3 Parallel and Perpendicular Lines

TEXAS

Before

In previous chapters, you learned the following skills, which you'll use in Chapter 3: describing angle pairs, using properties and postulates, using angle pair relationships, and sketching a diagram.

Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement.

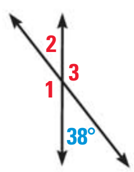
1. Adjacent angles share a common __?__.
2. Two angles are __?__ angles if the sum of their measures is 180°.

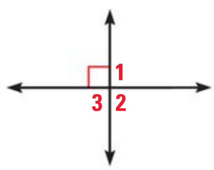
SKILLS AND ALGEBRA CHECK

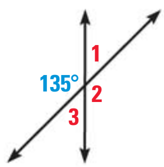
The midpoint of $\overline{AB}$ is M. Find AB. *(Review p. 15 for 3.2.)*

3. $AM = 5x - 2$, $MB = 2x + 7$
4. $AM = 4z + 1$, $MB = 6z - 11$

Find the measure of each numbered angle. *(Review p. 124 for 3.2, 3.3.)*

5. 

6. 

7. 

Sketch a diagram for each statement. *(Review pp. 2, 96 for 3.3.)*

8. $\overleftrightarrow{QR}$ is perpendicular to $\overleftrightarrow{WX}$.
9. Lines m and n intersect at point P.

TEXAS @HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 3, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 201. You will also use the key vocabulary listed below.

Big Ideas

1. Using properties of parallel and perpendicular lines
2. Proving relationships using angle measures
3. Making connections to lines in algebra

KEY VOCABULARY

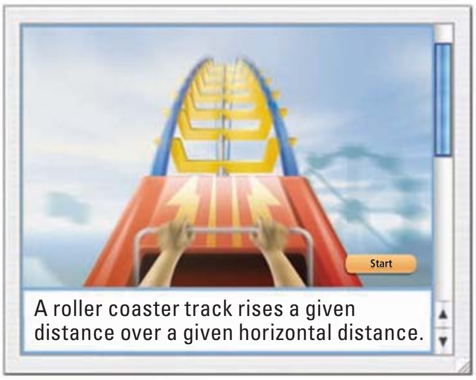
- parallel lines, *p. 147*
- skew lines, *p. 147*
- parallel planes, *p. 147*
- transversal, *p. 149*
- corresponding angles, *p. 149*
- alternate interior angles, *p. 149*
- alternate exterior angles, *p. 149*
- consecutive interior angles, *p. 149*
- paragraph proof, *p. 163*
- slope, *p. 171*
- slope-intercept form, *p. 180*
- standard form, *p. 182*
- distance from a point to a line, *p. 192*

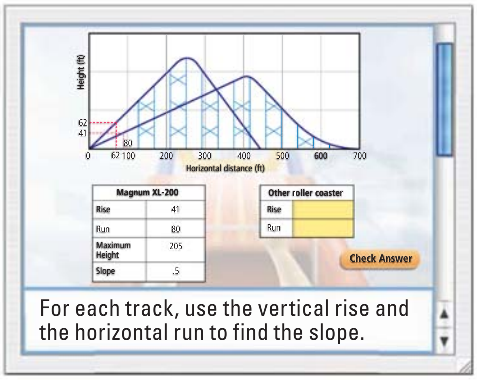
Why?

You can use slopes of lines to determine steepness of lines. For example, you can compare the slopes of roller coasters to determine which is steeper.

Animated Geometry

The animation illustrated below for Example 5 on page 174 helps you answer this question: How steep is a roller coaster?





Geometry at classzone.com

Animated **Geometry** at classzone.com

Other animations for Chapter 3: pages 148, 155, 163, and 181

Investigating Geometry ACTIVITY *Use before Lesson 3.1*

3.1 Draw and Interpret Lines

TEKS *a.2, G.2.A, G.6.C, G.9.D*

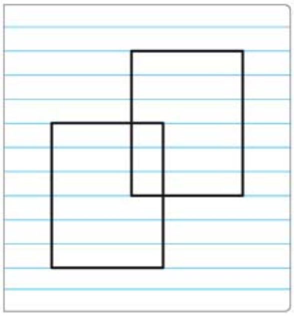
MATERIALS • pencil • straightedge • lined paper

QUESTION How are lines related in space?

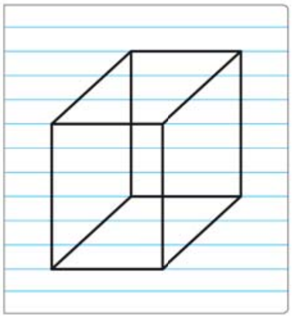
You can use a straightedge to draw a representation of a three-dimensional figure to explore lines in space.

