the key vocabulary listed below.

Big Ideas

1. **Using properties of special segments in triangles**
2. **Using triangle inequalities to determine what triangles are possible**
3. **Extending methods for justifying and proving relationships**

KEY VOCABULARY

- midsegment of a triangle, *p. 295*
- coordinate proof, *p. 296*
- perpendicular bisector, *p. 303*
- equidistant, *p. 303*
- point of concurrency, *p. 305*
- circumcenter, *p. 306*
- incenter, *p. 312*
- median of a triangle, *p. 319*
- centroid, *p. 319*
- altitude of a triangle, *p. 320*
- orthocenter, *p. 321*
- indirect proof, *p. 337*

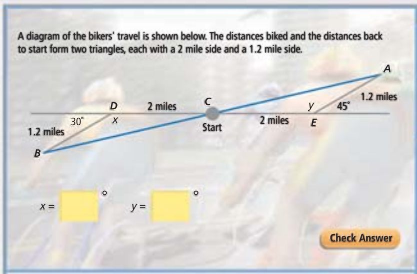
Why?

You can use triangle relationships to find and compare angle measures and distances. For example, if two sides of a triangle represent travel along two roads, then the third side represents the distance back to the starting point.

Animated Geometry

The animation illustrated below for Example 2 on page 336 helps you answer this question: After taking different routes, which group of bikers is farther from the camp?

Two groups of bikers head out from the same point and use different routes.



Enter values for *x* and *y*. Predict which bikers are farther from the start.

Geometry at classzone.com

Animated Geometry at classzone.com

Other animations for Chapter 5: pages 296, 304, 312, 321, and 330

5.1 Investigate Segments in Triangles

MATERIALS • graph paper • ruler • pencil TEKS *G.2.B, G.3.D, G.5.D, G.9.A*

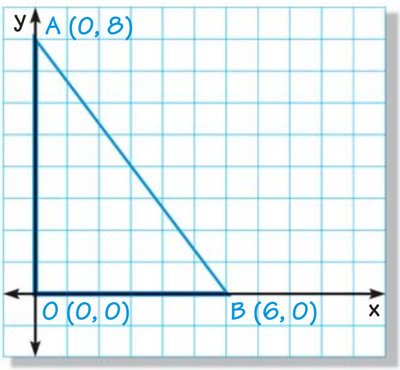
QUESTION How are the midsegments of a triangle related to the sides of the triangle?

A *midsegment* of a triangle connects the midpoints of two sides of a triangle.

EXPLORE Draw and find a midsegment

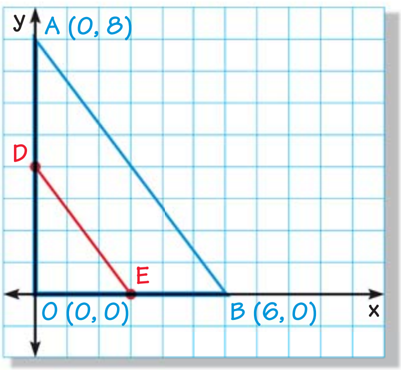
STEP 1 *Draw a right triangle*

Draw a right triangle with legs on the x-axis and the y-axis. Use vertices $A(0, 8)$, $B(6, 0)$, and $O(0, 0)$ as Case 1.



STEP 2 *Draw the midsegment*

Find the midpoints of $\overline{OA}$ and $\overline{OB}$. Plot the midpoints and label them D and E. Connect them to create the midsegment $\overline{DE}$.



STEP 3 *Make a table*

Draw the Case 2 triangle below. Copy and complete the table.

	Case 1	Case 2
O	(0, 0)	(0, 0)
A	(0, 8)	(0, 11)
B	(6, 0)	(5, 0)
D	?	?
E	?	?
Slope of $\overline{AB}$	?	?
Slope of $\overline{DE}$	?	?
Length of $\overline{AB}$	?	?
Length of $\overline{DE}$	?	?

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Choose two other right triangles with legs on the axes. Add these triangles as Cases 3 and 4 to your table.
2. Expand your table in Step 3 for Case 5 with $A(0, n)$, $B(k, 0)$, and $O(0, 0)$.
3. Expand your table in Step 3 for Case 6 with $A(0, 2n)$, $B(2k, 0)$, and $O(0, 0)$.
4. What do you notice about the slopes of $\overline{AB}$ and $\overline{DE}$? What do you notice about the lengths of $\overline{AB}$ and $\overline{DE}$?
5. In each case, is the midsegment $\overline{DE}$ parallel to $\overline{AB}$? *Explain.*
6. Are your observations true for the midsegment created by connecting the midpoints of $\overline{OA}$ and $\overline{AB}$? What about the midsegment connecting the midpoints of $\overline{AB}$ and $\overline{OB}$?
7. Make a conjecture about the relationship between a midsegment and a side of the triangle. Test your conjecture using an acute triangle.

5.1 Midsegment Theorem and Coordinate Proof

TEKS G.3.B, G.4, G.7.A, G.7.C

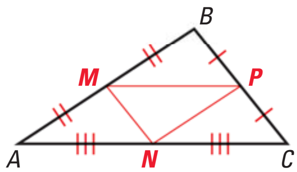
Before You used coordinates to show properties of figures.

Now You will use properties of midsegments and write coordinate proofs.

Why? So you can use indirect measure to find a height, as in Ex. 35.

Key Vocabulary
- **midsegment of a triangle**
- **coordinate proof**

A **midsegment of a triangle** is a segment that connects the midpoints of two sides of the triangle. Every triangle has three midsegments.



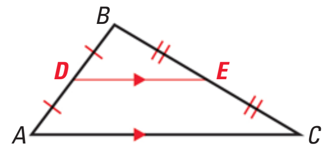
The midsegments of $\triangle ABC$ at the right are $\overline{MP}$, $\overline{MN}$, and $\overline{NP}$.

THEOREM — *For Your Notebook*

THEOREM 5.1 Midsegment Theorem

The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long as that side.

Proof: Example 5, p. 297; Ex. 41, p. 300

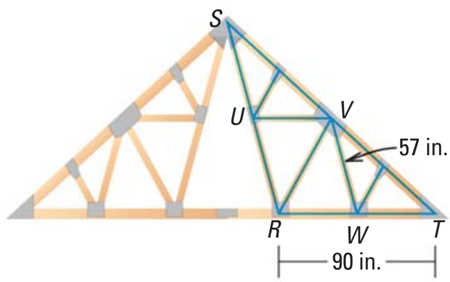


$\overline{DE} \parallel \overline{AC}$ and $DE = \frac{1}{2}AC$

EXAMPLE 1 Use the Midsegment Theorem to find lengths

READ DIAGRAMS
In the diagram for Example 1, midsegment $\overline{UV}$ can be called "the midsegment opposite $\overline{RT}$."

CONSTRUCTION Triangles are used for strength in roof trusses. In the diagram, $\overline{UV}$ and $\overline{VW}$ are midsegments of $\triangle RST$. Find UV and RS.



Solution

$$UV = \frac{1}{2} \cdot RT = \frac{1}{2}(90 \text{ in.}) = 45 \text{ in.}$$

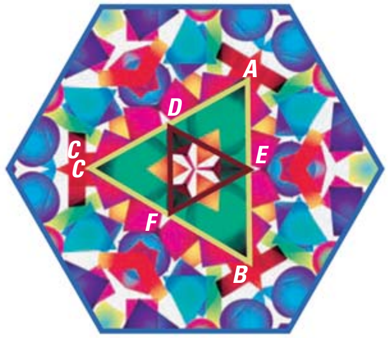
$$RS = 2 \cdot VW = 2(57 \text{ in.}) = 114 \text{ in.}$$

✓ GUIDED PRACTICE for Example 1

1. Copy the diagram in Example 1. Draw and name the third midsegment.
2. In Example 1, suppose the distance UW is 81 inches. Find VS.

EXAMPLE 2 Use the Midsegment Theorem

In the kaleidoscope image, $\overline{AE} \cong \overline{BE}$ and $\overline{AD} \cong \overline{CD}$. Show that $\overline{CB} \parallel \overline{DE}$.



Solution

Because $\overline{AE} \cong \overline{BE}$ and $\overline{AD} \cong \overline{CD}$, E is the midpoint of $\overline{AB}$ and D is the midpoint of $\overline{AC}$ by definition. Then $\overline{DE}$ is a midsegment of $\triangle ABC$ by definition and $\overline{CB} \parallel \overline{DE}$ by the Midsegment Theorem.

COORDINATE PROOF A **coordinate proof** involves placing geometric figures in a coordinate plane. When you use variables to represent the coordinates of a figure in a coordinate proof, the results are true for all figures of that type.

EXAMPLE 3 Place a figure in a coordinate plane

Place each figure in a coordinate plane in a way that is convenient for finding side lengths. Assign coordinates to each vertex.

a. A rectangle

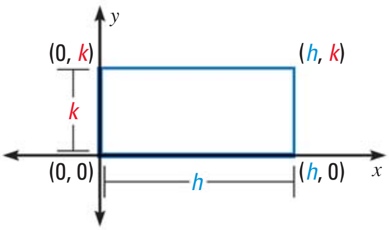
b. A scalene triangle

Solution

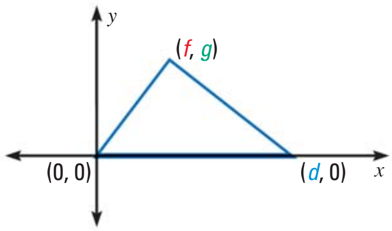
It is easy to find lengths of horizontal and vertical segments and distances from (0, 0), so place one vertex at the origin and one or more sides on an axis.

USE VARIABLES
The rectangle shown represents a general rectangle because the choice of coordinates is based only on the definition of a rectangle. If you use this rectangle to prove a result, the result will be true for all rectangles.

a. Let h represent the length and k represent the width.



b. Notice that you need to use three different variables.



AnimatedGeometry at classzone.com

✓ GUIDED PRACTICE for Examples 2 and 3

3. In Example 2, if F is the midpoint of $\overline{CB}$, what do you know about $\overline{DF}$?

4. Show another way to place the rectangle in part (a) of Example 3 that is convenient for finding side lengths. Assign new coordinates.

5. Is it possible to find any of the side lengths in part (b) of Example 3 without using the Distance Formula? *Explain.*

6. A square has vertices (0, 0), $(m, 0)$, and $(0, m)$. Find the fourth vertex.

EXAMPLE 4 Apply variable coordinates

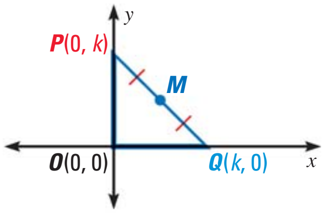
Place an isosceles right triangle in a coordinate plane. Then find the length of the hypotenuse and the coordinates of its midpoint M.

ANOTHER WAY

For an alternative method for solving the problem in Example 4, turn to page 302 for the **Problem Solving Workshop**.

Solution

Place $\triangle PQO$ with the right angle at the origin. Let the length of the legs be k. Then the vertices are located at $P(0, k)$, $Q(k, 0)$, and $O(0, 0)$.



Use the Distance Formula to find PQ.

$$PQ = \sqrt{(k-0)^2 + (0-k)^2} = \sqrt{k^2 + (-k)^2} = \sqrt{k^2 + k^2} = \sqrt{2k^2} = k\sqrt{2}$$

Use the Midpoint Formula to find the midpoint M of the hypotenuse.

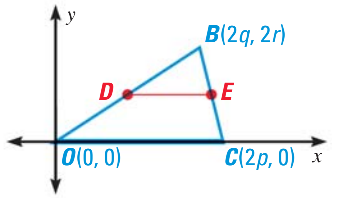
$$M\left(\frac{0+k}{2}, \frac{k+0}{2}\right) = M\left(\frac{k}{2}, \frac{k}{2}\right)$$

EXAMPLE 5 Prove the Midsegment Theorem

Write a coordinate proof of the Midsegment Theorem for one midsegment.

GIVEN ▶ $\overline{DE}$ is a midsegment of $\triangle OBC$.

PROVE ▶ $\overline{DE} \parallel \overline{OC}$ and $DE = \frac{1}{2}OC$



Solution

WRITE PROOFS

You can often assign coordinates in several ways, so choose a way that makes computation easier. In Example 5, you can avoid fractions by using $2p$, $2q$, and $2r$.

STEP 1 **Place** $\triangle OBC$ and assign coordinates. Because you are finding midpoints, use $2p$, $2q$, and $2r$. Then find the coordinates of D and E.

$$D\left(\frac{2q+0}{2}, \frac{2r+0}{2}\right) = D(q, r) \qquad E\left(\frac{2q+2p}{2}, \frac{2r+0}{2}\right) = E(q+p, r)$$

STEP 2 **Prove** $\overline{DE} \parallel \overline{OC}$. The y-coordinates of D and E are the same, so $\overline{DE}$ has a slope of 0. $\overline{OC}$ is on the x-axis, so its slope is 0.

▶ Because their slopes are the same, $\overline{DE} \parallel \overline{OC}$.

STEP 3 **Prove** $DE = \frac{1}{2}OC$. Use the Ruler Postulate to find $\overline{DE}$ and $\overline{OC}$.

$$DE = \left|(q+p) - q\right| = p \qquad OC = \left|2p - 0\right| = 2p$$

▶ So, the length of $\overline{DE}$ is half the length of $\overline{OC}$.

✓ GUIDED PRACTICE for Examples 4 and 5

7. In Example 5, find the coordinates of F, the midpoint of $\overline{OC}$. Then show that $\overline{EF} \parallel \overline{OB}$.

8. Graph the points $O(0, 0)$, $H(m, n)$, and $J(m, 0)$. Is $\triangle OHJ$ a right triangle? Find the side lengths and the coordinates of the midpoint of each side.

5.1 EXERCISES

HOMEWORK KEY

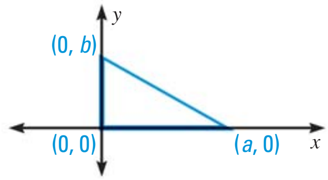
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 21, and 37

★ = **TAKS PRACTICE AND REASONING** Exs. 31, 39, 47, and 48

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: In $\triangle ABC$, D is the midpoint of $\overline{AB}$ and E is the midpoint of $\overline{AC}$. $\overline{DE}$ is a _?_ of $\triangle ABC$.

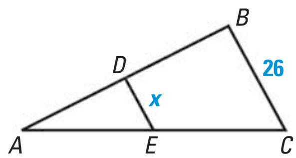
2. **WRITING** *Explain* why it is convenient to place a right triangle on the grid as shown when writing a coordinate proof. How might you want to relabel the coordinates of the vertices if the proof involves midpoints?



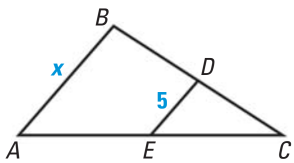
EXAMPLES 1 and 2 on pp. 295–296 for Exs. 3–11

FINDING LENGTHS $\overline{DE}$ **is a midsegment of** $\triangle ABC$**. Find the value of** x**.**

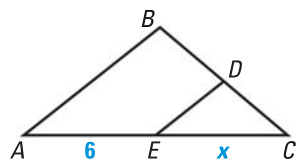
3.



4.



5.



USING THE MIDSEGMENT THEOREM **In** $\triangle XYZ$, $\overline{XJ} \cong \overline{JY}$, $\overline{YL} \cong \overline{LZ}$, **and** $\overline{XK} \cong \overline{KZ}$. **Copy and complete the statement.**

6. $\overline{JK} \parallel$ _?_

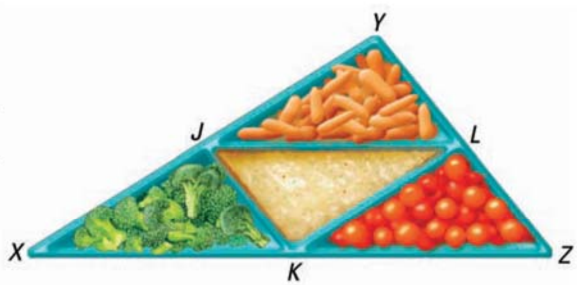
7. $\overline{JL} \parallel$ _?_

8. $\overline{XY} \parallel$ _?_

9. $\overline{YJ} \cong$ _?_ $\cong$ _?_

10. $\overline{JL} \cong$ _?_ $\cong$ _?_

11. $\overline{JK} \cong$ _?_ $\cong$ _?_



EXAMPLE 3 on p. 296 for Exs. 12–19

PLACING FIGURES **Place the figure in a coordinate plane in a convenient way. Assign coordinates to each vertex.**

12. Right triangle: leg lengths are 3 units and 2 units

13. Isosceles right triangle: leg length is 7 units

14. Square: side length is 3 units

15. Scalene triangle: one side length is $2m$

16. Rectangle: length is a and width is b

17. Square: side length is s

18. Isosceles right triangle: leg length is p

19. Right triangle: leg lengths are r and s

EXAMPLES 4 and 5 on p. 297 for Exs. 20–23

20. **COMPARING METHODS** Find the length of the hypotenuse in Exercise 19. Then place the triangle another way and use the new coordinates to find the length of the hypotenuse. Do you get the same result?

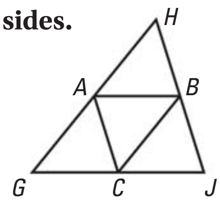
APPLYING VARIABLE COORDINATES **Sketch** $\triangle ABC$**. Find the length and the slope of each side. Then find the coordinates of each midpoint. Is** $\triangle ABC$ **a right triangle? Is it isosceles?** ***Explain.*** **(Assume all variables are positive,** $p \neq q$**, and** $m \neq n$**.)**

21. $A(0, 0)$, $B(p, q)$, $C(2p, 0)$

22. $A(0, 0)$, $B(h, h)$, $C(2h, 0)$

23. $A(0, n)$, $B(m, n)$, $C(m, 0)$

ALGEBRA **Use $\triangle GHJ$, where A, B, and C are midpoints of the sides.**

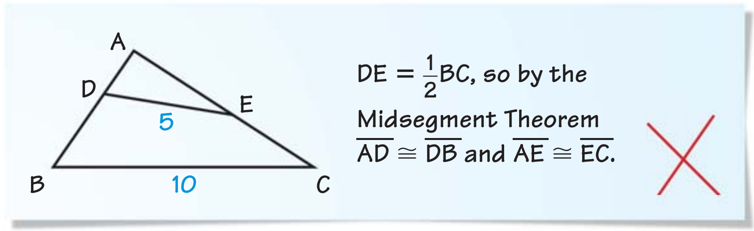


24. If $AB = 3x + 8$ and $GJ = 2x + 24$, what is AB?

25. If $AC = 3y - 5$ and $HJ = 4y + 2$, what is HB?

26. If $GH = 7z - 1$ and $BC = 4z - 3$, what is GH?

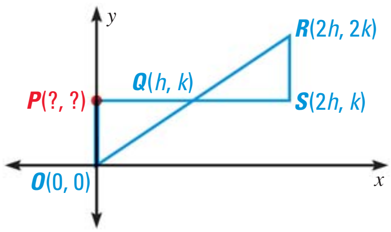
27. **ERROR ANALYSIS** *Explain* why the conclusion is incorrect.



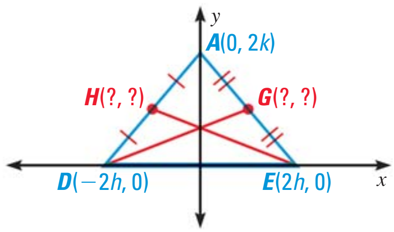
28. **FINDING PERIMETER** The midpoints of the three sides of a triangle are $P(2, 0)$, $Q(7, 12)$, and $R(16, 0)$. Find the length of each midsegment and the perimeter of $\triangle PQR$. Then find the perimeter of the original triangle.

APPLYING VARIABLE COORDINATES **Find the coordinates of the red point(s) in the figure. Then show that the given statement is true.**

29. $\triangle OPQ \cong \triangle RSQ$



30. slope of $\overline{HE} = -$(slope of $\overline{DG}$)



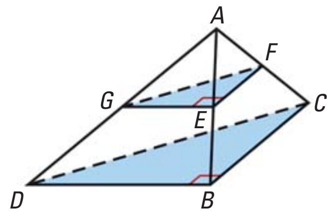
31. **TAKS REASONING** A rectangle with side lengths $3h$ and k has a vertex at $(-h, k)$. Which point *cannot* be a vertex of the rectangle?

Ⓐ (h, k) Ⓑ $(-h, 0)$ Ⓒ $(2h, 0)$ Ⓓ $(2h, k)$

32. **RECONSTRUCTING A TRIANGLE** The points $T(2, 1)$, $U(4, 5)$, and $V(7, 4)$ are the midpoints of the sides of a triangle. Graph the three midsegments. Then show how to use your graph and the properties of midsegments to draw the original triangle. Give the coordinates of each vertex.

33. **3-D FIGURES** Points A, B, C, and D are the vertices of a *tetrahedron* (a solid bounded by four triangles). $\overline{EF}$ is a midsegment of $\triangle ABC$, $\overline{GE}$ is a midsegment of $\triangle ABD$, and $\overline{FG}$ is a midsegment of $\triangle ACD$.

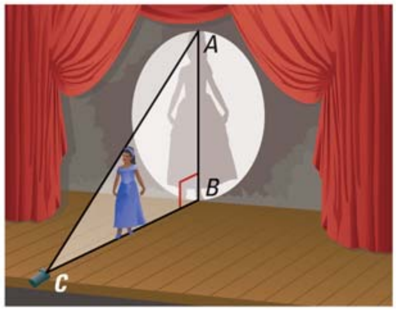
Show that Area of $\triangle EFG = \frac{1}{4} \cdot$ Area of $\triangle BCD$.



34. **CHALLENGE** In $\triangle PQR$, the midpoint of $\overline{PQ}$ is $K(4, 12)$, the midpoint of $\overline{QR}$ is $L(5, 15)$, and the midpoint of $\overline{PR}$ is $M(6.4, 10.8)$. Show how to find the vertices of $\triangle PQR$. *Compare* your work for this exercise with your work for Exercise 32. How were your methods different?

PROBLEM SOLVING

35. **FLOODLIGHTS** A floodlight on the edge of the stage shines upward onto the curtain as shown. Constance is 5 feet tall. She stands halfway between the light and the curtain, and the top of her head is at the midpoint of $\overline{AC}$. The edge of the light just reaches the top of her head. How tall is her shadow?