EXPLORE Draw lines in space

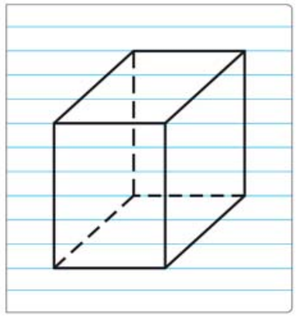
STEP 1 *Draw rectangles*
Use a straightedge to draw two identical rectangles.



STEP 2 *Connect corners*
Connect the corresponding corners of the rectangles.

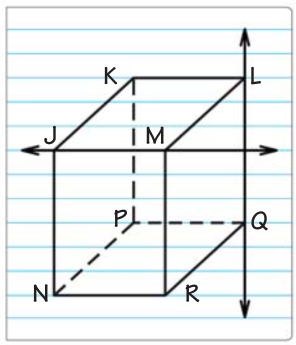


STEP 3 *Erase parts*
Erase parts of "hidden" lines to form dashed lines.



DRAW CONCLUSIONS Use your observations to complete these exercises

Using your sketch from the steps above, label the corners as shown at the right. Then extend $\overline{JM}$ and $\overline{LQ}$. Add lines to the diagram if necessary.



1. Will $\overleftrightarrow{JM}$ and $\overleftrightarrow{LQ}$ ever intersect in space? (Lines that intersect on the page do not necessarily intersect in space.)
2. Will the pair of lines intersect in space?
   a. $\overleftrightarrow{JK}$ and $\overleftrightarrow{NR}$ **b.** $\overleftrightarrow{QR}$ and $\overleftrightarrow{MR}$
   c. $\overleftrightarrow{LM}$ and $\overleftrightarrow{MR}$ **d.** $\overleftrightarrow{KL}$ and $\overleftrightarrow{NQ}$
3. Does the pair of lines lie in one plane?
   a. $\overleftrightarrow{JK}$ and $\overleftrightarrow{QR}$ **b.** $\overleftrightarrow{QR}$ and $\overleftrightarrow{MR}$
   c. $\overleftrightarrow{JN}$ and $\overleftrightarrow{LR}$ **d.** $\overleftrightarrow{JL}$ and $\overleftrightarrow{NQ}$
4. Do pairs of lines that intersect in space also lie in the same plane? *Explain* your reasoning.
5. Draw a rectangle that is not the same as the one you used in the Explore. Repeat the three steps of the Explore. Will any of your answers to Exercises 1–3 change?

3.1 Identify Pairs of Lines and Angles

TEKS a.2, G.1.A, G.5.B, G.9.A

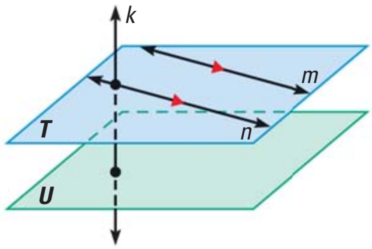
Before You identified angle pairs formed by two intersecting lines.

Now You will identify angle pairs formed by three intersecting lines.

Why? So you can classify lines in a real-world situation, as in Exs. 40–42.

Key Vocabulary
- parallel lines
- skew lines
- parallel planes
- transversal
- corresponding angles
- alternate interior angles
- alternate exterior angles
- consecutive interior angles

Two lines that do not intersect are either *parallel lines* or *skew lines*. Two lines are **parallel lines** if they do not intersect and are coplanar. Two lines are **skew lines** if they do not intersect and are not coplanar. Also, two planes that do not intersect are **parallel planes**.



Lines m and n are parallel lines ($m \parallel n$).

Lines m and k are skew lines.

Planes T and U are parallel planes ($T \parallel U$).

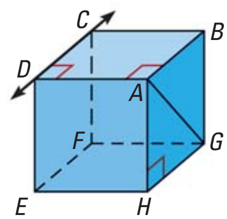
Lines k and n are intersecting lines, and there is a plane (not shown) containing them.

Small directed triangles, as shown on lines m and n above, are used to show that lines are parallel. The symbol $\parallel$ means "is parallel to," as in $m \parallel n$.

Segments and rays are parallel if they lie in parallel lines. A line is parallel to a plane if the line is in a plane parallel to the given plane. In the diagram above, line n is parallel to plane U.

EXAMPLE 1 Identify relationships in space

Think of each segment in the figure as part of a line. Which line(s) or plane(s) in the figure appear to fit the description?



a. Line(s) parallel to $\overleftrightarrow{CD}$ and containing point A

b. Line(s) skew to $\overleftrightarrow{CD}$ and containing point A

c. Line(s) perpendicular to $\overleftrightarrow{CD}$ and containing point A

d. Plane(s) parallel to plane EFG and containing point A

Solution

a. $\overleftrightarrow{AB}$, $\overleftrightarrow{HG}$, and $\overleftrightarrow{EF}$ all appear parallel to $\overleftrightarrow{CD}$, but only $\overleftrightarrow{AB}$ contains point A.