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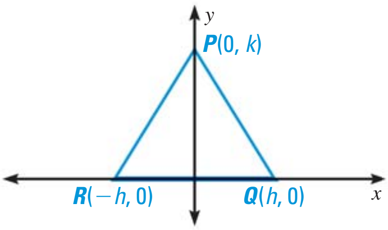
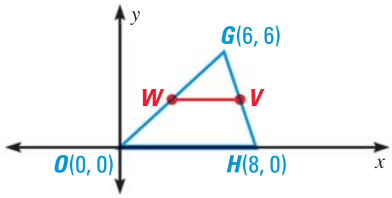
EXAMPLE 5 on p. 297 for Exs. 36–37

COORDINATE PROOF **Write a coordinate proof.**

36. **GIVEN** ▶ $P(0, k)$, $Q(h, 0)$, $R(-h, 0)$
PROVE ▶ $\triangle PQR$ is isosceles.



37. **GIVEN** ▶ $O(0, 0)$, $G(6, 6)$, $H(8, 0)$, $\overline{WV}$ is a midsegment.
PROVE ▶ $\overline{WV} \parallel \overline{OH}$ and $WV = \frac{1}{2}OH$



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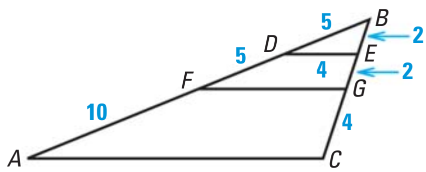
38. **CARPENTRY** In the set of shelves shown, the third shelf, labeled $\overline{CD}$, is closer to the bottom shelf, $\overline{EF}$, than midsegment $\overline{AB}$ is. If $\overline{EF}$ is 8 feet long, is it possible for $\overline{CD}$ to be 3 feet long? 4 feet long? 6 feet long? 8 feet long? *Explain.*

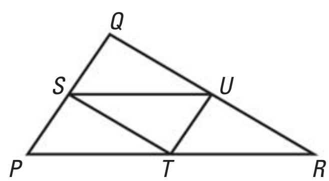
39. **TAKS REASONING** Use the information in the diagram at the right. What is the length of side $\overline{AC}$ of $\triangle ABC$? *Explain* your reasoning.



40. **PLANNING FOR PROOF** Copy and complete the plan for proof.

GIVEN ▶ $\overline{ST}$, $\overline{TU}$, and $\overline{SU}$ are midsegments of $\triangle PQR$.
PROVE ▶ $\triangle PST \cong \triangle SQU$

Use __?__ to show that $\overline{PS} \cong \overline{SQ}$. Use __?__ to show that $\angle QSU \cong \angle SPT$. Use __?__ to show that $\angle$ __?__ $\cong \angle$ __?__. Use __?__ to show that $\triangle PST \cong \triangle SQU$.



41. **PROVING THEOREM 5.1** Use the figure in Example 5. Draw the midpoint F of $\overline{OC}$. Prove that $\overline{DF}$ is parallel to $\overline{BC}$ and $DF = \frac{1}{2}BC$.

○ = WORKED-OUT SOLUTIONS on p. WS1

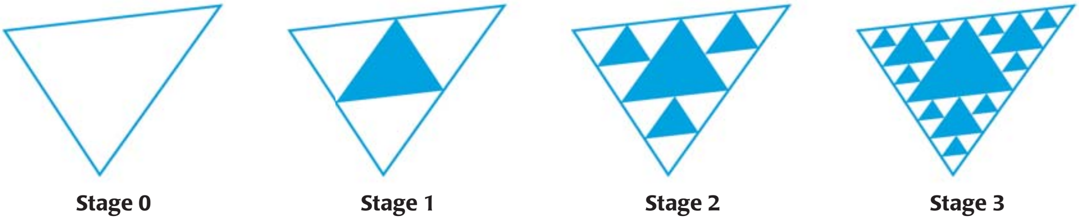
= TAKS PRACTICE AND REASONING

42. **COORDINATE PROOF** Write a coordinate proof.

GIVEN ▶ $\triangle ABD$ is a right triangle, with the right angle at vertex A. Point C is the midpoint of hypotenuse BD.

PROVE ▶ Point C is the same distance from each vertex of $\triangle ABD$.

43. **MULTI-STEP PROBLEM** To create the design below, shade the triangle formed by the three midsegments of a triangle. Then repeat the process for each unshaded triangle. Let the perimeter of the original triangle be 1.



a. What is the perimeter of the triangle that is shaded in Stage 1?

b. What is the total perimeter of all the shaded triangles in Stage 2?

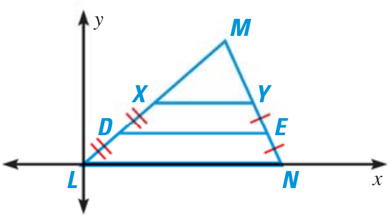
c. What is the total perimeter of all the shaded triangles in Stage 3?

RIGHT ISOSCELES TRIANGLES **In Exercises 44 and 45, write a coordinate proof.**

44. Any right isosceles triangle can be subdivided into a pair of congruent right isosceles triangles. (*Hint:* Draw the segment from the right angle to the midpoint of the hypotenuse.)

45. Any two congruent right isosceles triangles can be combined to form a single right isosceles triangle.

46. **CHALLENGE** XY is a midsegment of $\triangle LMN$. Suppose $\overline{DE}$ is called a "quarter-segment" of $\triangle LMN$. What do you think an "eighth-segment" would be? Make a conjecture about the properties of a quarter-segment and of an eighth-segment. Use variable coordinates to verify your conjectures.

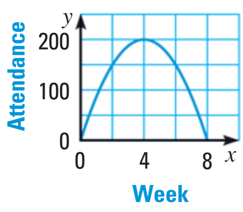


MIXED REVIEW FOR TAKS

TAKS PRACTICE at classzone.com

REVIEW
Skills Review Handbook p. 888;
TAKS Workbook

47. **TAKS PRACTICE** The attendance for a movie is shown in the graph. Which statement best describes the attendance? ***TAKS Obj. 5***



Ⓐ Attendance rapidly increased and then leveled off.

Ⓑ Attendance rapidly increased and then rapidly decreased.

Ⓒ Attendance rapidly increased and then gradually decreased.

Ⓓ Attendance gradually increased and then rapidly decreased.

REVIEW
TAKS Preparation p. 208;
TAKS Workbook

48. **TAKS PRACTICE** Which statement best describes the effect on the graph of $f(x) = 3x - 5$ if the y-intercept is changed to 4? ***TAKS Obj. 3***

Ⓕ The slope increases. Ⓖ The slope decreases.

Ⓗ The x-intercept increases. Ⓙ The x-intercept decreases.

Using ALTERNATIVE METHODS

TEKS G.4, G.7.A, G.7.B, G.7.C

Another Way to Solve Example 4, page 297

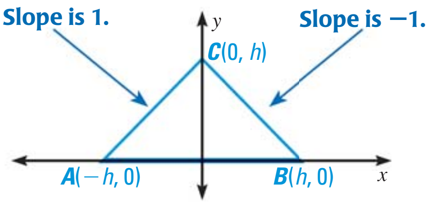
MULTIPLE REPRESENTATIONS When you write a coordinate proof, you often have several options for how to place the figure in the coordinate plane and how to assign variables.

PROBLEM

Place an isosceles right triangle in a coordinate plane. Then find the length of the hypotenuse and the coordinates of its midpoint M.

METHOD

Placing Hypotenuse on an Axis Place the triangle with point C at $(0, h)$ on the y-axis and the hypotenuse $\overline{AB}$ on the x-axis. To make $\angle ACB$ be a right angle, position A and B so that legs $\overline{CA}$ and $\overline{CB}$ have slopes of 1 and -1.



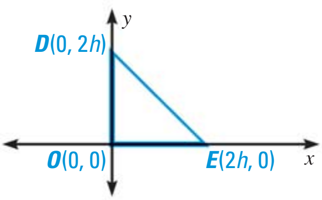
Length of hypotenuse $= 2h$

$$M = \left(\frac{-h+h}{2}, \frac{0+0}{2}\right) = (0, 0)$$

PRACTICE

1. **VERIFYING TRIANGLE PROPERTIES** Verify that $\angle C$ above is a right angle. Verify that $\triangle ABC$ is isosceles by showing $AC = BC$.

2. **MULTIPLES OF 2** Find the midpoint and length of each side using the placement below. What is the advantage of using $2h$ instead of h for the leg lengths?



3. **OTHER ALTERNATIVES** Graph $\triangle JKL$ and verify that it is an isosceles right triangle. Then find the length and midpoint of $\overline{JK}$.

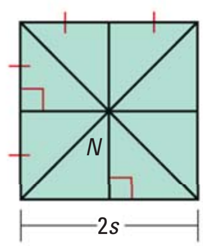
   a. $J(0, 0)$, $K(h, h)$, $L(h, 0)$

   b. $J(-2h, 0)$, $K(2h, 0)$, $L(0, 2h)$

4. **CHOOSE** Suppose you need to place a right isosceles triangle on a coordinate grid and assign variable coordinates. You know you will need to find all three side lengths and all three midpoints. How would you place the triangle? *Explain* your reasoning.

5. **RECTANGLES** Place rectangle $PQRS$ with length m and width n in the coordinate plane. Draw $\overline{PR}$ and $\overline{QS}$ connecting opposite corners of the rectangle. Then use coordinates to show that $\overline{PR} \cong \overline{QS}$.

6. **PARK** A square park has paths as shown. Use coordinates to determine whether a snack cart at point N is the same distance from each corner.



5.2 Use Perpendicular Bisectors

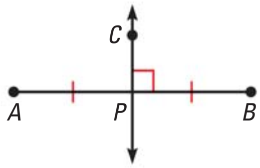
TEKS G.1.A, G.2.A, G.9.A, G.9.B

Before	You used segment bisectors and perpendicular lines.
Now	You will use perpendicular bisectors to solve problems.
Why?	So you can solve a problem in archaeology, as in Ex. 28.

Key Vocabulary
- **perpendicular bisector**
- **equidistant**
- **concurrent**
- **point of concurrency**
- **circumcenter**

In Lesson 1.3, you learned that a segment bisector intersects a segment at its midpoint. A segment, ray, line, or plane that is perpendicular to a segment at its midpoint is called a **perpendicular bisector**.

A point is **equidistant** from two figures if the point is the *same distance* from each figure. Points on the perpendicular bisector of a segment are equidistant from the segment's endpoints.



$\overleftrightarrow{CP}$ is a $\perp$ bisector of $\overline{AB}$.

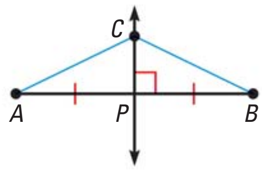
THEOREMS — *For Your Notebook*

THEOREM 5.2 Perpendicular Bisector Theorem

In a plane, if a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

If $\overleftrightarrow{CP}$ is the $\perp$ bisector of $\overline{AB}$, then $CA = CB$.

Proof: Ex. 26, p. 308

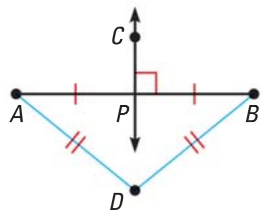


THEOREM 5.3 Converse of the Perpendicular Bisector Theorem

In a plane, if a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

If $DA = DB$, then D lies on the $\perp$ bisector of $\overline{AB}$.

Proof: Ex. 27, p. 308

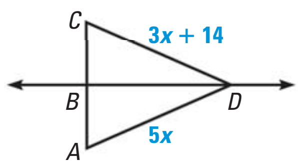


EXAMPLE 1 Use the Perpendicular Bisector Theorem

xy ALGEBRA $\overleftrightarrow{BD}$ **is the perpendicular bisector of** $\overline{AC}$**. Find** AD**.**

$AD = CD$	Perpendicular Bisector Theorem
$5x = 3x + 14$	Substitute.
$x = 7$	Solve for x.

▶ $AD = 5x = 5(7) = 35.$

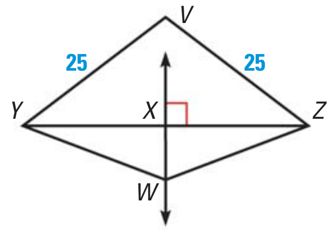


EXAMPLE 2 Use perpendicular bisectors

In the diagram, $\overleftrightarrow{WX}$ is the perpendicular bisector of $\overline{YZ}$.

a. What segment lengths in the diagram are equal?

b. Is V on $\overleftrightarrow{WX}$?



Solution

a. $\overleftrightarrow{WX}$ bisects $\overline{YZ}$, so $XY = XZ$. Because W is on the perpendicular bisector of $\overline{YZ}$, $WY = WZ$ by Theorem 5.2. The diagram shows that $VY = VZ = 25$.

b. Because $VY = VZ$, V is equidistant from Y and Z. So, by the Converse of the Perpendicular Bisector Theorem, V is on the perpendicular bisector of $\overline{YZ}$, which is $\overleftrightarrow{WX}$.

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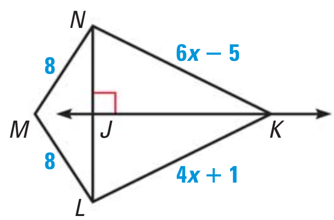
✓ GUIDED PRACTICE for Examples 1 and 2

In the diagram, $\overleftrightarrow{JK}$ is the perpendicular bisector of $\overline{NL}$.

1. What segment lengths are equal? *Explain* your reasoning.
2. Find NK.
3. *Explain* why M is on $\overleftrightarrow{JK}$.



ACTIVITY FOLD THE PERPENDICULAR BISECTORS OF A TRIANGLE

QUESTION Where do the perpendicular bisectors of a triangle meet?

Materials:
- paper
- scissors
- ruler

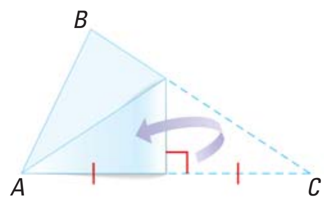
Follow the steps below and answer the questions about perpendicular bisectors of triangles.

STEP 1 Cut four large acute scalene triangles out of paper. Make each one different.

STEP 2 Choose one triangle. Fold it to form the perpendicular bisectors of the sides. Do the three bisectors intersect at the same point?

STEP 3 Repeat the process for the other three triangles. Make a conjecture about the perpendicular bisectors of a triangle.

STEP 4 Choose one triangle. Label the vertices A, B, and C. Label the point of intersection of the perpendicular bisectors as P. Measure $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$. What do you observe?



CONCURRENCY When three or more lines, rays, or segments intersect in the same point, they are called **concurrent** lines, rays, or segments. The point of intersection of the lines, rays, or segments is called the **point of concurrency**.

READ VOCABULARY
The perpendicular bisector of a side of a triangle can be referred to as a *perpendicular bisector of the triangle.*

As you saw in the Activity on page 304, the three perpendicular bisectors of a triangle are concurrent and the point of concurrency has a special property.

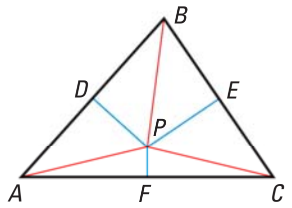
THEOREM *For Your Notebook*

THEOREM 5.4 Concurrency of Perpendicular Bisectors of a Triangle

The perpendicular bisectors of a triangle intersect at a point that is equidistant from the vertices of the triangle.

If $\overline{PD}$, $\overline{PE}$, and $\overline{PF}$ are perpendicular bisectors, then $PA = PB = PC$.

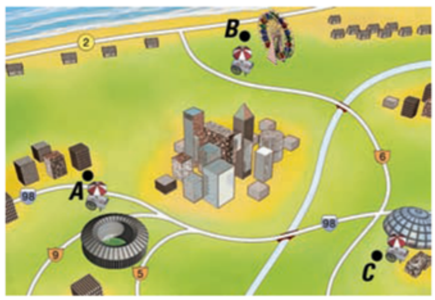
Proof: p. 933



EXAMPLE 3 Use the concurrency of perpendicular bisectors

FROZEN YOGURT Three snack carts sell frozen yogurt from points A, B, and C outside a city. Each of the three carts is the same distance from the frozen yogurt distributor.

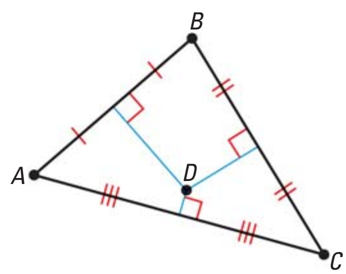
Find a location for the distributor that is equidistant from the three carts.



Solution

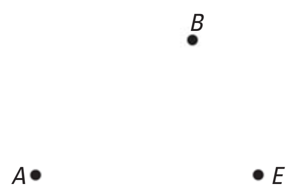
Theorem 5.4 shows you that you can find a point equidistant from three points by using the perpendicular bisectors of the triangle formed by those points.

Copy the positions of points A, B, and C and connect those points to draw $\triangle ABC$. Then use a ruler and protractor to draw the three perpendicular bisectors of $\triangle ABC$. The point of concurrency D is the location of the distributor.



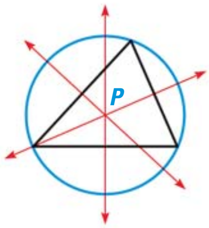
✓ **GUIDED PRACTICE for Example 3**

4. **WHAT IF?** Hot pretzels are sold from points A and B and also from a cart at point E. Where could the pretzel distributor be located if it is equidistant from those three points? Sketch the triangle and show the location.

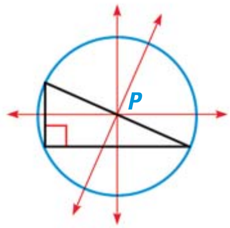


READ VOCABULARY
The prefix *circum-* means "around" or "about" as in *circumference* (distance around a circle).

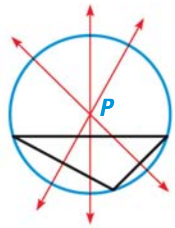
CIRCUMCENTER The point of concurrency of the three perpendicular bisectors of a triangle is called the **circumcenter** of the triangle. The circumcenter P is equidistant from the three vertices, so P is the center of a circle that passes through all three vertices.



Acute triangle
***P* is inside triangle.**



Right triangle
***P* is on triangle.**



Obtuse triangle
***P* is outside triangle.**

As shown above, the location of P depends on the type of triangle. The circle with the center P is said to be *circumscribed* about the triangle.

5.2 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 15, 17, and 25
★ = **TAKS PRACTICE AND REASONING** Exs. 9, 25, 28, 34, and 35

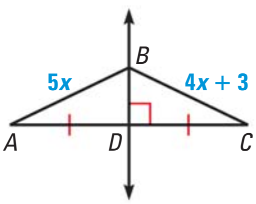
SKILL PRACTICE

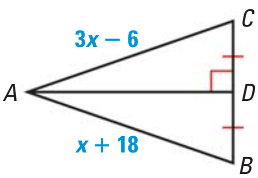
1. **VOCABULARY** Suppose you draw a circle with a compass. You choose three points on the circle to use as the vertices of a triangle. Copy and complete: The center of the circle is also the __?__ of the triangle.

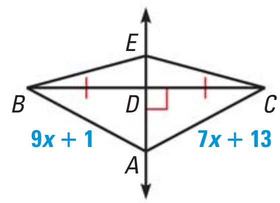
2. **WRITING** Consider $\overline{AB}$. How can you *describe* the set of all points in a plane that are equidistant from A and B?

EXAMPLES 1 and 2 on pp. 303–304 for Exs. 3–15

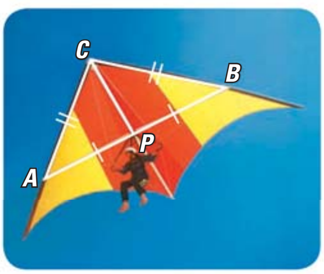
xy ALGEBRA Find the length of $\overline{AB}$.

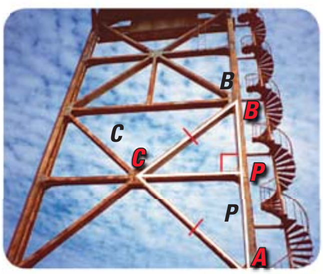
3. 

4. 

5. 

REASONING Tell whether the information in the diagram allows you to conclude that C is on the perpendicular bisector of $\overline{AB}$.

6. 

7. 

8. 

9. **TAKS REASONING** Point P is inside $\triangle ABC$ and is equidistant from points A and B. On which of the following segments must P be located?

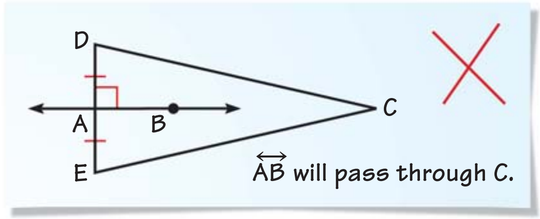
(A) $\overline{AB}$

(B) The perpendicular bisector of $\overline{AB}$

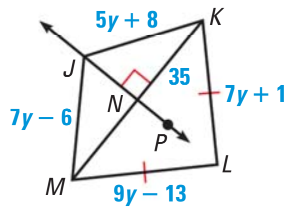
(C) The midsegment opposite $\overline{AB}$

(D) The perpendicular bisector of $\overline{AC}$

10. **ERROR ANALYSIS** *Explain* why the conclusion is not correct given the information in the diagram.



PERPENDICULAR BISECTORS **In Exercises 11–15, use the diagram. $\overleftrightarrow{JN}$ is the perpendicular bisector of $\overline{MK}$.**



11. Find NM.

12. Find JK.

13. Find KL.

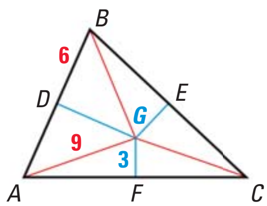
14. Find ML.