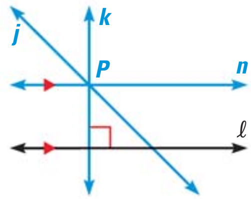
b. Both $\overleftrightarrow{AG}$ and $\overleftrightarrow{AH}$ appear skew to $\overleftrightarrow{CD}$ and contain point A.

c. $\overleftrightarrow{BC}$, $\overleftrightarrow{AD}$, $\overleftrightarrow{DE}$, and $\overleftrightarrow{FC}$ all appear perpendicular to $\overleftrightarrow{CD}$, but only $\overleftrightarrow{AD}$ contains point A.

d. Plane ABC appears parallel to plane EFG and contains point A.

PARALLEL AND PERPENDICULAR LINES Two lines in the same plane are either parallel or intersect in a point.

Through a point not on a line, there are infinitely many lines. Exactly one of these lines is parallel to the given line, and exactly one of them is perpendicular to the given line.

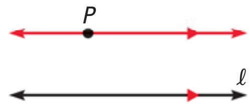


Animated Geometry at classzone.com

POSTULATES — *For Your Notebook*

POSTULATE 13 Parallel Postulate

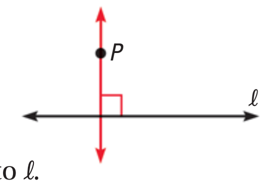
If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line.



There is exactly one line through P parallel to ℓ.

POSTULATE 14 Perpendicular Postulate

If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line.



There is exactly one line through P perpendicular to ℓ.

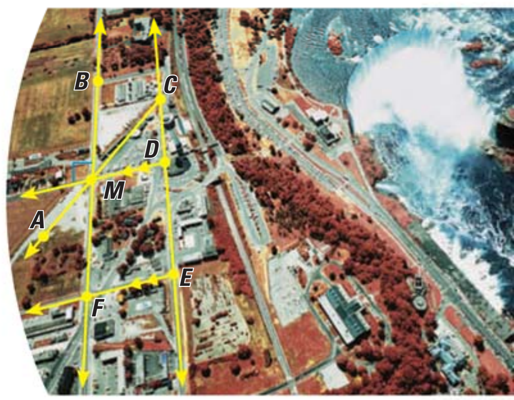
EXAMPLE 2 Identify parallel and perpendicular lines

PHOTOGRAPHY The given line markings show how the roads are related to one another.

a. Name a pair of parallel lines.

b. Name a pair of perpendicular lines.

c. Is $\overleftrightarrow{FE} \parallel \overleftrightarrow{AC}$? Explain.



Niagara Falls, New York

Solution

a. $\overleftrightarrow{MD} \parallel \overleftrightarrow{FE}$

b. $\overleftrightarrow{MD} \perp \overleftrightarrow{BF}$

c. $\overleftrightarrow{FE}$ is not parallel to $\overleftrightarrow{AC}$, because $\overleftrightarrow{MD}$ is parallel to $\overleftrightarrow{FE}$ and by the Parallel Postulate there is exactly one line parallel to $\overleftrightarrow{FE}$ through M.

✓ GUIDED PRACTICE for Examples 1 and 2

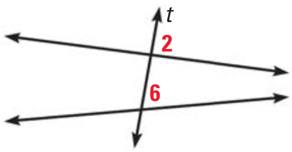
1. Look at the diagram in Example 1. Name the lines through point H that appear skew to $\overleftrightarrow{CD}$.

2. In Example 2, can you use the Perpendicular Postulate to show that $\overleftrightarrow{AC}$ is *not* perpendicular to $\overleftrightarrow{BF}$? *Explain* why or why not.

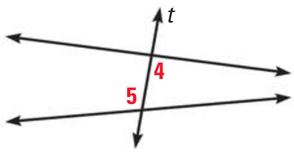
ANGLES AND TRANSVERSALS A **transversal** is a line that intersects two or more coplanar lines at different points.

KEY CONCEPT — *For Your Notebook*

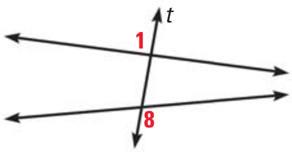
Angles Formed by Transversals



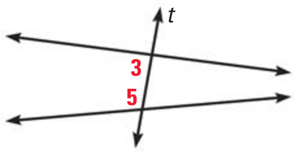
Two angles are **corresponding angles** if they have corresponding positions. For example, ∠2 and ∠6 are above the lines and to the right of the transversal *t*.



Two angles are **alternate interior angles** if they lie between the two lines and on opposite sides of the transversal.



Two angles are **alternate exterior angles** if they lie outside the two lines and on opposite sides of the transversal.



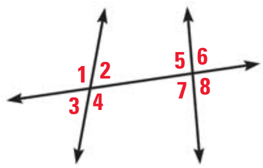
Two angles are **consecutive interior angles** if they lie between the two lines and on the same side of the transversal.

READ VOCABULARY
Another name for consecutive interior angles is **same-side interior angles**.