15. Is L on $\overleftrightarrow{JP}$? *Explain* your reasoning.

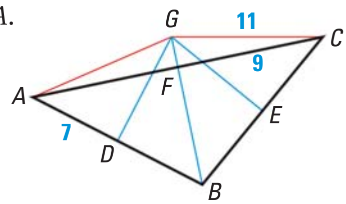
EXAMPLE 3
on p. 305
for Exs. 16–17

USING CONCURRENCY **In the diagram, the perpendicular bisectors of $\triangle ABC$ meet at point G and are shown in blue. Find the indicated measure.**

16. Find BG.



17. Find GA.



18. **CONSTRUCTING PERPENDICULAR BISECTORS** Use the construction shown on page 33 to construct the bisector of a segment. *Explain* why the bisector you constructed is actually the perpendicular bisector.

19. **CONSTRUCTION** Draw a right triangle. Use a compass and straightedge to find its circumcenter. Use a compass to draw the circumscribed circle.

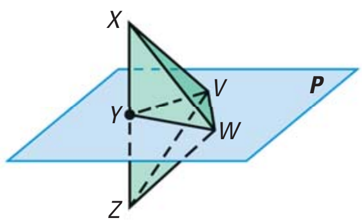
ANALYZING STATEMENTS **Copy and complete the statement with *always*, *sometimes*, or *never*. *Justify* your answer.**

20. The circumcenter of a scalene triangle is __?__ inside the triangle.

21. If the perpendicular bisector of one side of a triangle goes through the opposite vertex, then the triangle is __?__ isosceles.

22. The perpendicular bisectors of a triangle intersect at a point that is __?__ equidistant from the midpoints of the sides of the triangle.

23. **CHALLENGE** Prove the statements in parts (a) – (c).



GIVEN ▶ Plane P is a perpendicular bisector of $\overline{XZ}$ at Y.

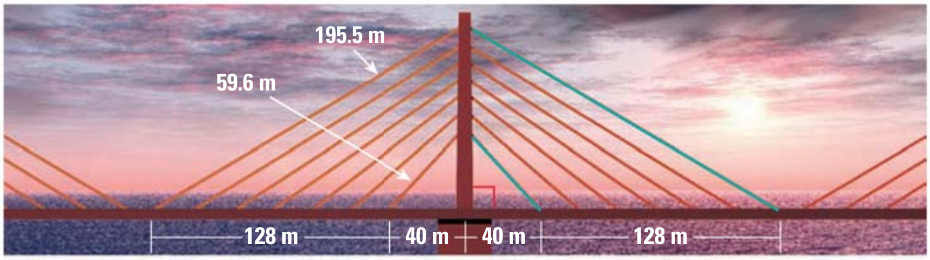
PROVE ▶ **a.** $\overline{XW} \cong \overline{ZW}$

b. $\overline{XV} \cong \overline{ZV}$

c. $\angle VXW \cong \angle VZW$

PROBLEM SOLVING

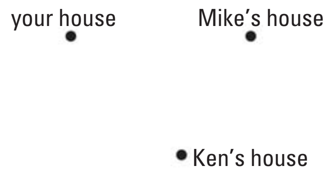
24. BRIDGE A cable-stayed bridge is shown below. Two cable lengths are given. Find the lengths of the blue cables. *Justify* your answer.



TEXAS @HomeTutor for problem solving help at classzone.com

EXAMPLE 3
on p. 305
for Exs. 25, 28

25. **TAKS REASONING** You and two friends plan to walk your dogs together. You want your meeting place to be the same distance from each person's house. *Explain* how you can use the diagram to locate the meeting place.



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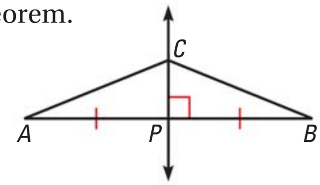
26. PROVING THEOREM 5.2 Prove the Perpendicular Bisector Theorem.

GIVEN ▶ $\overleftrightarrow{CP}$ is the perpendicular bisector of $\overline{AB}$.

PROVE ▶ $CA = CB$

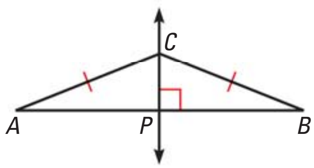
Plan for Proof Show that right triangles $\triangle APC$ and $\triangle BPC$ are congruent. Then show that $\overline{CA} \cong \overline{CB}$.



27. PROVING THEOREM 5.3 Prove the converse of Theorem 5.2. (*Hint:* Construct a line through C perpendicular to $\overline{AB}$.)

GIVEN ▶ $CA = CB$

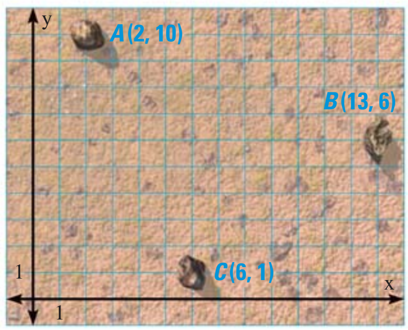
PROVE ▶ C is on the perpendicular bisector of $\overline{AB}$.



28. **TAKS REASONING** Archaeologists find three stones. They believe that the stones were once part of a circle of stones with a community firepit at its center. They mark the locations of Stones A, B, and C on a graph where distances are measured in feet.

a. *Explain* how the archaeologists can use a sketch to estimate the center of the circle of stones.

b. Copy the diagram and find the approximate coordinates of the point at which the archaeologists should look for the firepit.



29. TECHNOLOGY Use geometry drawing software to construct $\overline{AB}$. Find the midpoint C. Draw the perpendicular bisector of $\overline{AB}$ through C. Construct a point D along the perpendicular bisector and measure $\overline{DA}$ and $\overline{DB}$. Move D along the perpendicular bisector. What theorem does this construction demonstrate?

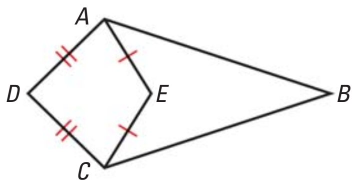
= **WORKED-OUT SOLUTIONS** on p. WS1

= **TAKS PRACTICE AND REASONING**

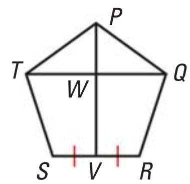
30. **COORDINATE PROOF** Where is the circumcenter located in any right triangle? Write a coordinate proof of this result.

PROOF **Use the information in the diagram to prove the given statement.**

31. $\overline{AB} \cong \overline{BC}$ if and only if D, E, and B are collinear.



32. $\overline{PV}$ is the perpendicular bisector of $\overline{TQ}$ for regular polygon $PQRST$.



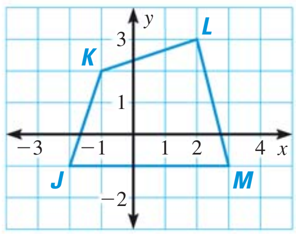
33. **CHALLENGE** The four towns on the map are building a common high school. They have agreed that the school should be an equal distance from each of the four towns. Is there a single point where they could agree to build the school? If so, find it. If not, *explain* why not. Use a diagram to *explain* your answer.

TAKS PRACTICE at classzone.com

MIXED REVIEW FOR TAKS

REVIEW
Lesson 4.8;
TAKS Workbook

34. **TAKS PRACTICE** Quadrilateral $JKLM$ is reflected across the x-axis to become quadrilateral $WXYZ$. What are the coordinates of Z? *TAKS Obj. 6*



Ⓐ (3, 1) Ⓑ (3, −1)

Ⓒ (−1, 3) Ⓓ (−3, −1)

REVIEW
Skills Review Handbook p. 894;
TAKS Workbook

35. **TAKS PRACTICE** Rachel is building a rectangular patio that measures 18 feet by 12 feet. She uses 1 foot by 1 foot square patio blocks that cost \$1.49 each. Which expression can Rachel use to find the total cost? *TAKS Obj. 10*

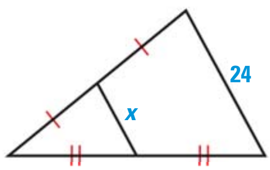
Ⓕ 18 • 12 + 1.49 Ⓖ 18 • 12 • 1.49

Ⓗ 18 + 12 + 1.49 Ⓙ (18 + 12) • 1.49

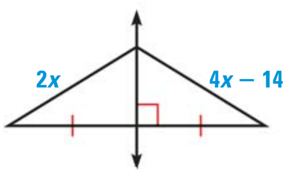
QUIZ *for Lessons 5.1–5.2*

Find the value of x. Identify the theorem used to find the answer. *(pp. 295, 303)*

1.



2.



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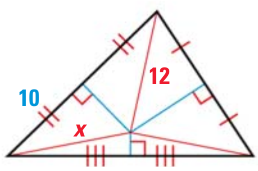


4. Graph the triangle $R(2a, 0)$, $S(0, 2b)$, $T(2a, 2b)$, where a and b are positive. Find RT and ST. Then find the slope of $\overline{SR}$ and the coordinates of the midpoint of $\overline{SR}$. *(p. 295)*

5.3 Use Angle Bisectors of Triangles

TEKS G.5.A, G.5.D, G.8.C, G.9.B

Before You used angle bisectors to find angle relationships.

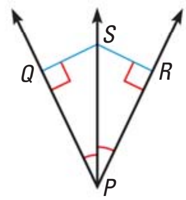
Now You will use angle bisectors to find distance relationships.

Why? So you can apply geometry in sports, as in Example 2.

Key Vocabulary
- **incenter**
- **angle bisector,** *p. 28*
- **distance from a point to a line,** *p. 192*

Remember that an *angle bisector* is a ray that divides an angle into two congruent adjacent angles. Remember also that the *distance from a point to a line* is the length of the perpendicular segment from the point to the line.

So, in the diagram, $\overrightarrow{PS}$ is the bisector of $\angle QPR$ and the distance from S to $\overrightarrow{PQ}$ is SQ, where $\overline{SQ} \perp \overrightarrow{PQ}$.

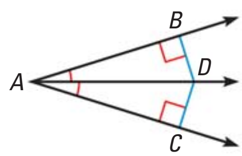


THEOREMS *For Your Notebook*

THEOREM 5.5 Angle Bisector Theorem

If a point is on the bisector of an angle, then it is equidistant from the two sides of the angle.

If $\overrightarrow{AD}$ bisects $\angle BAC$ and $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$, then $\boldsymbol{DB = DC}$.

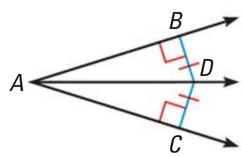


Proof: Ex. 34, p. 315

THEOREM 5.6 Converse of the Angle Bisector Theorem

If a point is in the interior of an angle and is equidistant from the sides of the angle, then it lies on the bisector of the angle.

If $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$ and $\boldsymbol{DB = DC}$, then $\overrightarrow{AD}$ bisects $\angle BAC$.

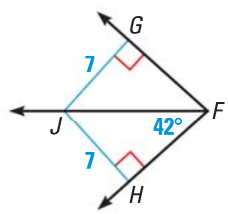


Proof: Ex. 35, p. 315

REVIEW DISTANCE
In Geometry, *distance* means the *shortest* length between two objects.

EXAMPLE 1 Use the Angle Bisector Theorems

Find the measure of $\angle GFJ$.

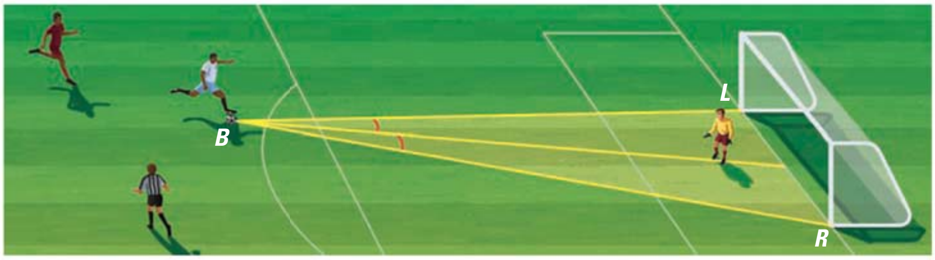


Solution

Because $\overline{JG} \perp \overrightarrow{FG}$ and $\overline{JH} \perp \overrightarrow{FH}$ and $JG = JH = 7$, $\overrightarrow{FJ}$ bisects $\angle GFH$ by the Converse of the Angle Bisector Theorem. So, $m\angle GFJ = m\angle HFJ = 42°$.

EXAMPLE 2 Solve a real-world problem

SOCCER A soccer goalie's position relative to the ball and goalposts forms congruent angles, as shown. Will the goalie have to move farther to block a shot toward the right goalpost R or the left goalpost L?



Solution

The congruent angles tell you that the goalie is on the bisector of $\angle LBR$. By the Angle Bisector Theorem, the goalie is equidistant from $\overrightarrow{BR}$ and $\overrightarrow{BL}$.

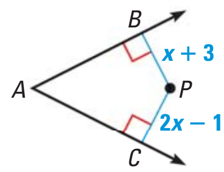
▶ So, the goalie must move the same distance to block either shot.

EXAMPLE 3 Use algebra to solve a problem

xy ALGEBRA **For what value of x does P lie on the bisector of $\angle A$?**

Solution

From the Converse of the Angle Bisector Theorem, you know that P lies on the bisector of $\angle A$ if P is equidistant from the sides of $\angle A$, so when $BP = CP$.



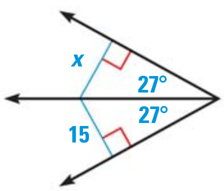
$BP = CP$	**Set segment lengths equal.**
$x + 3 = 2x - 1$	**Substitute expressions for segment lengths.**
$4 = x$	**Solve for x.**

▶ Point P lies on the bisector of $\angle A$ when $x = 4$.

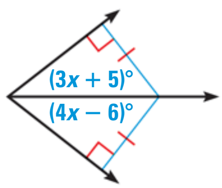
✓ GUIDED PRACTICE for Examples 1, 2, and 3

In Exercises 1–3, find the value of x.

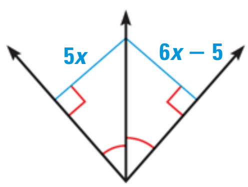
1.



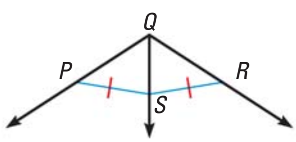
2.



3.



4. Do you have enough information to conclude that $\overrightarrow{QS}$ bisects $\angle PQR$? *Explain.*



READ VOCABULARY

An *angle bisector of a triangle* is the bisector of an interior angle of the triangle.

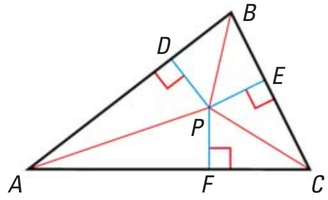
THEOREM — *For Your Notebook*

THEOREM 5.7 Concurrency of Angle Bisectors of a Triangle

The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle.

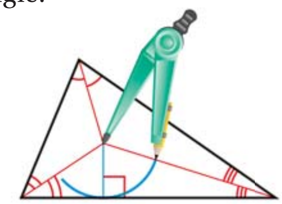
If $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$ are angle bisectors of $\triangle ABC$, then $PD = PE = PF$.

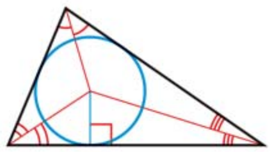
Proof: Ex. 36, p. 316



The point of concurrency of the three angle bisectors of a triangle is called the **incenter** of the triangle. The incenter always lies inside the triangle.

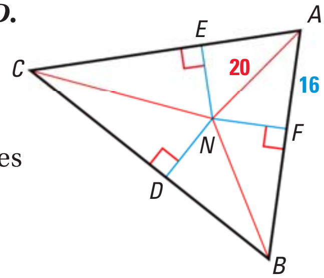
Because the incenter P is equidistant from the three sides of the triangle, a circle drawn using P as the center and the distance to one side as the radius will just touch the other two sides. The circle is said to be *inscribed* within the triangle.





EXAMPLE 4 Use the concurrency of angle bisectors

In the diagram, N is the incenter of $\triangle ABC$. Find ND.



Solution

By the Concurrency of Angle Bisectors of a Triangle Theorem, the incenter N is equidistant from the sides of $\triangle ABC$. So, to find ND, you can find NF in $\triangle NAF$. Use the Pythagorean Theorem stated on page 18.

REVIEW QUADRATIC EQUATIONS

For help with solving a quadratic equation by taking square roots, see page 882. Use only the positive square root when finding a distance, as in Example 4.

$c^2 = a^2 + b^2$	**Pythagorean Theorem**
$20^2 = NF^2 + 16^2$	**Substitute known values.**
$400 = NF^2 + 256$	**Multiply.**
$144 = NF^2$	**Subtract 256 from each side.**
$12 = NF$	**Take the positive square root of each side.**

▶ Because $NF = ND$, $ND = 12$.

Animated **Geometry** at classzone.com

✓ GUIDED PRACTICE for Example 4

5. **WHAT IF?** In Example 4, suppose you are not given AF or AN, but you are given that $BF = 12$ and $BN = 13$. Find ND.

5.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 15, and 29

★ = **TAKS PRACTICE AND REASONING** Exs. 18, 23, 30, 31, 39, 40, and 41

SKILL PRACTICE

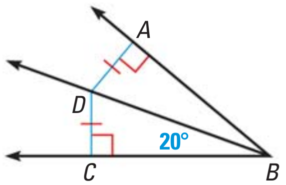
1. **VOCABULARY** Copy and complete: Point C is in the interior of $\angle ABD$. If $\angle ABC$ and $\angle DBC$ are congruent, then $\overrightarrow{BC}$ is the __?__ of $\angle ABD$.

2. **WRITING** How are perpendicular bisectors and angle bisectors of a triangle different? How are they alike?

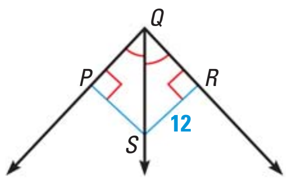
EXAMPLE 1 on p. 310 for Exs. 3–5

FINDING MEASURES Use the information in the diagram to find the measure.

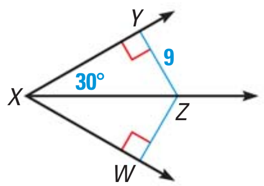
3. Find $m\angle ABD$.



4. Find PS.



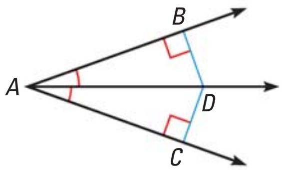
5. $m\angle YXW = 60°$. Find WZ.



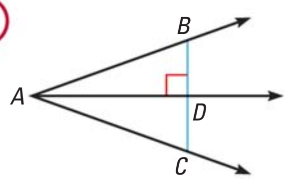
EXAMPLE 2 on p. 311 for Exs. 6–11

ANGLE BISECTOR THEOREM Is $DB = DC$? *Explain.*

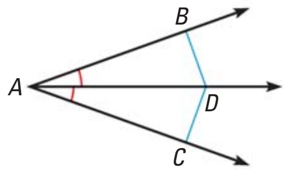
6.



7.

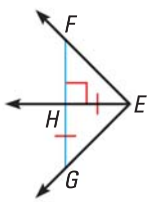


8.

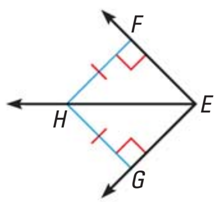


REASONING Can you conclude that $\overrightarrow{EH}$ bisects $\angle FEG$? *Explain.*

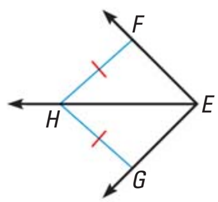
9.



10.



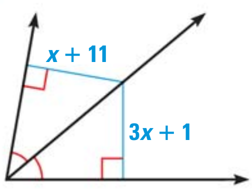
11.



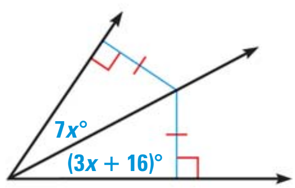
EXAMPLE 3 on p. 311 for Exs. 12–18

xy ALGEBRA Find the value of x.

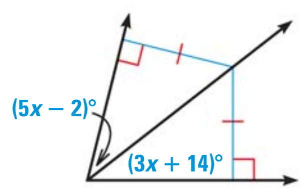
12.



13.

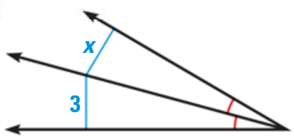


14.

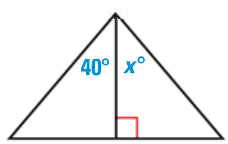


RECOGNIZING MISSING INFORMATION Can you find the value of x? *Explain.*

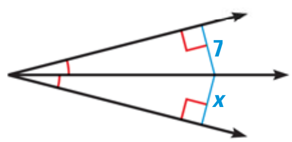
15.



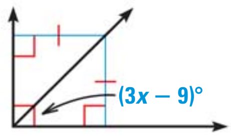
16.



17.



18. **TAKS REASONING** What is the value of x in the diagram?



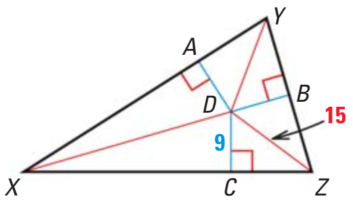
Ⓐ 13 Ⓑ 18

Ⓒ 33 Ⓓ Not enough information

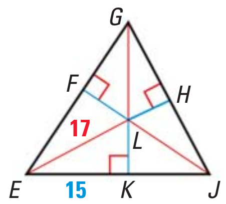
EXAMPLE 4
on p. 312
for Exs. 19–22

USING INCENTERS **Find the indicated measure.**

19. Point D is the incenter of $\triangle XYZ$. Find DB.

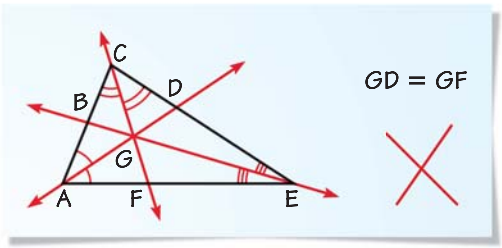


20. Point L is the incenter of $\triangle EGJ$. Find HL.

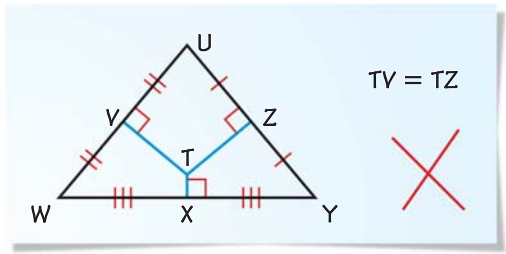


ERROR ANALYSIS ***Describe*** **the error in reasoning. Then state a correct conclusion about distances that can be deduced from the diagram.**

21.



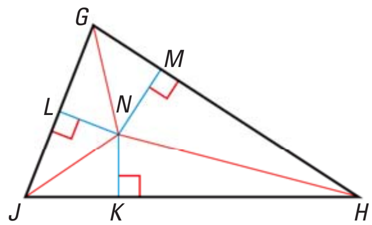
22.



23. **TAKS REASONING** In the diagram, N is the incenter of $\triangle GHJ$. Which statement cannot be deduced from the given information?

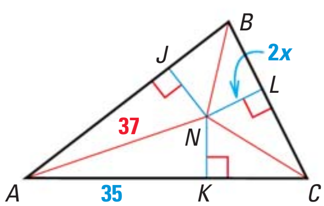
Ⓐ $\overline{NM} \cong \overline{NK}$ Ⓑ $\overline{NL} \cong \overline{NM}$

Ⓒ $\overline{NG} \cong \overline{NJ}$ Ⓓ $\overline{HK} \cong \overline{HM}$

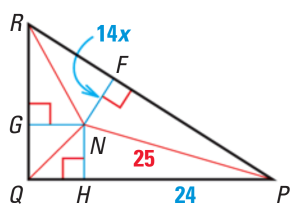


ALGEBRA **Find the value of x that makes N the incenter of the triangle.**

24.

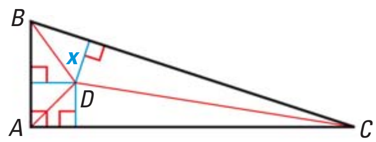


25.



26. **CONSTRUCTION** Use a compass and a straightedge to draw $\triangle ABC$ with incenter D. Label the angle bisectors and the perpendicular segments from D to each of the sides of $\triangle ABC$. Measure each segment. What do you notice? What theorem have you verified for your $\triangle ABC$?

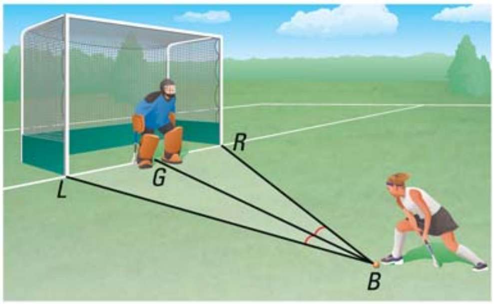
27. **CHALLENGE** Point D is the incenter of $\triangle ABC$. Write an expression for the length x in terms of the three side lengths AB, AC, and BC.



PROBLEM SOLVING

EXAMPLE 2
on p. 311
for Ex. 28

28. FIELD HOCKEY In a field hockey game, the goalkeeper is at point G and a player from the opposing team hits the ball from point B. The goal extends from left goalpost L to right goalpost R. Will the goalkeeper have to move farther to keep the ball from hitting L or R? *Explain.*



TEXAS @HomeTutor for problem solving help at classzone.com

29. KOI POND You are constructing a fountain in a triangular koi pond. You want the fountain to be the same distance from each edge of the pond. Where should you build the fountain? *Explain* your reasoning. Use a sketch to support your answer.

TEXAS @HomeTutor for problem solving help at classzone.com

30. TAKS REASONING What congruence postulate or theorem would you use to prove the Angle Bisector Theorem? to prove the Converse of the Angle Bisector Theorem? Use diagrams to show your reasoning.

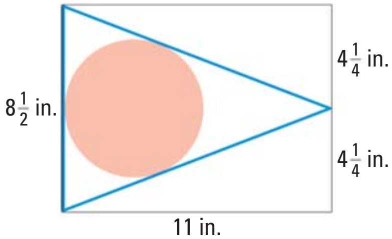
31. TAKS REASONING Suppose you are given a triangle and are asked to draw all of its perpendicular bisectors and angle bisectors.

a. For what type of triangle would you need the fewest segments? What is the minimum number of segments you would need? *Explain.*

b. For what type of triangle would you need the most segments? What is the maximum number of segments you would need? *Explain.*

CHOOSING A METHOD **In Exercises 32 and 33, tell whether you would use *perpendicular bisectors* or *angle bisectors*. Then solve the problem.**

32. BANNER To make a banner, you will cut a triangle from an $8\frac{1}{2}$ inch by 11 inch sheet of white paper and paste a red circle onto it as shown. The circle should just touch each side of the triangle. Use a model to decide whether the circle's radius should be *more* or *less* than $2\frac{1}{2}$ inches. Can you cut the circle from a 5 inch by 5 inch red square? *Explain.*



33. CAMP A map of a camp shows a pool at (10, 20), a nature center at (16, 2), and a tennis court at (2, 4). A new circular walking path will connect the three locations. Graph the points and find the approximate center of the circle. Estimate the radius of the circle if each unit on the grid represents 10 yards. Then use the formula $C = 2\pi r$ to estimate the length of the path.

PROVING THEOREMS 5.5 AND 5.6 **Use Exercise 30 to prove the theorem.**

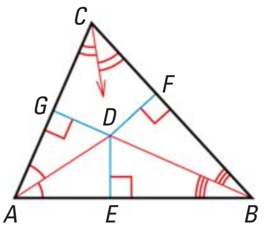
34. Angle Bisector Theorem

35. Converse of the Angle Bisector Theorem

36. **PROVING THEOREM 5.7** Write a proof of the Concurrency of Angle Bisectors of a Triangle Theorem.

GIVEN ▶ $\triangle ABC$, $\overline{AD}$ bisects $\angle CAB$, $\overline{BD}$ bisects $\angle CBA$, $\overline{DE} \perp \overline{AB}$, $\overline{DF} \perp \overline{BC}$, $\overline{DG} \perp \overline{CA}$

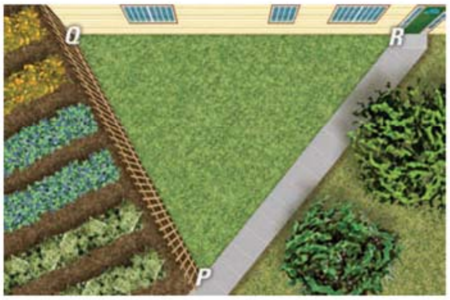
PROVE ▶ The angle bisectors intersect at D, which is equidistant from $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.



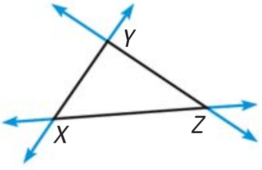
37. **CELEBRATION** You are planning a graduation party in the triangular courtyard shown. You want to fit as large a circular tent as possible on the site without extending into the walkway.

a. Copy the triangle and show how to place the tent so that it just touches each edge. Then *explain* how you can be sure that there is no place you could fit a larger tent on the site. Use sketches to support your answer.

b. Suppose you want to fit as large a tent as possible while leaving at least one foot of space around the tent. Would you put the center of the tent in the same place as you did in part (a)? *Justify* your answer.



38. **CHALLENGE** You have seen that there is a point inside any triangle that is equidistant from the three sides of the triangle. Prove that if you extend the sides of the triangle to form lines, you can find three points outside the triangle, each of which is equidistant from those three lines.

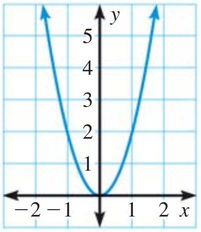


MIXED REVIEW FOR TAKS

TAKS PRACTICE at classzone.com

REVIEW
Skills Review Handbook p. 882; TAKS Workbook

39. **TAKS PRACTICE** What is the effect on the graph of the parabola $y = 2x^2$ shown when the coefficient of x^2 is increased to 4? *TAKS Obj. 5*



(A) The parabola will move 4 units down.

(B) The parabola will move 4 units up.

(C) The parabola will be wider.

(D) The parabola will be narrower.

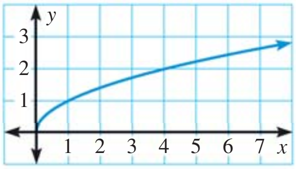
REVIEW
Lesson 1.7; TAKS Workbook

40. **TAKS PRACTICE** Ryan wants to paint the bottom of his circular pool. He knows the radius, r, and depth, d, of the pool. Which expression can he use to find the number of square feet that he needs to paint? *TAKS Obj. 7*

(F) rd (G) $2\pi r$ (H) $2\pi r^2$ (J) πr^2

REVIEW
TAKS Preparation p. 66; TAKS Workbook

41. **TAKS PRACTICE** Which relationship is best represented by the data in the graph? *TAKS Obj. 10*



(A) Comparing a circle's diameter to its area.

(B) Comparing a square's area to its side length.

(C) Comparing a cube's side length to its volume.

(D) Comparing a circle's radius to its circumference.

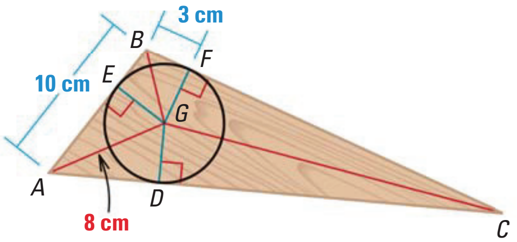
MIXED REVIEW FOR TEKS

TAKS PRACTICE
classzone.com

Lessons 5.1–5.3

MULTIPLE CHOICE

1. **CARPENTRY** A carpenter cuts as large a circle as possible from a triangular scrap of wood. The circle just touches each side of the triangle as shown below. What is the radius of the circle to the nearest tenth of a centimeter? *TEKS G.5.B*

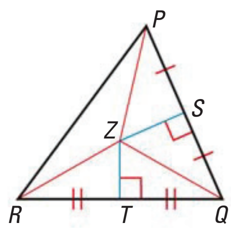


Ⓐ 3.9 cm Ⓑ 4.2 cm
Ⓒ 5.8 cm Ⓓ 6.2 cm

2. **LINES** Graph $\triangle GHJ$ with vertices $G(2, 2)$, $H(6, 8)$, and $J(10, 4)$. Draw its midsegments. Which line does not contain one of these midsegments? *TEKS G.7.B*

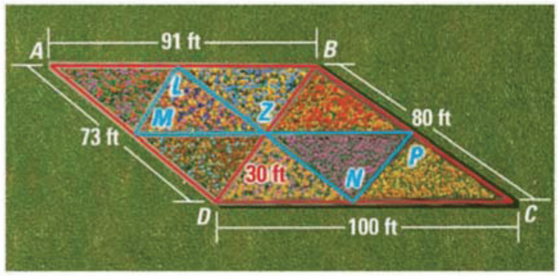
Ⓕ $y = \frac{1}{4}x + 4$ Ⓖ $y = \frac{3}{2}x - 6$
Ⓗ $y = -x + 9$ Ⓙ $y = -x + 14$

3. **DIAGRAM** Points S and T are the midpoints of two sides of $\triangle PQR$. Also, $\overline{ZS} \perp \overline{PQ}$ and $\overline{ZT} \perp \overline{QR}$. Which of the following is true? *TEKS G.11.A*



Ⓐ Point Z is equidistant from the sides of $\triangle PQR$.
Ⓑ Point Z lies along a midsegment of $\triangle PQR$.
Ⓒ Point Z is equidistant from the vertices of $\triangle PQR$.
Ⓓ Not enough information

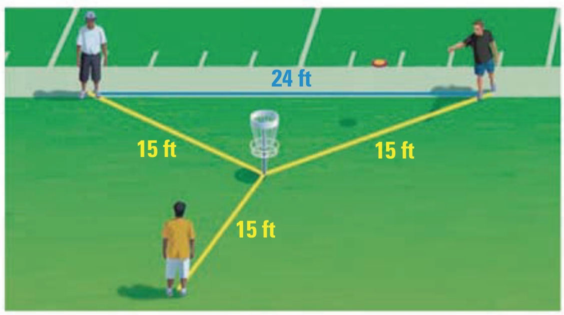
4. **LANDSCAPING** A landscaper is using recycled railroad ties to form a garden on a hillside. The garden consists of 8 triangular plots as shown below. The blue segments are the midsegments of the two red triangles, and $\overline{AB}$ is parallel to $\overline{DC}$. How many feet of railroad ties are needed to form the garden? *TEKS G.7.C*



Ⓕ 606 ft Ⓖ 636 ft
Ⓗ 684 ft Ⓙ 696 ft

GRIDDED ANSWER

5. **TARGET GAME** Three friends are playing a game in which a flat disk is thrown towards a target. Each player is 15 feet from the target. Two players are 24 feet from each other along one edge of the nearby football field. How many feet is the target from that edge of the football field? *TEKS G.7.C*



6. **ALGEBRA** Draw $\triangle JKL$ with midpoints T, U, and V. Make $\overline{VU}$ parallel to $\overline{JK}$ and $\overline{TV}$ parallel to $\overline{KL}$. Set $TU = 3x - 7$, $JL = 4x + 6$, $JT = x + 10$, and $KL = 8x - 18$. Find the length of $\overline{LU}$ to the nearest unit. *TEKS G.7.C*

5.4 Intersecting Medians

MATERIALS • cardboard • straightedge • scissors • metric ruler

TEKS *a.2, G.2.A, G.7.C, G.9.B*

QUESTION What is the relationship between segments formed by the medians of a triangle?

EXPLORE 1 Find the balance point of a triangle

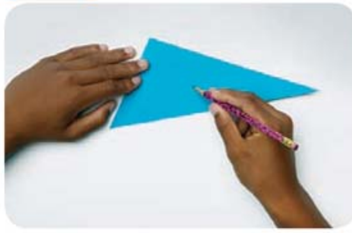
STEP 1

Cut out triangle Draw a triangle on a piece of cardboard. Then cut it out.

STEP 2

Balance the triangle Balance the triangle on the eraser end of a pencil.

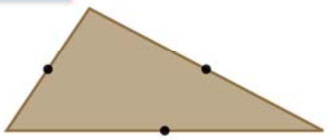
STEP 3



Mark the balance point Mark the point on the triangle where it balanced on the pencil.

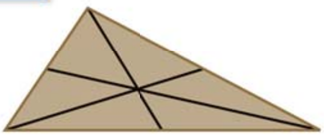
EXPLORE 2 Construct the medians of a triangle

STEP 1



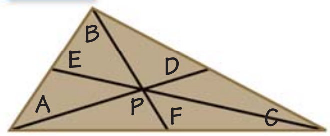
Find the midpoint Use a ruler to find the midpoint of each side of the triangle.

STEP 2



Draw medians Draw a segment, or *median*, from each midpoint to the vertex of the opposite angle.

STEP 3



Label points Label your triangle as shown. What do you notice about point *P* and the balance point in Explore 1?

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Copy and complete the table. Measure in millimeters.

Length of segment from vertex to midpoint of opposite side	$AD = ?$	$BF = ?$	$CE = ?$
Length of segment from vertex to P	$AP = ?$	$BP = ?$	$CP = ?$
Length of segment from P to midpoint	$PD = ?$	$PF = ?$	$PE = ?$

2. How does the length of the segment from a vertex to P compare with the length of the segment from P to the midpoint of the opposite side?

3. How does the length of the segment from a vertex to P compare with the length of the segment from the vertex to the midpoint of the opposite side?

5.4 Use Medians and Altitudes

TEKS G.1.A, G.2.B, G.3.E, G.5.B

Before You used perpendicular bisectors and angle bisectors of triangles.

Now You will use medians and altitudes of triangles.

Why? So you can find the balancing point of a triangle, as in Ex. 37.

Key Vocabulary
- **median of a triangle**
- **centroid**
- **altitude of a triangle**
- **orthocenter**

As shown by the Activity on page 318, a triangle will balance at a particular point. This point is the intersection of the *medians* of the triangle.

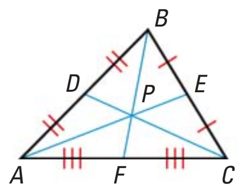
A **median of a triangle** is a segment from a vertex to the midpoint of the opposite side. The three medians of a triangle are concurrent. The point of concurrency, called the **centroid**, is inside the triangle.

Three medians meet at the centroid.

THEOREM *For Your Notebook*

THEOREM 5.8 Concurrency of Medians of a Triangle

The medians of a triangle intersect at a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.

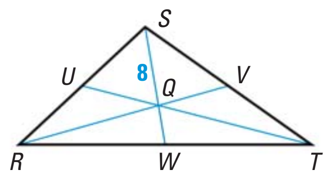


The medians of $\triangle ABC$ meet at P and $AP = \frac{2}{3}AE$, $BP = \frac{2}{3}BF$, and $CP = \frac{2}{3}CD$.

Proof: Ex. 32, p. 323; p. 934

EXAMPLE 1 Use the centroid of a triangle

In $\triangle RST$, Q is the centroid and $SQ = 8$. Find QW and SW.



Solution

$SQ = \frac{2}{3}SW$ — Concurrency of Medians of a Triangle Theorem

$8 = \frac{2}{3}SW$ — Substitute 8 for SQ.

$12 = SW$ — Multiply each side by the reciprocal, $\frac{3}{2}$.

Then $QW = SW - SQ = 12 - 8 = 4$.

▶ So, $QW = 4$ and $SW = 12$.

EXAMPLE 2 TAKS PRACTICE: Multiple Choice

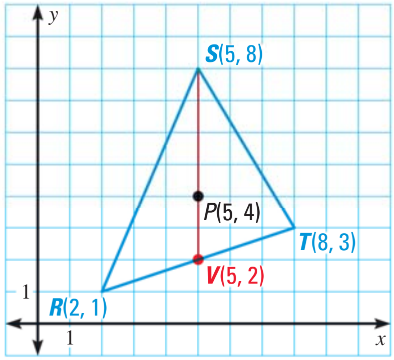
CHECK ANSWERS

Median $\overline{SV}$ was used in Example 2 because it is easy to find distances on a vertical segment. It is a good idea to check your answer by finding the centroid using a different median.

The vertices of $\triangle RST$ are $R(2, 1)$, $S(5, 8)$, and $T(8, 3)$. Which ordered pair gives the coordinates of the centroid P of $\triangle RST$?

Ⓐ (4, 4) Ⓑ (5, 4) Ⓒ (5, 6) Ⓓ (6,2)

Solution



Sketch $\triangle RST$. Then use the Midpoint Formula to find the midpoint V of $\overline{RT}$ and sketch median $\overline{SV}$.

$$V = \left(\frac{2+8}{2}, \frac{1+3}{2}\right) = (5, 2)$$

The centroid is two thirds of the distance from each vertex to the midpoint of the opposite side.

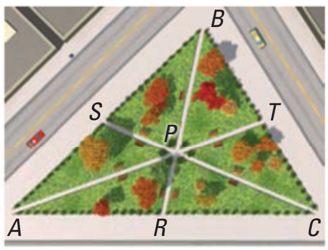
The distance from vertex $S(5, 8)$ to $V(5, 2)$ is $8 - 2 = 6$ units. So, the centroid is $\frac{2}{3}(6) = 4$ units down from S on $\overline{SV}$.

The coordinates of the centroid P are $(5, 8 - 4)$, or $(5, 4)$.

▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Examples 1 and 2

There are three paths through a triangular park. Each path goes from the midpoint of one edge to the opposite corner. The paths meet at point P.

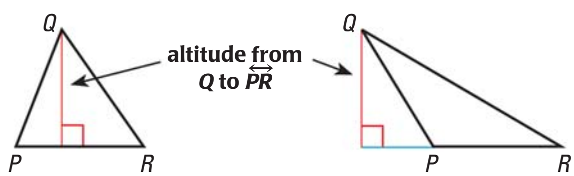


1. If $SC = 2100$ feet, find PS and PC.
2. If $BT = 1000$ feet, find TC and BC.
3. If $PT = 800$ feet, find PA and TA.

MEASURES OF TRIANGLES

In the area formula for a triangle, $A = \frac{1}{2}bh$, you can use the length of any side for the base b. The height h is the length of the altitude to that side from the opposite vertex.

ALTITUDES An **altitude of a triangle** is the perpendicular segment from a vertex to the opposite side or to the line that contains the opposite side.



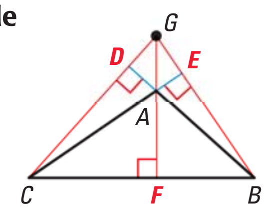
THEOREM — For Your Notebook

THEOREM 5.9 Concurrency of Altitudes of a Triangle

The lines containing the altitudes of a triangle are concurrent.

The lines containing $\overline{AF}$, $\overline{BE}$, and $\overline{CD}$ meet at G.

Proof: Exs. 29–31, p. 323; p. 936



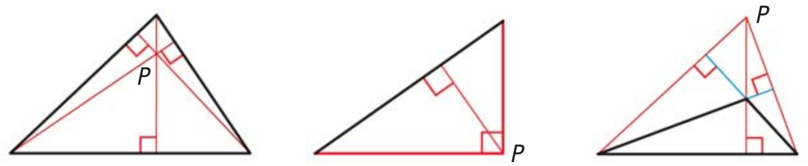
CONCURRENCY OF ALTITUDES The point at which the lines containing the three altitudes of a triangle intersect is called the **orthocenter** of the triangle.

EXAMPLE 3 Find the orthocenter

Find the orthocenter P in an acute, a right, and an obtuse triangle.

READ DIAGRAMS
The altitudes are shown in red. Notice that in the right triangle the legs are also altitudes. The altitudes of the obtuse triangle are extended to find the orthocenter.

Solution



Acute triangle
***P* is inside triangle.**

Right triangle
***P* is on triangle.**

Obtuse triangle
***P* is outside triangle.**

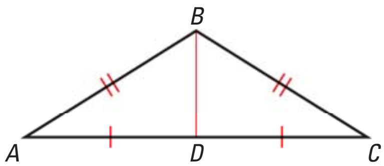
Animated Geometry at classzone.com

ISOSCELES TRIANGLES In an isosceles triangle, the perpendicular bisector, angle bisector, median, and altitude from the vertex angle to the base are all the same segment. In an equilateral triangle, this is true for the special segment from any vertex.

EXAMPLE 4 Prove a property of isosceles triangles

Prove that the median to the base of an isosceles triangle is an altitude.