EXAMPLE 3 Identify angle relationships

Identify all pairs of angles of the given type.

a. Corresponding **b.** Alternate interior
c. Alternate exterior **d.** Consecutive interior



Solution

a. ∠1 and ∠5
∠2 and ∠6
∠3 and ∠7
∠4 and ∠8

b. ∠2 and ∠7
∠4 and ∠5

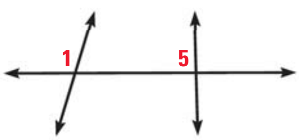
c. ∠1 and ∠8
∠3 and ∠6

d. ∠2 and ∠5
∠4 and ∠7

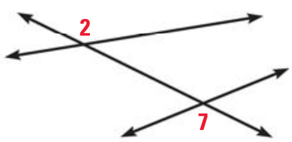
✓ GUIDED PRACTICE for Example 3

Classify the pair of numbered angles.

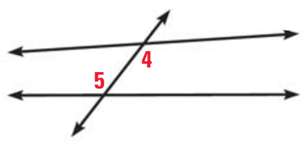
3.



4.



5.



3.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 25, and 35

★ = **TAKS PRACTICE AND REASONING** Exs. 28, 36, 37, 39, 45, and 46

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: A line that intersects two other lines is a __?__.

2. **WRITING** A table is set for dinner. Can the legs of the table and the top of the table lie in parallel planes? *Explain* why or why not.

EXAMPLE 1 on p. 147 for Exs. 3–6

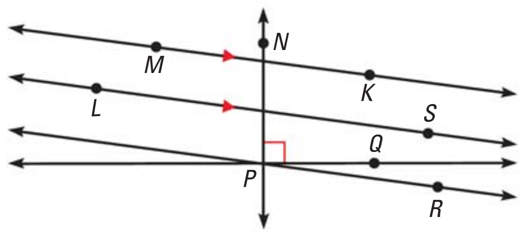
IDENTIFYING RELATIONSHIPS **Think of each segment in the diagram as part of a line. Which line(s) or plane(s) contain point *B* and appear to fit the description?**

3. Line(s) parallel to $\overleftrightarrow{CD}$
4. Line(s) perpendicular to $\overleftrightarrow{CD}$
5. Line(s) skew to $\overleftrightarrow{CD}$
6. Plane(s) parallel to plane *CDH*

EXAMPLE 2 on p. 148 for Exs. 7–10

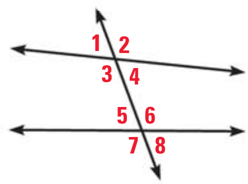
PARALLEL AND PERPENDICULAR LINES **Use the markings in the diagram.**

7. Name a pair of parallel lines.
8. Name a pair of perpendicular lines.
9. Is $\overleftrightarrow{PN} \parallel \overleftrightarrow{KM}$? *Explain.*
10. Is $\overleftrightarrow{PR} \perp \overleftrightarrow{NP}$? *Explain.*



EXAMPLE 3 on p. 149 for Exs. 11–15

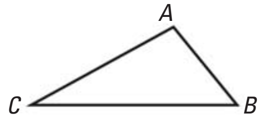
ANGLE RELATIONSHIPS **Identify all pairs of angles of the given type.**



11. Corresponding
12. Alternate interior
13. Alternate exterior
14. Consecutive interior

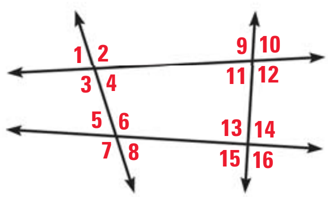
15. **ERROR ANALYSIS** *Describe* and correct the error in saying that ∠1 and ∠8 are corresponding angles in the diagram for Exercises 11–14.

APPLYING POSTULATES **How many lines can be drawn that fit each description? Copy the diagram and sketch all the lines.**



16. Lines through *B* and parallel to $\overleftrightarrow{AC}$
17. Lines through *A* and perpendicular to $\overleftrightarrow{BC}$

USING A DIAGRAM **Classify the angle pair as *corresponding, alternate interior, alternate exterior,* or *consecutive interior* angles.**



18. ∠5 and ∠1
19. ∠11 and ∠13
20. ∠6 and ∠13
21. ∠10 and ∠15
22. ∠2 and ∠11
23. ∠8 and ∠4

ANALYZING STATEMENTS **Copy and complete the statement with *sometimes*, *always*, or *never*. Sketch examples to *justify* your answer.**

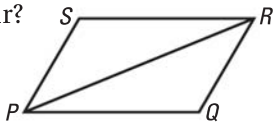
24. If two lines are parallel, then they are _?_ coplanar.

25. If two lines are not coplanar, then they _?_ intersect.

26. If three lines intersect at one point, then they are _?_ coplanar.

27. If two lines are skew to a third line, then they are _?_ skew to each other.

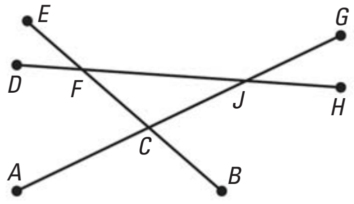
28. **TAKS REASONING** $\angle RPQ$ and $\angle PRS$ are what type of angle pair?