Solution



GIVEN ▶ $\triangle ABC$ is isosceles, with base $\overline{AC}$.
$\overline{BD}$ is the median to base $\overline{AC}$.

PROVE ▶ $\overline{BD}$ is an altitude of $\triangle ABC$.

Proof Legs $\overline{AB}$ and $\overline{BC}$ of isosceles $\triangle ABC$ are congruent. $\overline{CD} \cong \overline{AD}$ because $\overline{BD}$ is the median to $\overline{AC}$. Also, $\overline{BD} \cong \overline{BD}$. Therefore, $\triangle ABD \cong \triangle CBD$ by the SSS Congruence Postulate.

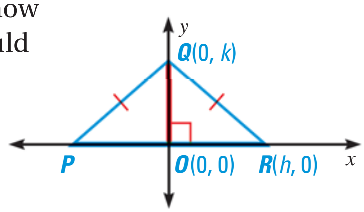
$\angle ADB \cong \angle CDB$ because corresponding parts of $\cong$ ⚠ are $\cong$. Also, $\angle ADB$ and $\angle CDB$ are a linear pair. $\overline{BD}$ and $\overline{AC}$ intersect to form a linear pair of congruent angles, so $\overline{BD} \perp \overline{AC}$ and $\overline{BD}$ is an altitude of $\triangle ABC$.

✓ GUIDED PRACTICE for Examples 3 and 4

4. Copy the triangle in Example 4 and find its orthocenter.

5. **WHAT IF?** In Example 4, suppose you wanted to show that median $\overline{BD}$ is also an angle bisector. How would your proof be different?

6. Triangle PQR is an isosceles triangle and segment $\overline{OQ}$ is an altitude. What else do you know about $\overline{OQ}$? What are the coordinates of P?



5.4 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on WS1 for Exs. 5, 21, and 39

★ = **TAKS PRACTICE AND REASONING** Exs. 7, 11, 12, 28, 40, 44, 46, and 47

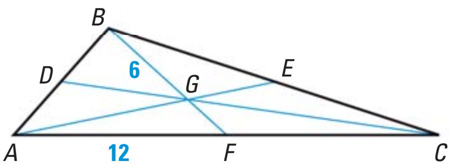
SKILL PRACTICE

1. **VOCABULARY** Name the four types of points of concurrency introduced in Lessons 5.2–5.4. When is each type inside the triangle? on the triangle? outside the triangle?

2. **WRITING** *Compare* a perpendicular bisector and an altitude of a triangle. *Compare* a perpendicular bisector and a median of a triangle.

EXAMPLE 1 on p. 319 for Exs. 3–7

FINDING LENGTHS ***G* is the centroid of $\triangle ABC$, $BG = 6$, $AF = 12$, and $AE = 15$. Find the length of the segment.**



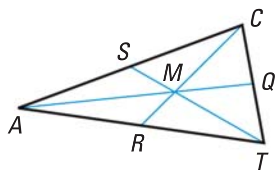
3. $\overline{FC}$

4. $\overline{BF}$

5. $\overline{AG}$

6. $\overline{GE}$

7. ★ **TAKS REASONING** In the diagram, M is the centroid of $\triangle ACT$, $CM = 36$, $MQ = 30$, and $TS = 56$. What is AM?



Ⓐ 15 Ⓑ 30

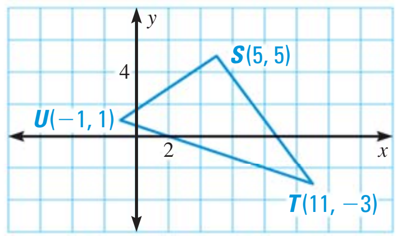
Ⓒ 36 Ⓓ 60

EXAMPLE 2 on p. 320 for Exs. 8–11

8. **FINDING A CENTROID** Use the graph shown.

   a. Find the coordinates of P, the midpoint of $\overline{ST}$. Use the median $\overline{UP}$ to find the coordinates of the centroid Q.

   b. Find the coordinates of R, the midpoint of $\overline{TU}$. Verify that $SQ = \frac{2}{3}SR$.



GRAPHING CENTROIDS **Find the coordinates of the centroid *P* of $\triangle ABC$.**

9. $A(-1, 2)$, $B(5, 6)$, $C(5, -2)$

10. $A(0, 4)$, $B(3, 10)$, $C(6, -2)$

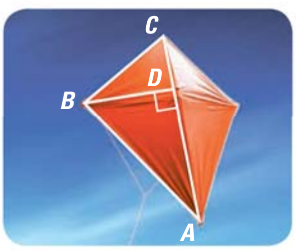
11. ★ **TAKS REASONING** Draw a large right triangle and find its centroid.

EXAMPLE 3 on p. 321 for Exs. 12–16

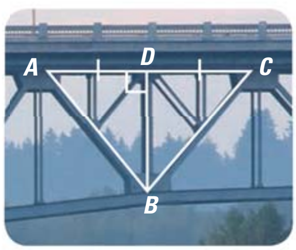
12. ★ **TAKS REASONING** Draw a large obtuse, scalene triangle and find its orthocenter.

IDENTIFYING SEGMENTS **Is $\overline{BD}$ a *perpendicular bisector* of $\triangle ABC$? Is $\overline{BD}$ a *median*? an *altitude*?**

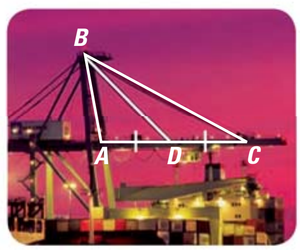
13.



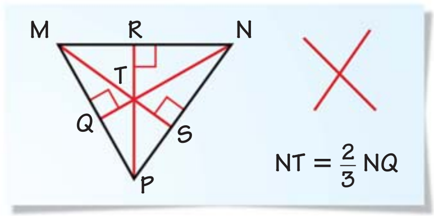
14.



15.

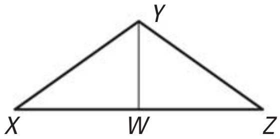


16. **ERROR ANALYSIS** A student uses the fact that T is a point of concurrency to conclude that $NT = \frac{2}{3}NQ$. *Explain* what is wrong with this reasoning.



EXAMPLE 4
on p. 321
for Exs. 17–22

REASONING Use the diagram shown and the given information to decide whether $\overline{YW}$ is a *perpendicular bisector*, an *angle bisector*, a *median*, or an *altitude* of $\triangle XYZ$. There may be more than one right answer.



17. $\overline{YW} \perp \overline{XZ}$

18. $\angle XYW \cong \angle ZYW$

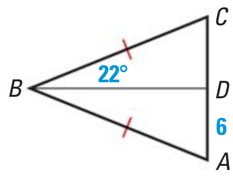
19. $\overline{XW} \cong \overline{ZW}$

20. $\overline{YW} \perp \overline{XZ}$ and $\overline{XW} \cong \overline{ZW}$

21. $\triangle XYW \cong \triangle ZYW$

22. $\overline{YW} \perp \overline{XZ}$ and $\overline{XY} \cong \overline{ZY}$

ISOSCELES TRIANGLES Find the measurements. *Explain* your reasoning.



23. Given that $\overline{DB} \perp \overline{AC}$, find DC and $m\angle ABD$.

24. Given that $AD = DC$, find $m\angle ADB$ and $m\angle ABD$.

RELATING LENGTHS Copy and complete the statement for $\triangle DEF$ with medians $\overline{DH}$, $\overline{EJ}$, and $\overline{FG}$, and centroid K.

25. $EJ =$ __?__ KJ

26. $DK =$ __?__ KH

27. $FG =$ __?__ KF

28. **TAKS REASONING** Any isosceles triangle can be placed in the coordinate plane with its base on the x-axis and the opposite vertex on the y-axis as in Guided Practice Exercise 6 on page 321. *Explain* why.

CONSTRUCTION Verify the Concurrency of Altitudes of a Triangle by drawing a triangle of the given type and constructing its altitudes. (*Hint:* To construct an altitude, use the construction in Exercise 25 on page 195.)

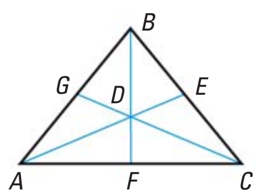
29. Equilateral triangle

30. Right scalene triangle

31. Obtuse isosceles triangle

32. **VERIFYING THEOREM 5.8** Use Example 2 on page 320. Verify that Theorem 5.8, the Concurrency of Medians of a Triangle, holds for the median from vertex F and for the median from vertex H.

xy ALGEBRA Point D is the centroid of $\triangle ABC$. Use the given information to find the value of x.

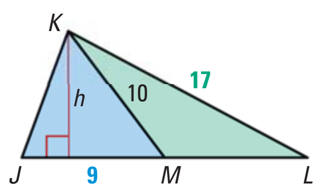


33. $BD = 4x + 5$ and $BF = 9x$

34. $GD = 2x - 8$ and $GC = 3x + 3$

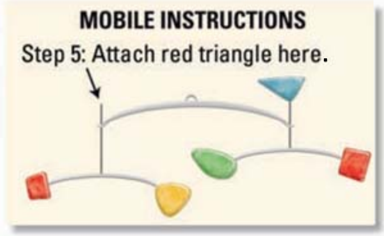
35. $AD = 5x$ and $DE = 3x - 2$

36. **CHALLENGE** $\overline{KM}$ is a median of $\triangle JKL$. Find the areas of $\triangle JKM$ and $\triangle LKM$. Compare the areas. Do you think that the two areas will always compare in this way, regardless of the shape of the triangle? *Explain*.



PROBLEM SOLVING

37. **MOBILES** To complete the mobile, you need to balance the red triangle on the tip of a metal rod. Copy the triangle and decide if you should place the rod at A or B. *Explain.*

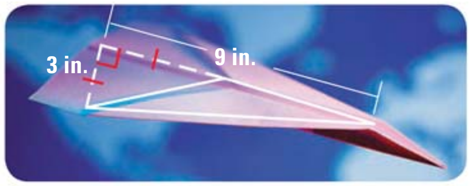


TEXAS @HomeTutor for problem solving help at classzone.com

38. **DEVELOPING PROOF** Show two different ways that you can place an isosceles triangle with base $2n$ and height h on the coordinate plane. Label the coordinates for each vertex.

TEXAS @HomeTutor for problem solving help at classzone.com

39. **PAPER AIRPLANE** Find the area of the triangular part of the paper airplane wing that is outlined in red. Which special segment of the triangle did you use?



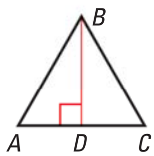
40. **TAKS REASONING** In what type(s) of triangle can a vertex of the triangle be one of the points of concurrency of the triangle? *Explain.*

41. **COORDINATE GEOMETRY** Graph the lines on the same coordinate plane and find the centroid of the triangle formed by their intersections.

$y_1 = 3x - 4$ $\qquad$ $y_2 = \frac{3}{4}x + 5$ $\qquad$ $y_3 = -\frac{3}{2}x - 4$

EXAMPLE 4 on p. 321 for Ex. 42

42. **PROOF** Write proofs using different methods.



GIVEN ▶ $\triangle ABC$ is equilateral.
$\overline{BD}$ is an altitude of $\triangle ABC$.

PROVE ▶ $\overline{BD}$ is also a perpendicular bisector of $\overline{AC}$.

a. Write a proof using congruent triangles.

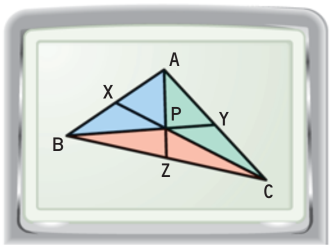
b. Write a proof using the Perpendicular Postulate on page 148.

43. **TECHNOLOGY** Use geometry drawing software.

a. Construct a triangle and its medians. Measure the areas of the blue, green, and red triangles.

b. What do you notice about the triangles?

c. If a triangle is of uniform thickness, what can you conclude about the weight of the three interior triangles? How does this support the idea that a triangle will balance on its centroid?



44. **TAKS REASONING** Use $P(0, 0)$, $Q(8, 12)$, and $R(14, 0)$.

a. What is the slope of the altitude from R to $\overline{PQ}$?

b. Write an equation for each altitude of $\triangle PQR$. Find the orthocenter by finding the ordered pair that is a solution of the three equations.

c. How would your steps change if you were finding the circumcenter?

45. CHALLENGE Prove the results in parts (a) – (c).

GIVEN ▶ $\overline{LP}$ and $\overline{MQ}$ are medians of scalene $\triangle LMN$. Point R is on $\overrightarrow{LP}$ such that $\overline{LP} \cong \overline{PR}$. Point S is on $\overrightarrow{MQ}$ such that $\overline{MQ} \cong \overline{QS}$.

PROVE ▶ **a.** $\overline{NS} \cong \overline{NR}$

b. $\overline{NS}$ and $\overline{NR}$ are both parallel to $\overline{LM}$.

c. R, N, and S are collinear.

TAKS PRACTICE at classzone.com

MIXED REVIEW FOR TAKS

REVIEW
Lesson 3.5;
TAKS Workbook

46. TAKS PRACTICE Sylvia makes and sells wind chimes. The table shows her profit given the number of wind chimes she sells. Which equation represents Sylvia's profit, p, when she sells w wind chimes? *TAKS Obj. 4*

Number of Wind Chimes Sold	Profit (in dollars)
1	−12
2	1
3	14
4	27

Ⓐ $p = w - 13$ Ⓑ $p = -12w$

Ⓒ $p = 10w - 2$ Ⓓ $p = 13w - 25$

REVIEW
TAKS Preparation p. 140;
TAKS Workbook

47. TAKS PRACTICE A pattern exists among the digits in the ones place when 7 is raised to different powers, as shown in the table. Which digit is in the ones place in 7^{51}? *TAKS Obj. 10*

Power of 7	Number in Ones Place	Power of 7	Number in Ones Place
7^1	7	7^5	7
7^2	9	7^6	9
7^3	3	7^7	3
7^4	1	7^8	1

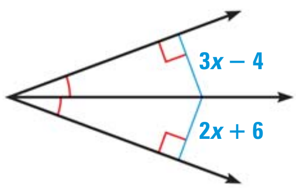
Ⓕ 1 Ⓖ 3

Ⓗ 7 Ⓙ 9

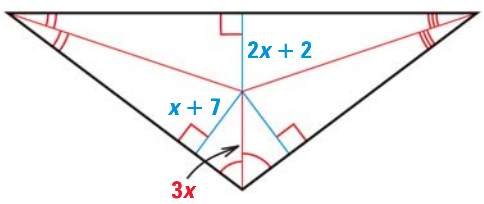
QUIZ for Lessons 5.3–5.4

Find the value of x. Identify the theorem used to find the answer. *(p. 310)*

1.



2.

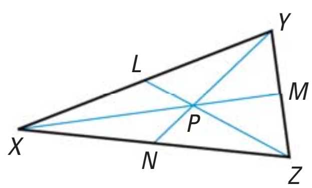


In the figure, P is the centroid of $\triangle XYZ$, $YP = 12$, $LX = 15$, and $LZ = 18$. *(p. 319)*

3. Find the length of $\overline{LY}$.

4. Find the length of $\overline{YN}$.

5. Find the length of $\overline{LP}$.



5.4 Investigate Points of Concurrency

MATERIALS • graphing calculator or computer

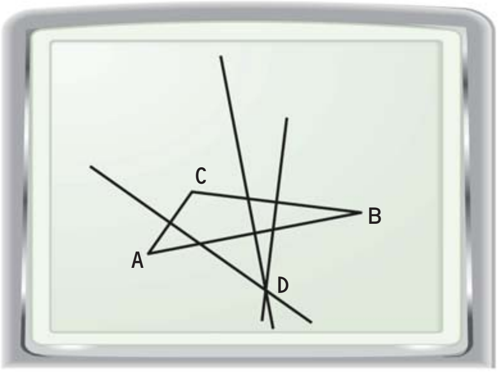
TEKS *a.5, G.1.B, G.2.A, G.3.D*

QUESTION How are the points of concurrency in a triangle related?

You can use geometry drawing software to investigate concurrency.

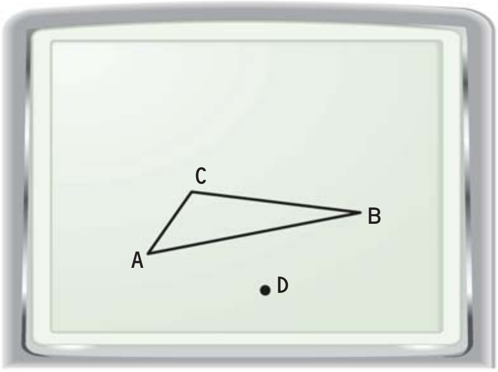
EXAMPLE 1 Draw the perpendicular bisectors of a triangle

STEP 1



Draw perpendicular bisectors Draw a line perpendicular to each side of a $\triangle ABC$ at the midpoint. Label the point of concurrency D.

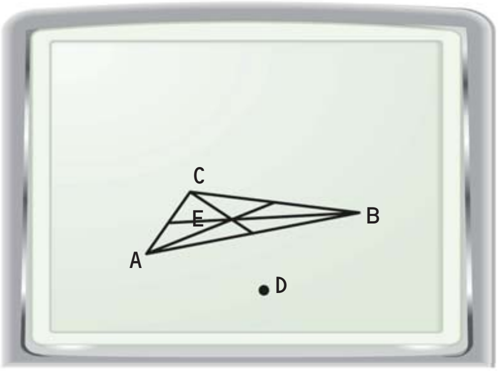
STEP 2



Hide the lines Use the *HIDE* feature to hide the perpendicular bisectors. Save as "EXAMPLE1."

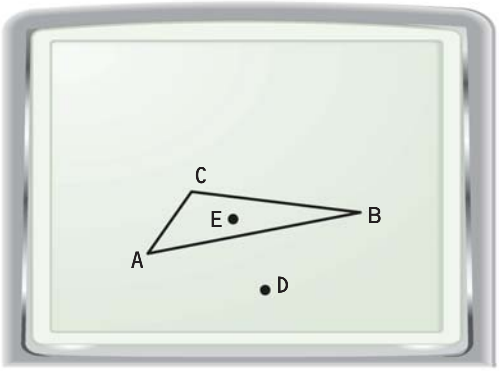
EXAMPLE 2 Draw the medians of the triangle

STEP 1



Draw medians Start with the figure you saved as "EXAMPLE1." Draw the medians of $\triangle ABC$. Label the point of concurrency E.

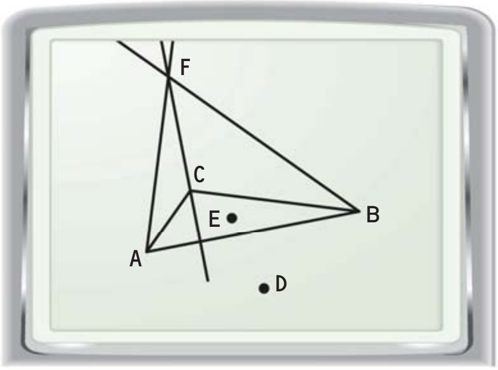
STEP 2



Hide the lines Use the *HIDE* feature to hide the medians. Save as "EXAMPLE2."

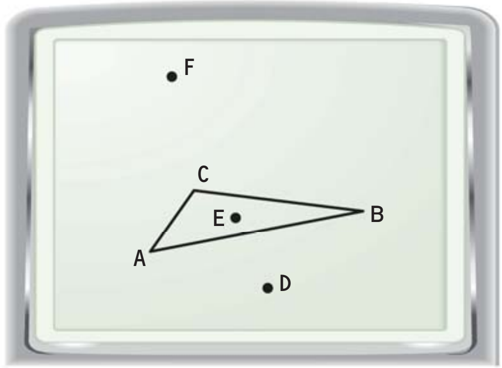
EXAMPLE 3 Draw the altitudes of the triangle

STEP 1



Draw altitudes Start with the figure you saved as "EXAMPLE2." Draw the altitudes of $\triangle ABC$. Label the point of concurrency *F*.

STEP 2



Hide the lines Use the *HIDE* feature to hide the altitudes. Save as "EXAMPLE3."

PRACTICE

1. Try to draw a line through points *D*, *E*, and *F*. Are the points collinear?
2. Try dragging point *A*. Do points *D*, *E*, and *F* remain collinear?

In Exercises 3–5, use the triangle you saved as "EXAMPLE3."

3. Draw the angle bisectors. Label the point of concurrency as point *G*.
4. How does point *G* relate to points *D*, *E*, and *F*?
5. Try dragging point *A*. What do you notice about points *D*, *E*, *F*, and *G*?

DRAW CONCLUSIONS

In 1765, Leonhard Euler (pronounced "oi′-ler") proved that the circumcenter, the centroid, and the orthocenter are all collinear. The line containing these three points is called *Euler's line*. Save the triangle from Exercise 5 as "EULER" and use that for Exercises 6–8.

6. Try moving the triangle's vertices. Can you verify that the same three points lie on Euler's line whatever the shape of the triangle? *Explain*.
7. Notice that some of the four points can be outside of the triangle. Which points lie outside the triangle? Why? What happens when you change the shape of the triangle? Are there any points that never lie outside the triangle? Why?
8. Draw the three midsegments of the triangle. Which, if any, of the points seem contained in the triangle formed by the midsegments? Do those points stay there when the shape of the large triangle is changed?

5.5 Use Inequalities in a Triangle

TEKS G.1.A, G.2.A, G.3.E, G.9.B

Before You found what combinations of angles are possible in a triangle.

Now You will find possible side lengths of a triangle.

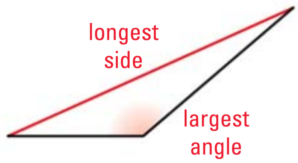
Why? So you can find possible distances, as in Ex. 39.

Key Vocabulary
- **side opposite,** *p. 241*
- **inequality,** *p. 876*

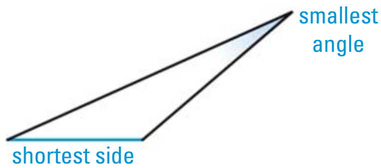
EXAMPLE 1 Relate side length and angle measure

Draw an obtuse scalene triangle. Find the largest angle and longest side and mark them in red. Find the smallest angle and shortest side and mark them in blue. What do you notice?