Ⓐ Corresponding Ⓑ Alternate interior

Ⓒ Alternate exterior Ⓓ Consecutive interior

ANGLE RELATIONSHIPS **Copy and complete the statement. List all possible correct answers.**



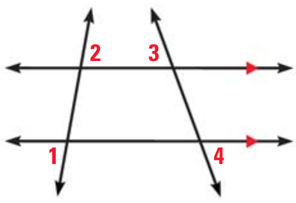
29. $\angle BCG$ and _?_ are corresponding angles.

30. $\angle BCG$ and _?_ are consecutive interior angles.

31. $\angle FCJ$ and _?_ are alternate interior angles.

32. $\angle FCA$ and _?_ are alternate exterior angles.

33. **CHALLENGE** Copy the diagram at the right and extend the lines.



a. Measure $\angle 1$ and $\angle 2$.

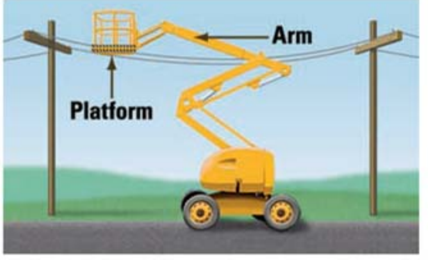
b. Measure $\angle 3$ and $\angle 4$.

c. Make a conjecture about alternate exterior angles formed when parallel lines are cut by transversals.

PROBLEM SOLVING

EXAMPLE 2
on p. 148
for Exs. 34–35

CONSTRUCTION **Use the picture of the cherry-picker for Exercises 34 and 35.**



34. Is the platform *perpendicular*, *parallel*, or *skew* to the ground?

TEXAS @HomeTutor for problem solving help at classzone.com

35. Is the arm *perpendicular*, *parallel*, or *skew* to a telephone pole?

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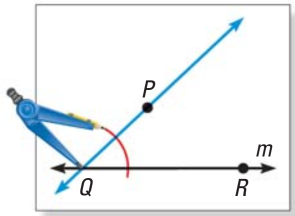
36. **TAKS REASONING** *Describe* two lines in your classroom that are parallel, and two lines that are skew.

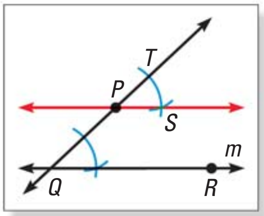
37. **TAKS REASONING** What is the best description of the horizontal bars in the photo?

Ⓐ Parallel Ⓑ Perpendicular

Ⓒ Skew Ⓓ Intersecting

38. **CONSTRUCTION** Use these steps to construct a line through a given point P that is parallel to a given line m.





STEP 1 Draw points Q and R on m. Draw $\overleftrightarrow{PQ}$. Draw an arc with the compass point at Q so it crosses $\overleftrightarrow{QP}$ and $\overleftrightarrow{QR}$.

STEP 2 Copy $\angle PQR$ on $\overleftrightarrow{QP}$. Be sure the two angles are corresponding. Label the new angle $\angle TPS$. Draw $\overleftrightarrow{PS}$. $\overleftrightarrow{PS} \parallel \overleftrightarrow{QR}$.

39. **TAKS REASONING** Two lines are cut by a transversal. Suppose the measure of a pair of alternate interior angles is 90°. *Explain* why the measure of all four interior angles must be 90°.

TREE HOUSE In Exercises 40–42, use the photo to decide whether the statement is *true* or *false*.

40. The plane containing the floor of the tree house is parallel to the ground.

41. All of the lines containing the railings of the staircase, such as $\overleftrightarrow{AB}$, are skew to the ground.

42. All of the lines containing the *balusters*, such as $\overleftrightarrow{CD}$, are perpendicular to the plane containing the floor of the tree house.

CHALLENGE Draw the figure described.

43. Lines ℓ and m are skew, lines ℓ and n are skew, and lines m and n are parallel.

44. Line ℓ is parallel to plane A, plane A is parallel to plane B, and line ℓ is not parallel to plane B.

TAKS PRACTICE at classzone.com

MIXED REVIEW FOR TAKS

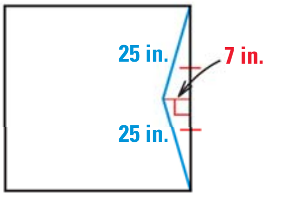
REVIEW
Skills Review Handbook p. 872; TAKS Workbook

45. **TAKS PRACTICE** Simplify the expression $-6(4 - x) + 3(2x + 5)$. *TAKS Obj. 2*

Ⓐ 39 Ⓑ $12x - 9$ Ⓒ $12x + 39$ Ⓓ $5x - 19$

REVIEW
Lessons 1.3, 1.7; TAKS Workbook

46. **TAKS PRACTICE** What is the approximate perimeter of the square? *TAKS Obj. 10*



Ⓕ 96 in. Ⓖ 144 in.