Solution



The longest side and largest angle are opposite each other.



The shortest side and smallest angle are opposite each other.

The relationships in Example 1 are true for all triangles as stated in the two theorems below. These relationships can help you to decide whether a particular arrangement of side lengths and angle measures in a triangle may be possible.

AVOID ERRORS

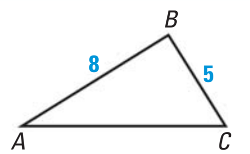
Be careful not to confuse the symbol $\angle$ meaning *angle* with the symbol $<$ meaning *is less than*. Notice that the bottom edge of the angle symbol is horizontal.

THEOREMS *For Your Notebook*

THEOREM 5.10

If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.

Proof: p. 329

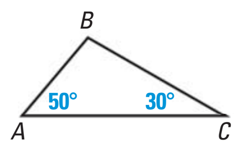


$AB > BC$, so $m\angle C > m\angle A$.

THEOREM 5.11

If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

Proof: Ex. 24, p. 340



$m\angle A > m\angle C$, so $BC > AB$.

EXAMPLE 2 TAKS PRACTICE: Multiple Choice

STAGE PROP Jenny is constructing a stage prop that shows a large triangular mountain. The bottom edge of the mountain is 32 feet long, the left slope is 24 feet long, and the right slope is 26 feet long. Jenny is told that one of the angles measures 53° and one measures 48°. What is the angle measure of the peak of the mountain?

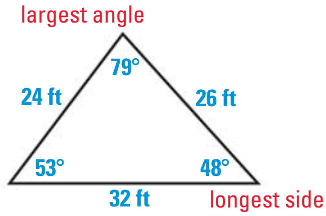
Ⓐ 48° Ⓑ 53° Ⓒ 79° Ⓓ 89°

ELIMINATE CHOICES

You can eliminate choice D because a triangle with a 53° angle and a 48° angle cannot have an 89° angle. The sum of the three angles in a triangle must be 180°, but the sum of 53, 48, and 89 is 190, not 180.

Solution

Draw a diagram and label the side lengths. The peak angle is opposite the longest side. So, by Theorem 5.10, the peak angle is the largest angle.

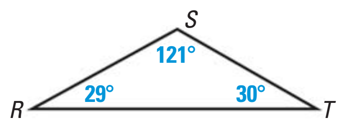


The angle measures sum to 180°, so the third angle measure is $180° - (53° + 48°) = 79°$. You can now label the angle measures in your diagram.

▶ The greatest angle measure is 79°, so the correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Examples 1 and 2

1. List the sides of $\triangle RST$ in order from shortest to longest.

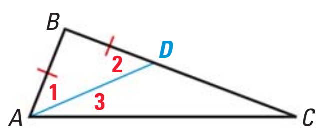


2. Another stage prop is a right triangle with sides that are 6, 8, and 10 feet long and angles of 90°, about 37°, and about 53°. Sketch and label a diagram with the shortest side on the bottom and the right angle at the left.

PROOF Theorem 5.10

GIVEN ▶ $BC > AB$

PROVE ▶ $m\angle BAC > m\angle C$



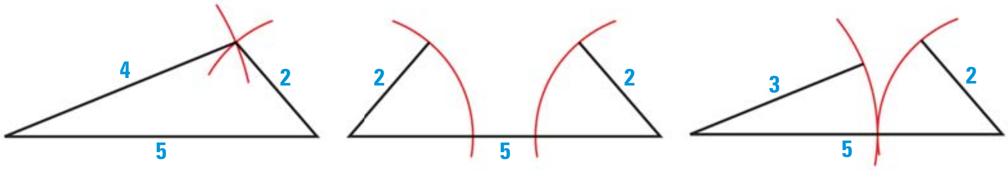
Locate a point D on $\overline{BC}$ such that $DB = BA$. Then draw $\overline{AD}$. In the isosceles triangle $\triangle ABD$, $\angle 1 \cong \angle 2$.

Because $m\angle BAC = m\angle 1 + m\angle 3$, it follows that $m\angle BAC > m\angle 1$. Substituting $m\angle 2$ for $m\angle 1$ produces $m\angle BAC > m\angle 2$.

By the Exterior Angle Theorem, $m\angle 2 = m\angle 3 + m\angle C$, so it follows that $m\angle 2 > m\angle C$ (see Exercise 27, page 332). Finally, because $m\angle BAC > m\angle 2$ and $m\angle 2 > m\angle C$, you can conclude that $m\angle BAC > m\angle C$.

THE TRIANGLE INEQUALITY Not every group of three segments can be used to form a triangle. The lengths of the segments must fit a certain relationship.

For example, three attempted triangle constructions for sides with given lengths are shown below. Only the first set of side lengths forms a triangle.



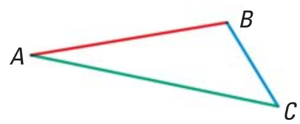
If you start with the longest side and attach the other two sides at its endpoints, you can see that the other two sides are not long enough to form a triangle in the second and third figures. This leads to the *Triangle Inequality Theorem*.

Animated Geometry at classzone.com

THEOREM — For Your Notebook

THEOREM 5.12 Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.



$AB + BC > AC$ $\quad$ $AC + BC > AB$ $\quad$ $AB + AC > BC$

Proof: Ex. 47, p. 334

EXAMPLE 3 Find possible side lengths

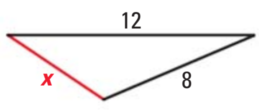
xy ALGEBRA A triangle has one side of length 12 and another of length 8. Describe the possible lengths of the third side.

Solution

Let x represent the length of the third side. Draw diagrams to help visualize the small and large values of x. Then use the Triangle Inequality Theorem to write and solve inequalities.

USE SYMBOLS
You can combine the two inequalities, $x > 4$ and $x < 20$, to write the compound inequality $4 < x < 20$. This can be read as *x is between 4 and 20.*

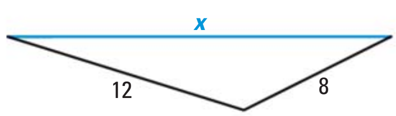
Small values of x



$x + 8 > 12$

$x > 4$

Large values of x



$8 + 12 > x$

$20 > x$, or $x < 20$

▶ The length of the third side must be greater than 4 and less than 20.

GUIDED PRACTICE for Example 3

3. A triangle has one side of 11 inches and another of 15 inches. *Describe* the possible lengths of the third side.

5.5 EXERCISES

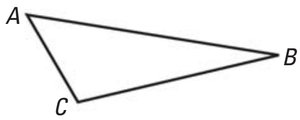
HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 17, and 39

★ = **TAKS PRACTICE AND REASONING** Exs. 12, 20, 30, 39, 45, and 49

SKILL PRACTICE

1. **VOCABULARY** Use the diagram at the right. For each angle, name the side that is *opposite* that angle.



2. **WRITING** How can you tell from the angle measures of a triangle which side of the triangle is the longest? the shortest?

EXAMPLE 1 on p. 328 for Exs. 3–5

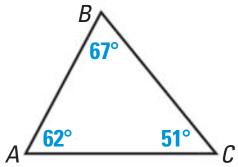
MEASURING Use a ruler and protractor to draw the given type of triangle. Mark the largest angle and longest side in red and the smallest angle and shortest side in blue. What do you notice?

3. Acute scalene
4. Right scalene
5. Obtuse isosceles

EXAMPLE 2 on p. 329 for Exs. 6–15

WRITING MEASUREMENTS IN ORDER List the sides and the angles in order from smallest to largest.

6.



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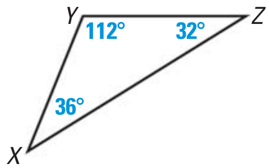

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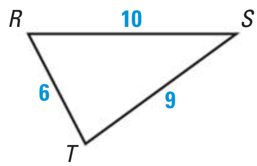

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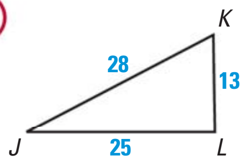

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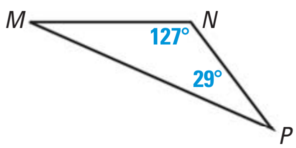
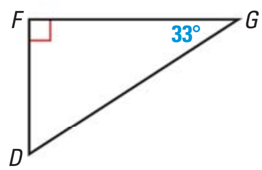
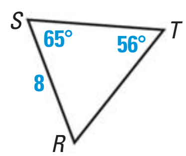


11.



12. ★ **TAKS REASONING** In $\triangle RST$, which is a possible side length for ST?



Ⓐ 7 Ⓑ 8

Ⓒ 9 Ⓓ Cannot be determined

DRAWING TRIANGLES Sketch and label the triangle described.

13. Side lengths: about 3 m, 7 m, and 9 m, with longest side on the bottom
Angle measures: 16°, 41°, and 123°, with smallest angle at the left

14. Side lengths: 37 ft, 35 ft, and 12 ft, with shortest side at the right
Angle measures: about 71°, about 19°, and 90°, with right angle at the top

15. Side lengths: 11 in., 13 in., and 14 in., with middle-length side at the left
Two angle measures: about 48° and 71°, with largest angle at the top

EXAMPLE 3 on p. 330 for Exs. 16–26

IDENTIFYING POSSIBLE TRIANGLES Is it possible to construct a triangle with the given side lengths? If not, *explain* why not.

16. 6, 7, 11
17. 3, 6, 9
18. 28, 34, 39
19. 35, 120, 125

20. **TAKS REASONING** Which group of side lengths can be used to construct a triangle?

Ⓐ 3 yd, 4 ft, 5 yd Ⓑ 3 yd, 5 ft, 8 ft

Ⓒ 11 in., 16 in., 27 in. Ⓓ 2 ft, 11 in., 12 in.

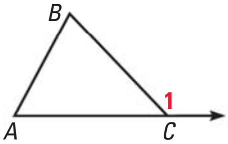
POSSIBLE SIDE LENGTHS ***Describe*** **the possible lengths of the third side of the triangle given the lengths of the other two sides.**

21. 5 inches, 12 inches **22.** 3 meters, 4 meters **23.** 12 feet, 18 feet

24. 10 yards, 23 yards **25.** 2 feet, 40 inches **26.** 25 meters, 25 meters

27. EXTERIOR ANGLE INEQUALITY Another triangle inequality relationship is given by the Exterior Inequality Theorem. It states:

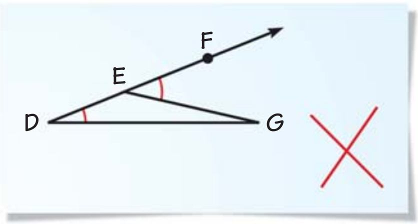
The measure of an exterior angle of a triangle is greater than the measure of either of the nonadjacent interior angles.



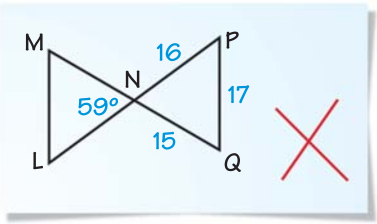
Use a relationship from Chapter 4 to *explain* how you know that $m\angle 1 > m\angle A$ and $m\angle 1 > m\angle B$ in $\triangle ABC$ with exterior angle $\angle 1$.

ERROR ANALYSIS **Use Theorems 5.10–5.12 and the theorem in Exercise 27 to** ***explain*** **why the diagram must be incorrect.**

28.



29.



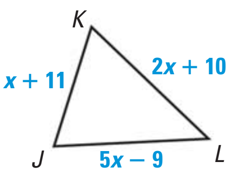
30. **TAKS REASONING** *Explain* why the hypotenuse of a right triangle must always be longer than either leg.

ORDERING MEASURES **Is it possible to build a triangle using the given side lengths? If so, order the angles measures of the triangle from least to greatest.**

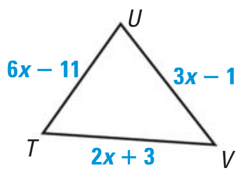
31. $PQ = \sqrt{58}$, $QR = 2\sqrt{13}$, $PR = 5\sqrt{2}$ **32.** $ST = \sqrt{29}$, $TU = 2\sqrt{17}$, $SU = 13.9$

xy ALGEBRA ***Describe*** **the possible values of *x*.**

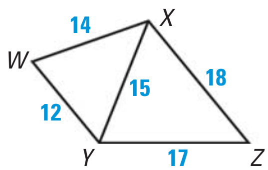
33.



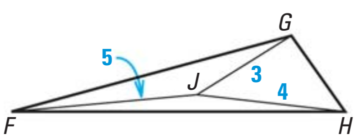
34.



35. USING SIDE LENGTHS Use the diagram at the right. Suppose $\overline{XY}$ bisects $\angle WYZ$. List all six angles of $\triangle XYZ$ and $\triangle WXY$ in order from smallest to largest. *Explain* your reasoning.

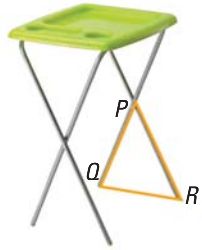


36. CHALLENGE The perimeter of $\triangle HGF$ must be between what two integers? *Explain* your reasoning.



PROBLEM SOLVING

37. TRAY TABLE In the tray table shown, $\overline{PQ} \cong \overline{PR}$ and $QR < PQ$. Write two inequalities about the angles in $\triangle PQR$. What other angle relationship do you know?



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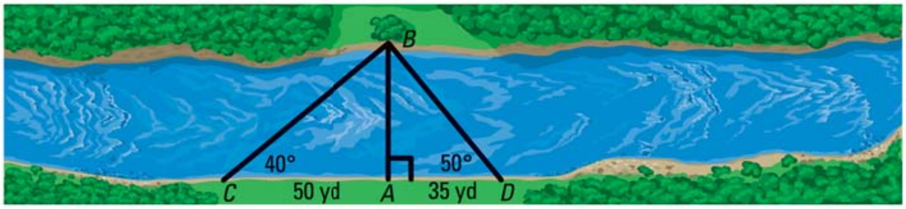
38. INDIRECT MEASUREMENT You can estimate the width of the river at point A by taking several sightings to the tree across the river at point B. The diagram shows the results for locations C and D along the riverbank. Using $\triangle BCA$ and $\triangle BDA$, what can you conclude about AB, the width of the river at point A? What could you do if you wanted a closer estimate?



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EXAMPLE 3
on p. 330
for Ex. 39

39. TAKS REASONING You are planning a vacation to Montana. You want to visit the destinations shown in the map.

a. A brochure states that the distance between Granite Peak and Fort Peck Lake is 1080 kilometers. *Explain* how you know that this distance is a misprint.

b. Could the distance from Granite Peak to Fort Peck Lake be 40 kilometers? *Explain*.

c. Write two inequalities to represent the range of possible distances from Granite Peak to Fort Peck Lake.

d. What can you say about the distance between Granite Peak and Fort Peck Lake if you know that $m\angle 2 < m\angle 1$ and $m\angle 2 < m\angle 3$?

FORMING TRIANGLES **In Exercises 40–43, you are given a 24 centimeter piece of string. You want to form a triangle out of the string so that the length of each side is a whole number. Draw figures accurately.**

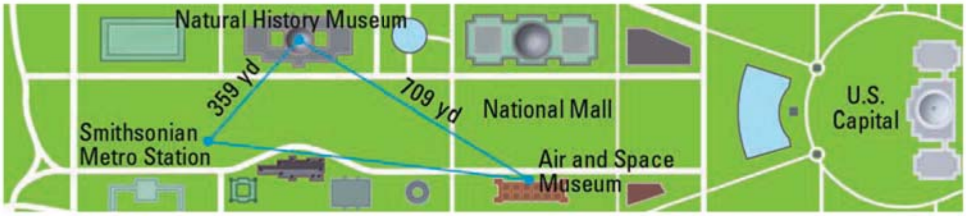
40. Can you decide if three side lengths form a triangle without checking all three inequalities shown for Theorem 5.12? If so, *describe* your shortcut.

41. Draw four possible isosceles triangles and label each side length. Tell whether each of the triangles you formed is *acute*, *right*, or *obtuse*.

42. Draw three possible scalene triangles and label each side length. Try to form at least one scalene acute triangle and one scalene obtuse triangle.

43. List three combinations of side lengths that will not produce triangles.

44. SIGHTSEEING You get off the Washington, D.C., subway system at the Smithsonian Metro station. First you visit the Museum of Natural History. Then you go to the Air and Space Museum. You record the distances you walk on your map as shown. *Describe* the range of possible distances you might have to walk to get back to the Smithsonian Metro station.



45. TAKS REASONING Your house is 2 miles from the library. The library is $\frac{3}{4}$ mile from the grocery store. What do you know about the distance from your house to the grocery store? *Explain*. Include the special case when the three locations are all in a straight line.

46. ISOSCELES TRIANGLES For what combinations of angle measures in an isosceles triangle are the congruent sides shorter than the base of the triangle? longer than the base of the triangle?

47. PROVING THEOREM 5.12 Prove the Triangle Inequality Theorem.

GIVEN ▶ $\triangle ABC$

PROVE ▶ (1) $AB + BC > AC$
(2) $AC + BC > AB$
(3) $AB + AC > BC$

Plan for Proof One side, say BC, is longer than or at least as long as each of the other sides. Then (1) and (2) are true. To prove (3), extend $\overline{AC}$ to D so that $\overline{AB} \cong \overline{AD}$ and use Theorem 5.11 to show that $DC > BC$.

48. CHALLENGE Prove the following statements.

a. The length of any one median of a triangle is less than half the perimeter of the triangle.

b. The sum of the lengths of the three medians of a triangle is greater than half the perimeter of the triangle.

MIXED REVIEW FOR TAKS

TAKS **PRACTICE** at classzone.com

REVIEW
Skills Review Handbook p. 893; TAKS Workbook

49. TAKS PRACTICE A bag contains the six tiles shown below. Jane randomly draws a tile, replaces it, and then randomly draws another tile. What is the probability Jane first draws a star tile and then draws a blue tile? *TAKS Obj. 9*

Ⓐ $\frac{1}{12}$ Ⓑ $\frac{1}{6}$ Ⓒ $\frac{1}{2}$ Ⓓ $\frac{2}{3}$

5.6 Inequalities in Two Triangles and Indirect Proof

TEKS G.3.A, G.3.C, G.3.E, G.5.B

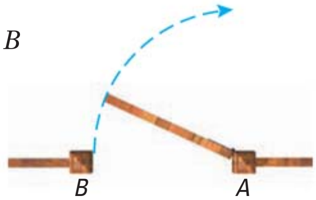
Before You used inequalities to make comparisons in one triangle.

Now You will use inequalities to make comparisons in two triangles.

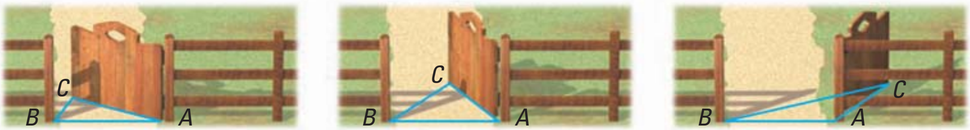
Why? So you can compare the distances hikers traveled, as in Ex. 22.

Key Vocabulary
- **indirect proof**
- **included angle,** *p. 240*

Imagine a gate between fence posts A and B that has hinges at A and swings open at B.



As the gate swings open, you can think of $\triangle ABC$, with side $\overline{AC}$ formed by the gate itself, side $\overline{AB}$ representing the distance between the fence posts, and side $\overline{BC}$ representing the opening between post B and the outer edge of the gate.

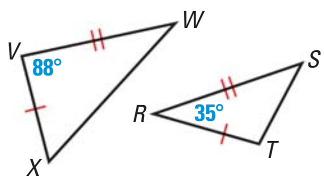


Notice that as the gate opens wider, both the measure of $\angle A$ and the distance CB increase. This suggests the *Hinge Theorem*.

THEOREMS — *For Your Notebook*

THEOREM 5.13 Hinge Theorem

If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first is longer than the third side of the second.

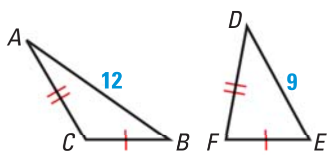


$WX > ST$

Proof: Ex. 28, p. 341

THEOREM 5.14 Converse of the Hinge Theorem

If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first is longer than the third side of the second, then the included angle of the first is larger than the included angle of the second.

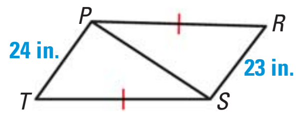


$m\angle C > m\angle F$

Proof: Example 4, p. 338

EXAMPLE 1 Use the Converse of the Hinge Theorem

Given that $\overline{ST} \cong \overline{PR}$, how does $\angle PST$ compare to $\angle SPR$?



Solution

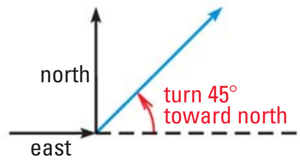
You are given that $\overline{ST} \cong \overline{PR}$ and you know that $\overline{PS} \cong \overline{PS}$ by the Reflexive Property. Because 24 inches > 23 inches, $PT > RS$. So, two sides of $\triangle STP$ are congruent to two sides of $\triangle PRS$ and the third side in $\triangle STP$ is longer.

▶ By the Converse of the Hinge Theorem, $m\angle PST > m\angle SPR$.

TAKS

EXAMPLE 2 TAKS Reasoning: Multi-Step Problem

BIKING Two groups of bikers leave the same camp heading in opposite directions. Each group goes 2 miles, then changes direction and goes 1.2 miles. Group A starts due east and then turns 45° toward north as shown. Group B starts due west and then turns 30° toward south.



Which group is farther from camp? Explain your reasoning.

Solution

Draw a diagram and mark the given measures. The distances biked and the distances back to camp form two triangles, with congruent 2 mile sides and congruent 1.2 mile sides. Add the third sides of the triangles to your diagram.

Next use linear pairs to find and mark the included angles of 150° and 135°.

▶ Because 150° > 135°, Group B is farther from camp by the Hinge Theorem.

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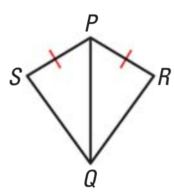
✓ **GUIDED PRACTICE** for Examples 1 and 2

Use the diagram at the right.



1. If $PR = PS$ and $m\angle QPR > m\angle QPS$, which is longer, $\overline{SQ}$ or $\overline{RQ}$?
2. If $PR = PS$ and $RQ < SQ$, which is larger, $\angle RPQ$ or $\angle SPQ$?
3. **WHAT IF?** In Example 2, suppose Group C leaves camp and goes 2 miles due north. Then they turn 40° toward east and continue 1.2 miles. *Compare* the distances from camp for all three groups.

INDIRECT REASONING Suppose a student looks around the cafeteria, concludes that hamburgers are not being served, and explains as follows.

> *At first I assumed that we are having hamburgers because today is Tuesday and Tuesday is usually hamburger day.*
>
> *There is always ketchup on the table when we have hamburgers, so I looked for the ketchup, but I didn't see any.*
>
> *So, my assumption that we are having hamburgers must be false.*

The student used *indirect* reasoning. So far in this book, you have reasoned *directly* from given information to prove desired conclusions.

In an **indirect proof**, you start by making the temporary assumption that the desired conclusion is false. By then showing that this assumption leads to a logical impossibility, you prove the original statement true *by contradiction*.

KEY CONCEPT *For Your Notebook*

How to Write an Indirect Proof

STEP 1 **Identify** the statement you want to prove. **Assume** temporarily that this statement is false by assuming that its opposite is true.

STEP 2 **Reason** logically until you reach a contradiction.

STEP 3 **Point out** that the desired conclusion must be true because the contradiction proves the temporary assumption false.

EXAMPLE 3 Write an indirect proof

Write an indirect proof that an odd number is not divisible by 4.

GIVEN ▶ x is an odd number.

PROVE ▶ x is not divisible by 4.

Solution

STEP 1 Assume temporarily that x is divisible by 4. This means that $\frac{x}{4} = n$ for some whole number n. So, multiplying both sides by 4 gives $x = 4n$.

STEP 2 If x is odd, then, by definition, x cannot be divided evenly by 2. However, $x = 4n$ so $\frac{x}{2} = \frac{4n}{2} = 2n$. We know that $2n$ is a whole number because n is a whole number, so x *can* be divided evenly by 2. This contradicts the given statement that x is odd.

STEP 3 Therefore, the assumption that x is divisible by 4 must be false, which proves that x is not divisible by 4.

READ VOCABULARY
You have reached a *contradiction* when you have two statements that cannot both be true at the same time.

✓ **GUIDED PRACTICE** for Example 3

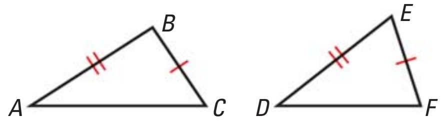
4. Suppose you wanted to prove the statement "If $x + y \neq 14$ and $y = 5$, then $x \neq 9$." What temporary assumption could you make to prove the conclusion indirectly? How does that assumption lead to a contradiction?

EXAMPLE 4 Prove the Converse of the Hinge Theorem

Write an indirect proof of Theorem 5.14.

GIVEN ▶ $\overline{AB} \cong \overline{DE}$
$\overline{BC} \cong \overline{EF}$
$AC > DF$

PROVE ▶ $m\angle B > m\angle E$



Proof Assume temporarily that $m\angle B \not> m\angle E$. Then, it follows that either $m\angle B = m\angle E$ or $m\angle B < m\angle E$.

Case 1 If $m\angle B = m\angle E$, then $\angle B \cong \angle E$. So, $\triangle ABC \cong \triangle DEF$ by the SAS Congruence Postulate and $AC = DF$.

Case 2 If $m\angle B < m\angle E$, then $AC < DF$ by the Hinge Theorem.

Both conclusions contradict the given statement that $AC > DF$. So, the temporary assumption that $m\angle B \not> m\angle E$ cannot be true. This proves that $m\angle B > m\angle E$.

✓ GUIDED PRACTICE for Example 4

5. Write a temporary assumption you could make to prove the Hinge Theorem indirectly. What two cases does that assumption lead to?

5.6 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 7, and 23
★ = **TAKS PRACTICE AND REASONING** Exs. 9, 19, 25, and 29

SKILL PRACTICE

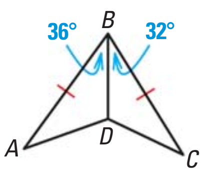
1. **VOCABULARY** Why is indirect proof also called *proof by contradiction*?

2. **WRITING** *Explain* why the name "Hinge Theorem" is used for Theorem 5.13.

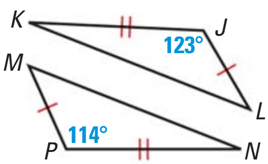
EXAMPLE 1 on p. 336 for Exs. 3–10

APPLYING THEOREMS **Copy and complete with <, >, or =. *Explain*.**

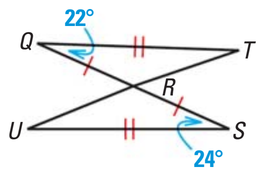
3. AD __?__ CD



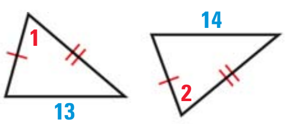
4. MN __?__ LK



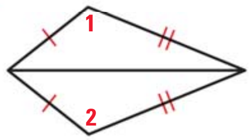
5. TR __?__ UR



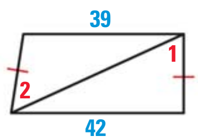
6. $m\angle 1$ __?__ $m\angle 2$



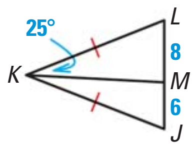
7. $m\angle 1$ __?__ $m\angle 2$



8. $m\angle 1$ __?__ $m\angle 2$



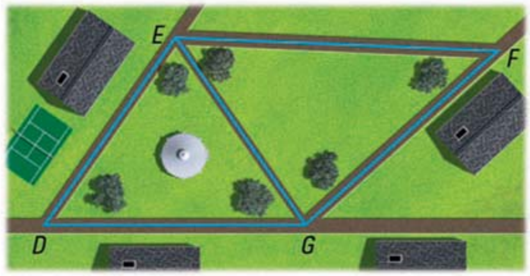
9. **TAKS REASONING** Which is a possible measure for $\angle JKM$?



Ⓐ 20° Ⓑ 25°

Ⓒ 30° Ⓓ Cannot be determined

10. **USING A DIAGRAM** The path from E to F is longer than the path from E to D. The path from G to D is the same length as the path from G to F. What can you conclude about the angles of the paths? *Explain* your reasoning.



EXAMPLES 3 and 4 on p. 337–338 for Exs. 11–13

STARTING AN INDIRECT PROOF **In Exercises 11 and 12, write a temporary assumption you could make to prove the conclusion indirectly.**

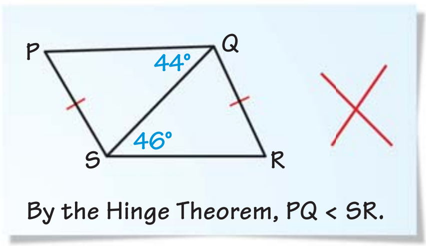
11. If x and y are odd integers, then xy is odd.

12. In $\triangle ABC$, if $m\angle A = 100°$, then $\angle B$ is not a right angle.

13. **REASONING** Your study partner is planning to write an indirect proof to show that $\angle A$ is an obtuse angle. She states "Assume temporarily that $\angle A$ is an acute angle." What has your study partner overlooked?

ERROR ANALYSIS ***Explain*** **why the student's reasoning is not correct.**

14.



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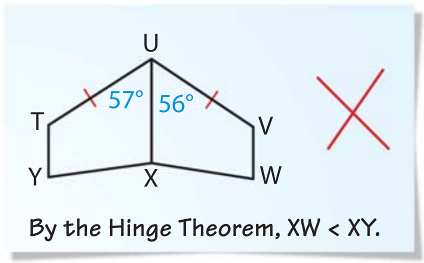
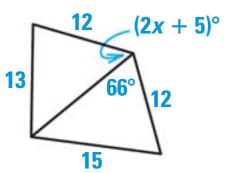


xy **ALGEBRA** **Use the Hinge Theorem or its converse and properties of triangles to write and solve an inequality to describe a restriction on the value of x.**

16.



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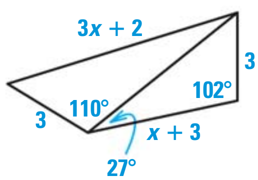

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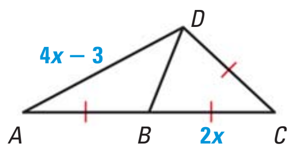

19. **TAKS REASONING** If $\overline{NR}$ is a median of $\triangle NPQ$ and $NQ > NP$, *explain* why $\angle NRQ$ is obtuse.

20. **ANGLE BISECTORS** In $\triangle EFG$, the bisector of $\angle F$ intersects the bisector of $\angle G$ at point H. *Explain* why $\overline{FG}$ must be longer than $\overline{FH}$ or $\overline{HG}$.

21. **CHALLENGE** In $\triangle ABC$, the altitudes from B and C meet at D. What is true about $\triangle ABC$ if $m\angle BAC > m\angle BDC$? *Justify* your answer.

PROBLEM SOLVING

EXAMPLE 2 on p. 336 for Ex. 22

22. **HIKING** Two hikers start at the visitor center. The first hikes 4 miles due west, then turns 40° toward south and hikes 1.8 miles. The second hikes 4 miles due east, then turns 52° toward north and and hikes 1.8 miles. Which hiker is farther from camp? *Explain* how you know.

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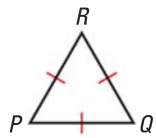
EXAMPLES 3 and 4 on pp. 337–338 for Exs. 23–24

23. **INDIRECT PROOF** Arrange statements A–E in order to write an indirect proof of the corollary: If $\triangle ABC$ is *equilateral*, then it is *equiangular*.



GIVEN ▶ $\triangle PQR$ is equilateral.

A. That means that for some pair of vertices, say P and Q, $m\angle P > m\angle Q$.

B. But this contradicts the given statement that $\triangle PQR$ is equilateral.

C. The contradiction shows that the temporary assumption that $\triangle PQR$ is not equiangular is false. This proves that $\triangle PQR$ is equiangular.

D. Then, by Theorem 5.11, you can conclude that $QR > PR$.

E. Temporarily assume that $\triangle PQR$ is not equiangular.

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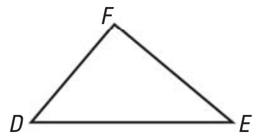
24. **PROVING THEOREM 5.11** Write an indirect proof of Theorem 5.11, page 328.

GIVEN ▶ $m\angle D > m\angle E$

PROVE ▶ $EF > DF$



Plan for Proof In Case 1, assume that $EF < DF$. In Case 2, assume that $EF = DF$.

25. **TAKS REASONING** A scissors lift can be used to adjust the height of a platform.

a. Interpret As the mechanism expands, $\overline{KL}$ gets longer. As KL increases, what happens to $m\angle LNK$? to $m\angle KNM$?

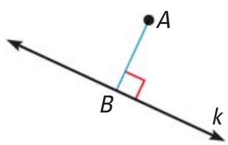
b. Apply Name a distance that decreases as $\overline{KL}$ gets longer.

c. Writing *Explain* how the adjustable mechanism illustrates the Hinge Theorem.

26. **PROOF** Write a proof that the shortest distance from a point to a line is the length of the perpendicular segment from the point to the line.

GIVEN ▶ Line k; point A not on k; point B on k such that $\overline{AB} \perp k$

PROVE ▶ $\overline{AB}$ is the shortest segment from A to k.



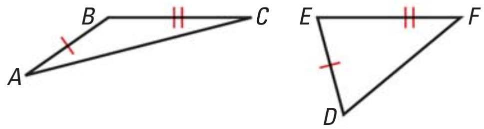
Plan for Proof Assume that there is a shorter segment from A to k and use Theorem 5.10 to show that this leads to a contradiction.

27. USING A CONTRAPOSITIVE Because the contrapositive of a conditional is equivalent to the original statement, you can prove the statement by proving its contrapositive. Look back at the conditional in Example 3 on page 337. Write a proof of the contrapositive that uses direct reasoning. How is your proof similar to the indirect proof of the original statement?

28. CHALLENGE Write a proof of Theorem 5.13, the Hinge Theorem.

GIVEN ▶ $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, $m\angle ABC > m\angle DEF$

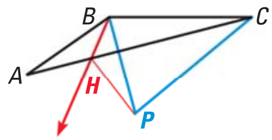
PROVE ▶ $AC > DF$



Plan for Proof

1. Because $m\angle ABC > m\angle DEF$, you can locate a point P in the interior of $\angle ABC$ so that $\angle CBP \cong \angle FED$ and $\overline{BP} \cong \overline{ED}$. Draw $\overline{BP}$ and show that $\triangle PBC \cong \triangle DEF$.
2. Locate a point H on $\overline{AC}$ so that $\overrightarrow{BH}$ bisects $\angle PBA$ and show that $\triangle ABH \cong \triangle PBH$.
3. Give reasons for each statement below to show that $AC > DF$.

$AC = AH + HC = PH + HC > PC = DF$

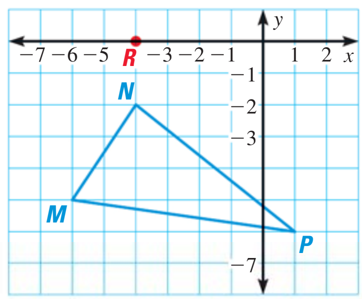


TAKS PRACTICE at classzone.com

MIXED REVIEW FOR TAKS

REVIEW
Lesson 4.8;
TAKS Workbook

29. TAKS PRACTICE A translation of $\triangle MNP$ results in the image $\triangle RST$, with point M mapped to point R. What are the coordinates of S and T? *TAKS Obj. 6*



Ⓐ $S(-2, 3)$ and $T(3, -1)$

Ⓑ $S(-6, 3)$ and $T(1, 4)$

Ⓒ $S(-6, -3)$ and $T(1, -4)$

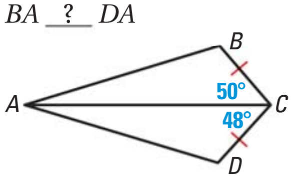
Ⓓ $S(-11, 1)$ and $T(-9, 4)$

QUIZ for Lessons 5.5–5.6

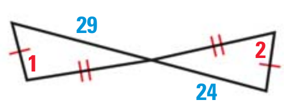
1. Is it possible to construct a triangle with side lengths 5, 6, and 12? If not, *explain* why not. *(p. 328)*
2. The lengths of two sides of a triangle are 15 yards and 27 yards. *Describe* the possible lengths of the third side of the triangle. *(p. 328)*
3. In $\triangle PQR$, $m\angle P = 48°$ and $m\angle Q = 79°$. List the sides of $\triangle PQR$ in order from shortest to longest. *(p. 328)*

Copy and complete with <, >, or =. *(p. 335)*

4. BA __?__ DA



5. $m\angle 1$ __?__ $m\angle 2$



MIXED REVIEW FOR TEKS

TAKS PRACTICE
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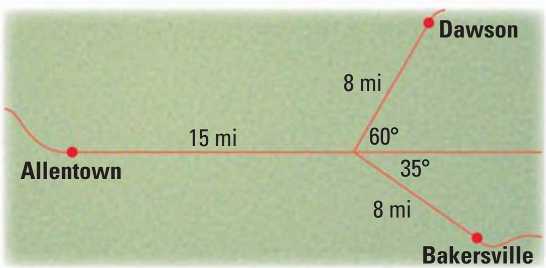
Lessons 5.4–5.6

MULTIPLE CHOICE

1. **PEDIMENT** The peaked cap pediment shown below forms an isosceles triangle with base 13.5 feet and height 4.25 feet. Which is closest to the perimeter of $\triangle JKL$? ***TEKS G.5.B***

Ⓐ 7.97 ft. Ⓑ 17.75 ft.
Ⓒ 29.5 ft. Ⓓ 41.8 ft.

2. **DRIVING** The map shows Eliza's driving route from Allentown to Bakersville and from Allentown to Dawson. The distance between Allentown and Dawson is about 20 miles. Which is a possible distance from Allentown to Bakersville? ***TEKS G.5.B***



Ⓕ 18 miles
Ⓖ 20 miles
Ⓗ 22 miles
Ⓙ Not enough information

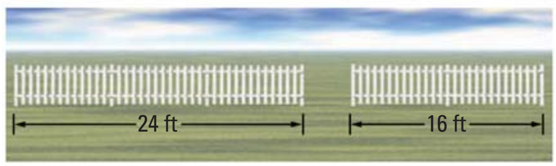
3. **MEDIANS** One side of $\triangle TUV$ measures 4 units and another side measures 6 units. Which is not a possible length for any median of $\triangle TUV$? ***TEKS G.5.B***

Ⓐ 2 units Ⓑ 5 units
Ⓒ 7 units Ⓓ 9 units

4. **ARCHITECTURE** In the figure below, $DE = 8.8$ m and $EF = 38.3$ m. Which is a possible length of $\overline{DF}$? ***TEKS G.5.B***

Ⓕ 27.3 m Ⓖ 29.3 m
Ⓗ 39.3 m Ⓙ 47.3 m

5. **FENCING** Tony wants to create a triangular fenced pen for his dog. He already has the two pieces of fencing shown and plans to use these for two sides of the pen. Which compound inequality represents the possible lengths (in feet) of the third side of the fence, x? ***TEKS G.5.B***

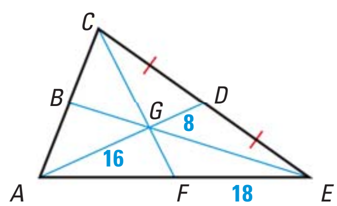


Ⓐ $8 < x < 24$ Ⓑ $8 < x < 40$
Ⓒ $16 < x < 24$ Ⓓ $16 < x < 40$

GRIDDED ANSWER

0 1 2 3 4 5 6 7 8 9

6. **TRIANGLE PROPERTIES** Find the length of $\overline{AF}$ in the diagram below. ***TEKS G.5.B***



5 CHAPTER SUMMARY

BIG IDEAS *For Your Notebook*

Big Idea 1

TEKS G.5.B

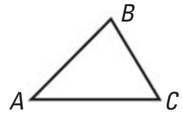
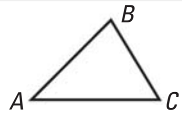
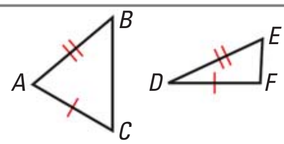
Using Properties of Special Segments in Triangles

Special segment	***Properties to remember***
Midsegment	Parallel to side opposite it and half the length of side opposite it
Perpendicular bisector	Concurrent at the circumcenter, which is: • equidistant from 3 vertices of △ • center of *circumscribed* circle that passes through 3 vertices of △
Angle bisector	Concurrent at the incenter, which is: • equidistant from 3 sides of △ • center of *inscribed* circle that just touches each side of △
Median (connects vertex to midpoint of opposite side)	Concurrent at the centroid, which is: • located two thirds of the way from vertex to midpoint of opposite side • balancing point of △
Altitude (perpendicular to side of △ through opposite vertex)	Concurrent at the orthocenter Used in finding area: If b is length of any side and h is length of altitude to that side, then $A = \frac{1}{2}bh$.

Big Idea 2

TEKS G.9.B

Using Triangle Inequalities to Determine What Triangles are Possible

Sum of lengths of any two sides of a △ is greater than length of third side.		$AB + BC > AC$ $AB + AC > BC$ $BC + AC > AB$
In a △, longest side is opposite largest angle and shortest side is opposite smallest angle.		If $AC > AB > BC$, then $m\angle B > m\angle C > m\angle A$. If $m\angle B > m\angle C > m\angle A$, then $AC > AB > BC$.
If two sides of a △ are ≅ to two sides of another △, then the △ with longer third side also has larger included angle.		If $BC > EF$, then $m\angle A > m\angle D$. If $m\angle A > m\angle D$, then $BC > EF$.

Big Idea 3

TEKS G.7.A, G.3.E

Extending Methods for Justifying and Proving Relationships

Coordinate proof uses the coordinate plane and variable coordinates. *Indirect proof* involves assuming the conclusion is false and then showing that the assumption leads to a contradiction.

5 CHAPTER REVIEW

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• Multi-Language Glossary
• Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- midsegment of a triangle, *p. 295*
- coordinate proof, *p. 296*
- perpendicular bisector, *p. 303*
- equidistant, *p. 303*
- concurrent, *p. 305*
- point of concurrency, *p. 305*
- circumcenter, *p. 306*
- incenter, *p. 312*
- median of a triangle, *p. 319*
- centroid, *p. 319*
- altitude of a triangle, *p. 320*
- orthocenter, *p. 321*
- indirect proof, *p. 337*

VOCABULARY EXERCISES

1. Copy and complete: A perpendicular bisector is a segment, ray, line, or plane that is perpendicular to a segment at its __?__.

2. **WRITING** *Explain* how to draw a circle that is circumscribed about a triangle. What is the center of the circle called? *Describe* its radius.

In Exercises 3–5, match the term with the correct definition.

3. Incenter **A.** The point of concurrency of the medians of a triangle

4. Centroid **B.** The point of concurrency of the angle bisectors of a triangle

5. Orthocenter **C.** The point of concurrency of the altitudes of a triangle

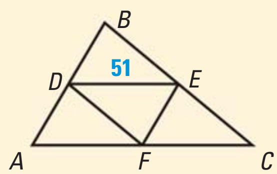
REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 5.

5.1 Midsegment Theorem and Coordinate Proof *pp. 295–301*

EXAMPLE

In the diagram, $\overline{DE}$ is a midsegment of $\triangle ABC$. Find AC.



By the Midsegment Theorem, $DE = \frac{1}{2}AC$.

So, $AC = 2DE = 2(51) = 102$.

EXERCISES

EXAMPLES 1, 4, and 5 on pp. 295, 297 for Exs. 6–8

Use the diagram above where $\overline{DF}$ and $\overline{EF}$ are midsegments of $\triangle ABC$.

6. If $AB = 72$, find EF.

7. If $DF = 45$, find EC.

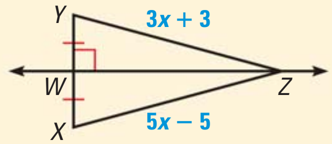
8. Graph $\triangle PQR$, with vertices $P(2a, 2b)$, $Q(2a, 0)$, and $O(0, 0)$. Find the coordinates of midpoint S of $\overline{PQ}$ and midpoint T of $\overline{QO}$. Show $\overline{ST} \parallel \overline{PO}$.