Ⓗ 192 in. Ⓙ 200 in.

Investigating Geometry ACTIVITY *Use before Lesson 3.2*

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classzone.com
Keystrokes

3.2 Parallel Lines and Angles

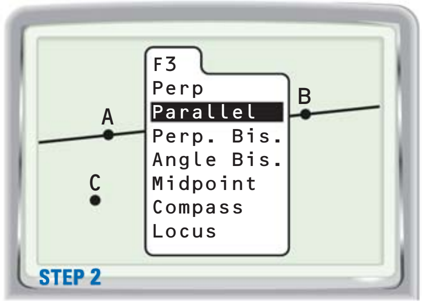
TEKS *a.5, G.2.B, G.3.D, G.9.A*

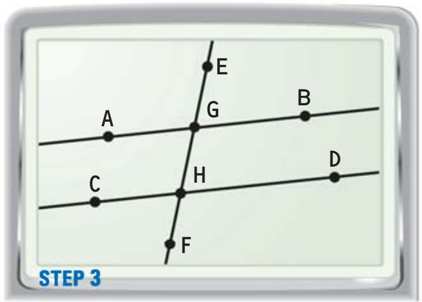
MATERIALS • graphing calculator or computer

QUESTION **What are the relationships among the angles formed by two parallel lines and a transversal?**

You can use geometry drawing software to explore parallel lines.

EXPLORE **Draw parallel lines and a transversal**





STEP 1 ***Draw line*** Draw and label two points *A* and *B*. Draw $\overleftrightarrow{AB}$.

STEP 2 ***Draw parallel line*** Draw a point not on $\overleftrightarrow{AB}$. Label it *C*. Choose Parallel from the F3 menu and select $\overleftrightarrow{AB}$. Then select *C* to draw a line through *C* parallel to $\overleftrightarrow{AB}$. Draw a point on the parallel line you constructed. Label it *D*.

STEP 3 ***Draw transversal*** Draw two points *E* and *F* outside the parallel lines. Draw transversal $\overleftrightarrow{EF}$. Find the intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{EF}$ by choosing Point from the F2 menu. Then choose Intersection. Label the intersection *G*. Find and label the intersection *H* of $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$.

STEP 4 ***Measure angle*** Measure all eight angles formed by the three lines by choosing Measure from the F5 menu, then choosing Angle.

DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. Record the angle measures from Step 4 in a table like the one shown. Which angles are congruent?

Angle	∠*AGE*	∠*EGB*	∠*AGH*	∠*BGH*	∠*CHG*	∠*GHD*	∠*CHF*	∠*DHF*
Measure 1	?	?	?	?	?	?	?	?

2. Drag point *E* or *F* to change the angle the transversal makes with the parallel lines. Be sure *E* and *F* stay outside the parallel lines. Record the new angle measures as row "Measure 2" in your table.

3. Make a conjecture about the measures of the given angles when two parallel lines are cut by a transversal.

a. Corresponding angles **b.** Alternate interior angles

4. **REASONING** Make and test a conjecture about the sum of the measures of two consecutive interior angles when two parallel lines are cut by a transversal.

3.2 Use Parallel Lines and Transversals

TEKS G.1.A, G.2.A, G.3.C, G.9.A

Before You identified angle pairs formed by a transversal.

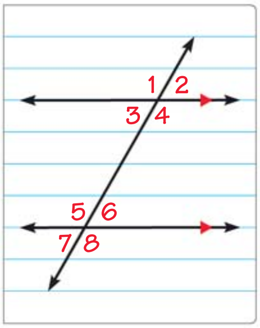
Now You will use angles formed by parallel lines and transversals.

Why? So you can understand angles formed by light, as in Example 4.

Key Vocabulary
- **corresponding angles,** *p. 149*
- **alternate interior angles,** *p. 149*
- **alternate exterior angles,** *p. 149*
- **consecutive interior angles,** *p. 149*

ACTIVITY EXPLORE PARALLEL LINES

Materials: lined paper, tracing paper, straightedge



STEP 1 **Draw** a pair of parallel lines cut by a nonperpendicular transversal on lined paper. Label the angles as shown.

STEP 2 **Trace** your drawing onto tracing paper.

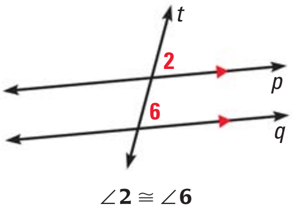
STEP 3 **Move** the tracing paper to position ∠1 of the traced figure over ∠5 of the original figure. Compare the angles. Are they congruent?

STEP 4 **Compare** the eight angles and list all the congruent pairs. What do you notice about the special angle pairs formed by the transversal?