5.2 Use Perpendicular Bisectors

pp. 303–309

EXAMPLE

Use the diagram at the right to find XZ.



$\overleftrightarrow{WZ}$ is the perpendicular bisector of $\overline{XY}$.

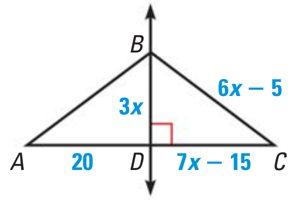
$5x - 5 = 3x + 3$	**By the Perpendicular Bisector Theorem, $ZX = ZY$.**
$x = 4$	**Solve for x.**

▶ So, $XZ = 5x - 5 = 5(4) - 5 = 15$.

EXERCISES

EXAMPLES 1 and 2 on pp. 303–304 for Exs. 9–11

In the diagram, $\overleftrightarrow{BD}$ is the perpendicular bisector of $\overline{AC}$.



9. What segment lengths are equal?

10. What is the value of x?

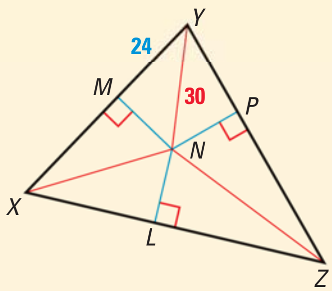
11. Find AB.

5.3 Use Angle Bisectors of Triangles

pp. 310–316

EXAMPLE

In the diagram, N is the incenter of $\triangle XYZ$. Find NL.



Use the Pythagorean Theorem to find NM in $\triangle NMY$.

$c^2 = a^2 + b^2$	**Pythagorean Theorem**
$30^2 = NM^2 + 24^2$	**Substitute known values.**
$900 = NM^2 + 576$	**Multiply.**
$324 = NM^2$	**Subtract 576 from each side.**
$18 = NM$	**Take positive square root of each side.**

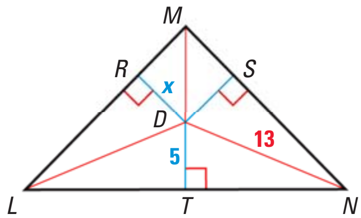
▶ By the Concurrency of Angle Bisectors of a Triangle, the incenter N of $\triangle XYZ$ is equidistant from all three sides of $\triangle XYZ$. So, because $NM = NL$, $NL = 18$.

EXERCISES

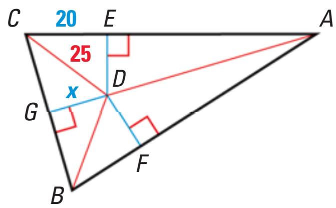
EXAMPLE 4 on p. 312 for Exs. 12–13

Point D is the incenter of the triangle. Find the value of x.

12.



13.

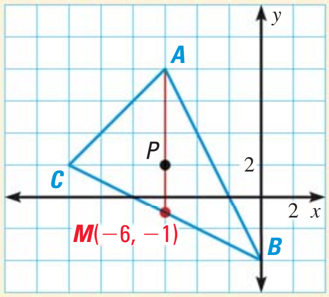


5 CHAPTER REVIEW

5.4 Use Medians and Altitudes
pp. 319–325

EXAMPLE

The vertices of $\triangle ABC$ are $A(-6, 8)$, $B(0, -4)$, and $C(-12, 2)$. Find the coordinates of its centroid P.



Sketch $\triangle ABC$. Then find the midpoint M of $\overline{BC}$ and sketch median $\overline{AM}$.

$$M\left(\frac{-12+0}{2}, \frac{2+(-4)}{2}\right) = M(-6, -1)$$

The centroid is two thirds of the distance from a vertex to the midpoint of the opposite side.

The distance from vertex $A(-6, 8)$ to midpoint $M(-6, -1)$ is $8 - (-1) = 9$ units.

So, the centroid P is $\frac{2}{3}(9) = 6$ units down from A on $\overline{AM}$.

▶ The coordinates of the centroid P are $(-6, 8 - 6)$, or $(-6, 2)$.

EXERCISES

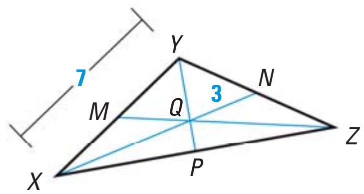
EXAMPLES 1, 2, and 3 on pp. 319–321 for Exs. 14–18

Find the coordinates of the centroid D of $\triangle RST$.

14. $R(-4, 0)$, $S(2, 2)$, $T(2, -2)$

15. $R(-6, 2)$, $S(-2, 6)$, $T(2, 4)$

Point Q is the centroid of $\triangle XYZ$.



16. Find XQ.

17. Find XM.

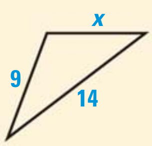
18. Draw an obtuse $\triangle ABC$. Draw its three altitudes. Then label its orthocenter D.

5.5 Use Inequalities in a Triangle
pp. 328–334

EXAMPLE

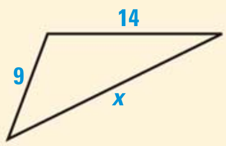
A triangle has one side of length 9 and another of length 14. Describe the possible lengths of the third side.

Let x represent the length of the third side. Draw diagrams and use the Triangle Inequality Theorem to write inequalities involving x.



$x + 9 > 14$

$x > 5$



$9 + 14 > x$

$23 > x$, or $x < 23$

▶ The length of the third side must be greater than 5 and less than 23.

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Chapter Review Practice

EXERCISES

EXAMPLES 1, 2, and 3 on pp. 328–330 for Exs. 19–24

***Describe* the possible lengths of the third side of the triangle given the lengths of the other two sides.**

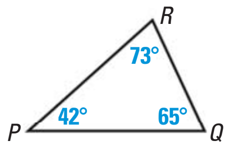
19. 4 inches, 8 inches

20. 6 meters, 9 meters

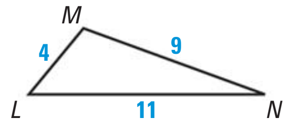
21. 12 feet, 20 feet

List the sides and the angles in order from smallest to largest.

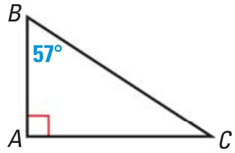
22.



23.



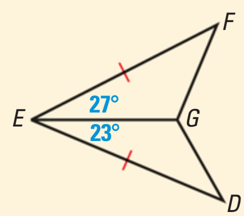
24.



5.6 Inequalities in Two Triangles and Indirect Proof *pp. 335–341*

EXAMPLE

How does the length of $\overline{DG}$ compare to the length of $\overline{FG}$?



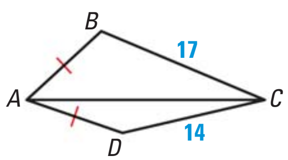
▶ Because $27° > 23°$, $m\angle GEF > m\angle GED$. You are given that $\overline{DE} \cong \overline{FE}$ and you know that $\overline{EG} \cong \overline{EG}$. Two sides of $\triangle GEF$ are congruent to two sides of $\triangle GED$ and the included angle is larger so, by the Hinge Theorem, $FG > DG$.

EXERCISES

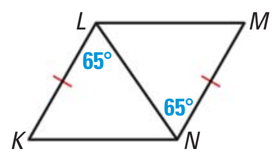
EXAMPLES 1, 3, and 4 on pp. 336–338 for Exs. 25–27

Copy and complete with <, >, or =.

25. $m\angle BAC$ _?_ $m\angle DAC$



26. LM _?_ KN



27. Arrange statements A–D in correct order to write an indirect proof of the statement: *If two lines intersect, then their intersection is exactly one point.*

GIVEN ▶ Intersecting lines m and n

PROVE ▶ The intersection of lines m and n is exactly one point.

A. But this contradicts Postulate 5, which states that through any two points there is exactly one line.

B. Then there are two lines (m and n) through points P and Q.

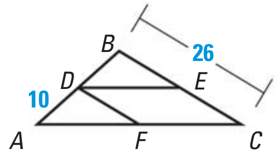
C. Assume that there are two points, P and Q, where m and n intersect.

D. It is false that m and n can intersect in two points, so they must intersect in exactly one point.

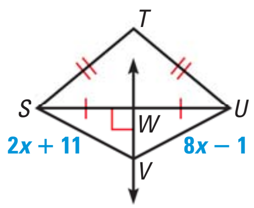
5 CHAPTER TEST

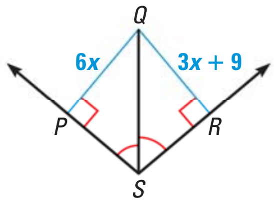
Two midsegments of $\triangle ABC$ are $\overline{DE}$ and $\overline{DF}$.

1. Find DB.
2. Find DF.
3. What can you conclude about $\overline{EF}$?



Find the value of x. *Explain* your reasoning.

4.


5.


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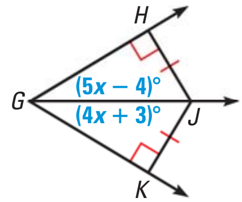
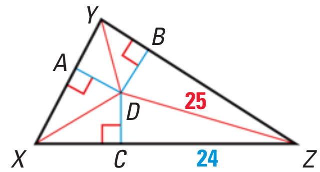
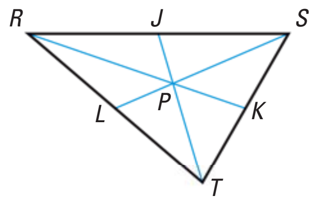


7. In Exercise 4, is point T on the perpendicular bisector of $\overline{SU}$? *Explain.*
8. In the diagram at the right, the angle bisectors of $\triangle XYZ$ meet at point D. Find DB.



In the diagram at the right, P is the centroid of $\triangle RST$.

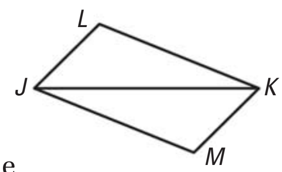
9. If $LS = 36$, find PL and PS.
10. If $TP = 20$, find TJ and PJ.
11. If $JR = 25$, find JS and RS.



12. Is it possible to construct a triangle with side lengths 9, 12, and 22? If not, *explain* why not.
13. In $\triangle ABC$, $AB = 36$, $BC = 18$, and $AC = 22$. Sketch and label the triangle. List the angles in order from smallest to largest.

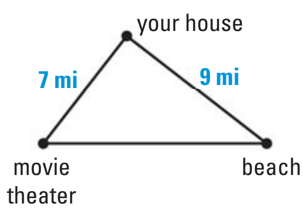
In the diagram for Exercises 14 and 15, $JL = MK$.

14. If $m\angle JKM > m\angle LJK$, which is longer, $\overline{LK}$ or $\overline{MJ}$? *Explain.*
15. If $MJ < LK$, which is larger, $\angle LJK$ or $\angle JKM$? *Explain.*
16. Write a temporary assumption you could make to prove the conclusion indirectly: *If $RS + ST \neq 12$ and $ST = 5$, then $RS \neq 7$.*



Use the diagram in Exercises 17 and 18.

17. *Describe* the range of possible distances from the beach to the movie theater.
18. A market is the same distance from your house, the movie theater, and the beach. Copy the diagram and locate the market.



5 ALGEBRA REVIEW

Animated Algebra classzone.com

USE RATIOS AND PERCENT OF CHANGE

EXAMPLE 1 *Write a ratio in simplest form*

A team won 18 of its 30 games and lost the rest. Find its win-loss ratio.

The ratio of a to b, $b \neq 0$, can be written as a to b, $a:b$, and $\frac{a}{b}$.

$$\frac{\text{wins}}{\text{losses}} = \frac{18}{30 - 18}$$ **To find losses, subtract wins from total.**

$$= \frac{18}{12} = \frac{3}{2}$$ **Simplify.**

▶ The team's win-loss ratio is 3 : 2.

EXAMPLE 2 *Find and interpret a percent of change*

A \$50 sweater went on sale for \$28. What is the percent of change in price? The new price is what percent of the old price?

$$\text{Percent of change} = \frac{\text{Amount of increase or decrease}}{\text{Original amount}} = \frac{50 - 28}{50} = \frac{22}{50} = 0.44$$

▶ The price went down, so the change is a decrease. The percent of decrease is 44%. So, the new price is 100%−44% = 56% of the original price.

EXERCISES

EXAMPLE 1 for Exs. 1–3

1. A team won 12 games and lost 4 games. Write each ratio in simplest form.
   a. wins to losses
   b. losses out of total games
2. A scale drawing that is 2.5 feet long by 1 foot high was used to plan a mural that is 15 feet long by 6 feet high. Write each ratio in simplest form.
   a. length to height of mural
   b. length of scale drawing to length of mural
3. There are 8 males out of 18 members in the school choir. Write the ratio of females to males in simplest form.

EXAMPLE 2 for Exs. 4–13

Find the percent of change.

4. From 75 campsites to 120 campsites
5. From 150 pounds to 136.5 pounds
6. From \$480 to \$408
7. From 16 employees to 18 employees
8. From 24 houses to 60 houses
9. From 4000 ft^2 to 3990 ft^2

Write the percent comparing the new amount to the original amount. Then find the new amount.

10. 75 feet increased by 4%
11. 45 hours decreased by 16%
12. \$16,500 decreased by 85%
13. 80 people increased by 7.5%

5 TAKS PREPARATION

TAKS Obj. 1
TEKS A.1.C, A.1.D
TAKS Obj. 6
TEKS 8.7.D

REVIEWING MODELING WITH EQUATIONS AND INEQUALITIES

When modeling real-life situations with equations and inequalities, it is helpful to follow these steps.

STEP 1 **Read** the problem carefully to figure out what you need to find.

STEP 2 **Develop** a plan.

STEP 3 **Make** a model you can use to solve the equation or inequality.

EXAMPLE

COTTON AND HAY Jason is packing a truck with bales of cotton and hay for shipping. Each bale of hay weighs about 750 pounds and each bale of cotton weighs about 500 pounds. The truck can carry no more than 30 tons. Write an inequality to represent how many bales of hay, h, and how many bales of cotton, c, Jason can put in the truck. Then, find the maximum number of cotton bales he can put in the truck with 25 bales of hay.

Solution

STEP 1 **Figure** out what you need to find. Find the maximum number of bales of cotton that can be put in the truck with 25 bales of hay.

STEP 2 **Develop** a plan. First, make a model to write an inequality. Then, use the inequality to solve the problem.

STEP 3 **Make** a model.

Verbal Model

Weight per bale of hay	•	Number of bales of hay	+	Weight per bale of cotton	•	Number of bales of cotton	≤	Maximum weight on truck

Labels

Weight per bale of hay = **750** **(pounds per bale of hay)**

Number of bales of hay = h **(bales of hay)**

Weight per bale of cotton = **500** **(pounds per bale of cotton)**

Number of bales of cotton = c **(bales of cotton)**

Maximum weight on truck = 30 tons = **60,000** **(pounds)**

Algebraic Model

$$750h + 500c \leq 60{,}000$$

To find the maximum number of bales of cotton that can be put in a truck with 25 bales of hay, substitute 25 for h. Solve the inequality for c.

$750(25) + 500c \leq 60000$ or $c \leq 82.5$

▶ So, the maximum number of bales of cotton is 82.

MODELING WITH EQUATIONS AND INEQUALITIES ON TAKS

Below are examples of modeling with equations and inequalities in multiple choice format. Try solving the problems before looking at the solutions. (Cover the solutions with a piece of paper.) Then check your solutions against the ones given.

1. Alicia is making flower arrangements. Vases cost \$3.75 each and flowers cost \$3.50 per bundle. Alicia has a budget of \$40. Which inequality describes the possible number of vases, v, and bundles of flowers, b, that she can buy on her budget?

A $40 \geq 3.75v + 3.5b$

B $40 \geq 3.5v + 3.75b$

C $40 \leq 3.75v + 3.5b$

D $40 \leq 3.5v + 3.75b$

Solution

Write a verbal model to write an inequality.

Total cost	≥	Cost per vase	•	Number of vases	+	Cost per bundle	•	Bundles of flowers
40	≥	3.75	•	v	+	3.5	•	b

So, the correct answer is A.

A B C D

2. Thomas earns \$5.25 per hour waiting tables at a cafe plus an average tip of \$2.50 for each customer he serves. Which equation describes his total earnings, e, for h hours worked and c customers served?

F $e = 2.5h + 5.25c$

G $e = 2.5h - 5.25c$

H $e = 5.25h + 2.5c$

J $e = 5.25h - 2.5c$

Solution

Write a verbal model to write an inequality.

Total earnings	=	Hourly rate	•	Number of hours	+	Tip per customer	•	Customers served
e	=	5.25	•	h	+	2.5	•	c

So, the correct answer is H.

F G H J

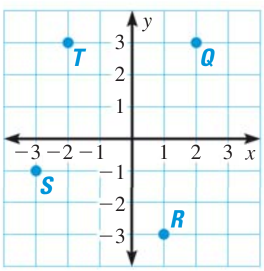
3. Which point on the grid satisfies the conditions $x < -1$ and $y \geq 3$?

A Q

B R

C S

D T



Solution

Look at the graph.

Only points S and T satisfy the condition that $x < -1$. Of those two points, only T satisfies the condition that $y \geq 3$.

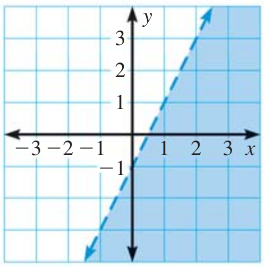
So, the correct answer is D.

A B C D

5 TAKS PRACTICE

PRACTICE FOR TAKS OBJECTIVES 1 AND 6

1. Which inequality best describes the shaded area of the graph?



A $y \le 2x - 1$

B $y < 2x - 1$

C $y > 2x - 1$

D $y \ge 2x - 1$

2. A candle company sells candles and glass candle holders. Each candle weighs 14 ounces and each glass holder weighs 8 ounces. Which equation represents the total weight, w, in ounces, of c candles and h candle holders?

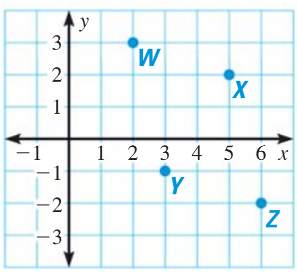
F $w = 8c + 14h$

G $w = (8 + 14)ch$

H $w = 14c + 8h$

J $w = 14c - 8h$

3. Which point on the grid satisfies the conditions $x > 4$ and $y \ge 2$?



A W

B X

C Y

D Z

4. Joe is packaging cookies to sell at a fundraiser. Each empty container weighs 90 grams. Each cookie weighs at least 35 grams. Which inequality represents the total weight, w, in grams, of a container of cookies in terms of c, the number of cookies?

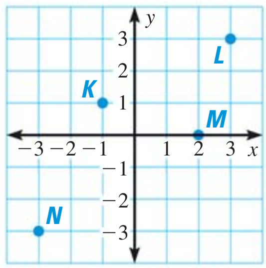
F $w \ge 35c + 90$

G $w \ge 90c + 35$

H $w \le 35c + 90$

J $w \le 90c + 35$

5. Which point on the grid satisfies the conditions $x \le -\frac{1}{2}$ and $y > -\frac{5}{2}$?



A K

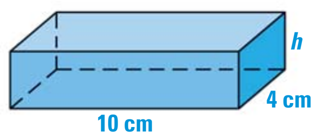
B L

C M

D N

MIXED TAKS PRACTICE

6. The volume of the rectangular prism shown is 100 cubic centimeters. What is the height of the prism? *TAKS Obj. 8*



F 2.5 cm

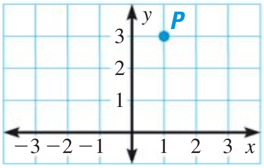
G 4 cm

H 5 cm

J 10 cm

MIXED TAKS PRACTICE

7. What are the coordinates of the image of P after the translation $(x, y) \to (x - 3, y - 4)$? ***TAKS Obj. 6***



A (1, 3)

B (4, 7)

C (−3, 0)

D (−2, −1)

8. George and Kate buy art supplies. George buys 3 paint brushes and 5 jars of paint for \$27.75. Kate buys 2 paint brushes and 3 jars of paint for \$17.20. Which system of equations can be used to determine the cost of 1 paint brush, x, and 1 jar of paint, y? ***TAKS Obj. 4***

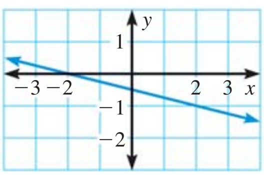
F $3x + 5y = 27.75$
$2x + 3y = 17.2$

G $3x + 5y = 17.2$
$2x + 3y = 27.75$

H $3x + 2y = 27.75$
$5x + 3y = 17.2$

J $3x + 2y = 17.2$
$5x + 3y = 27.75$

9. What is the rate of change of the graph? ***TAKS Obj. 3***



A 4

B −4

C 0.25

D −0.25

10. How does the graph of $y = x^2$ differ from the graph of $y = 2x^2$? ***TAKS Obj. 5***

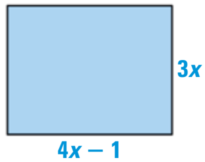
F The graph of $y = 2x^2$ is wider than $y = x^2$.

G The graph of $y = 2x^2$ is narrower than $y = x^2$.

H The graph of $y = 2x^2$ is shifted 2 units above $y = x^2$.

J The graph of $y = 2x^2$ is shifted 2 units to the right of $y = x^2$.

11. Which expression represents the area of the rectangle below? ***TAKS Obj. 2***



A $12x^2 - 3x$

B $12x^2 - 1$

C $14x - 2$

D $10x - 1$

12. **GRIDDED ANSWER** A bag contains blue, red, green, and purple tiles. Julia randomly draws one tile from the bag, replaces it, then draws another tile from the bag, replaces it, and so on. The table shows the results after she draws 50 tiles. Based on these results, what is the experimental probability of randomly drawing a blue tile from the bag? Write your answer as a decimal. ***TAKS Obj. 9***

Outcome	Frequency
Blue	8
Red	10
Green	20
Purple	12

Record your answer and fill in the bubbles on your answer document. Be sure to use the correct place value.