POSTULATE *For Your Notebook*

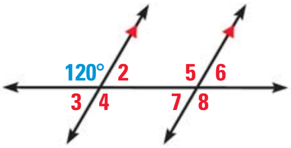
POSTULATE 15 Corresponding Angles Postulate

If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.



EXAMPLE 1 Identify congruent angles

The measure of three of the numbered angles is 120°. Identify the angles. Explain your reasoning.



Solution

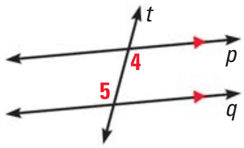
By the Corresponding Angles Postulate, $m\angle 5 = 120°$.
Using the Vertical Angles Congruence Theorem, $m\angle 4 = 120°$.
Because ∠4 and ∠8 are corresponding angles, by the Corresponding Angles Postulate, you know that $m\angle 8 = 120°$.

THEOREMS — *For Your Notebook*

THEOREM 3.1 Alternate Interior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

Proof: Example 3, p. 156

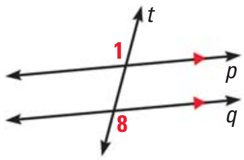


$\angle 4 \cong \angle 5$

THEOREM 3.2 Alternate Exterior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.

Proof: Ex. 37, p. 159

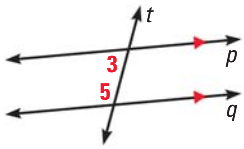


$\angle 1 \cong \angle 8$

THEOREM 3.3 Consecutive Interior Angles Theorem

If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.

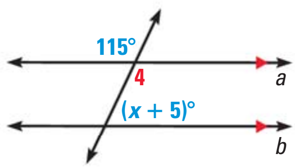
Proof: Ex. 41, p. 159



$\angle 3$ and $\angle 5$ are supplementary.

EXAMPLE 2 Use properties of parallel lines

xy ALGEBRA **Find the value of *x*.**



Solution

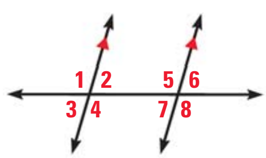
By the Vertical Angles Congruence Theorem, $m\angle 4 = 115°$. Lines *a* and *b* are parallel, so you can use the theorems about parallel lines.

$m\angle 4 + (x + 5)° = 180°$	Consecutive Interior Angles Theorem
$115° + (x + 5)° = 180°$	Substitute 115° for $m\angle 4$.
$x + 120 = 180$	Combine like terms.
$x = 60$	Subtract 120 from each side.

Animated Geometry at classzone.com

✓ GUIDED PRACTICE for Examples 1 and 2

Use the diagram at the right.



1. If $m\angle 1 = 105°$, find $m\angle 4$, $m\angle 5$, and $m\angle 8$. Tell which postulate or theorem you use in each case.

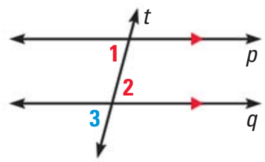
2. If $m\angle 3 = 68°$ and $m\angle 8 = (2x + 4)°$, what is the value of x? Show your steps.

EXAMPLE 3 Prove the Alternate Interior Angles Theorem

Prove that if two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

Solution

Draw a diagram. Label a pair of alternate interior angles as ∠1 and ∠2. You are looking for an angle that is related to both ∠1 and ∠2. Notice that one angle is a vertical angle with ∠2 and a corresponding angle with ∠1. Label it ∠3.



WRITE PROOFS
You can use the information from the diagram in your proof. Find any special angle pairs. Then decide what you know about those pairs.

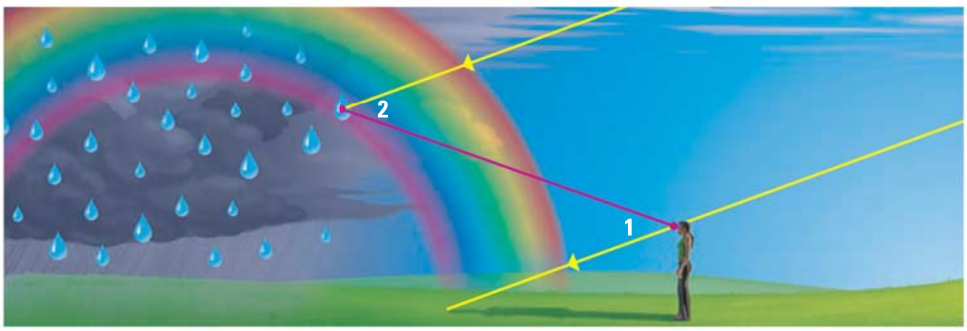
GIVEN ▶ $p \parallel q$

PROVE ▶ $\angle 1 \cong \angle 2$

STATEMENTS	REASONS
1. $p \parallel q$	**1.** Given
2. $\angle 1 \cong \angle 3$	**2.** Corresponding Angles Postulate
3. $\angle 3 \cong \angle 2$	**3.** Vertical Angles Congruence Theorem
4. $\angle 1 \cong \angle 2$	**4.** Transitive Property of Congruence

EXAMPLE 4 Solve a real-world problem

SCIENCE When sunlight enters a drop of rain, different colors of light leave the drop at different angles. This process is what makes a rainbow. For violet light, $m\angle 2 = 40°$. What is $m\angle 1$? How do you know?



Solution

Because the sun's rays are parallel, ∠1 and ∠2 are alternate interior angles. By the Alternate Interior Angles Theorem, $\angle 1 \cong \angle 2$. By the definition of congruent angles, $m\angle 1 = m\angle 2 = 40°$.

✓ GUIDED PRACTICE for Examples 3 and 4

3. In the proof in Example 3, if you use the third statement before the second statement, could you still prove the theorem? *Explain*.

4. **WHAT IF?** Suppose the diagram in Example 4 shows yellow light leaving a drop of rain. Yellow light leaves the drop at an angle of 41°. What is $m\angle 1$ in this case? How do you know?

3.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 9, and 39

★ = **TAKS PRACTICE AND REASONING** Exs. 3, 21, 33, 39, 40, 44, 45, and 46

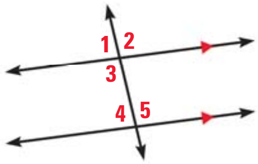
SKILL PRACTICE

1. **VOCABULARY** Draw a pair of parallel lines and a transversal. Label a pair of *corresponding angles*.

2. **WRITING** Two parallel lines are cut by a transversal. Which pairs of angles are congruent? Which pairs of angles are supplementary?

EXAMPLES 1 and 2 on pp. 154–155 for Exs. 3–16

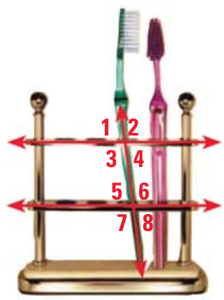
3. ★ **TAKS REASONING** In the figure at the right, which angle has the same measure as $\angle 1$?



Ⓐ $\angle 2$ Ⓑ $\angle 3$

Ⓒ $\angle 4$ Ⓓ $\angle 5$

USING PARALLEL LINES **Find the angle measure. Tell which postulate or theorem you use.**



4. If $m\angle 4 = 65°$, then $m\angle 1 = \underline{\ ?\ }$.

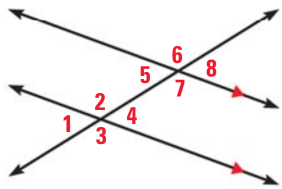
5. If $m\angle 7 = 110°$, then $m\angle 2 = \underline{\ ?\ }$.

6. If $m\angle 5 = 71°$, then $m\angle 4 = \underline{\ ?\ }$.

7. If $m\angle 3 = 117°$, then $m\angle 5 = \underline{\ ?\ }$.

8. If $m\angle 8 = 54°$, then $m\angle 1 = \underline{\ ?\ }$.

USING POSTULATES AND THEOREMS **What postulate or theorem justifies the statement about the diagram?**



9. $\angle 1 \cong \angle 5$

10. $\angle 4 \cong \angle 5$

11. $\angle 2 \cong \angle 7$

12. $\angle 2$ and $\angle 5$ are supplementary.

13. $\angle 3 \cong \angle 6$

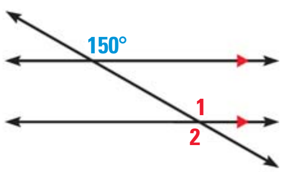
14. $\angle 3 \cong \angle 7$

15. $\angle 1 \cong \angle 8$

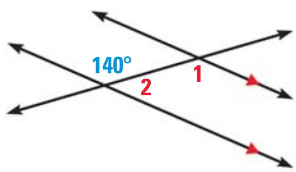
16. $\angle 4$ and $\angle 7$ are supplementary.

USING PARALLEL LINES **Find $m\angle 1$ and $m\angle 2$. *Explain* your reasoning.**

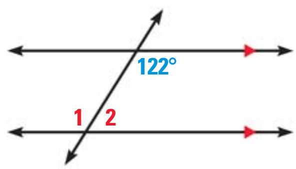
17.



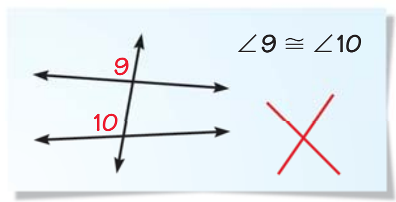
18.



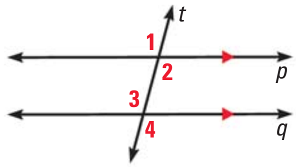
19.



20. **ERROR ANALYSIS** A student concludes that $\angle 9 \cong \angle 10$ by the Corresponding Angles Postulate. Describe and correct the error in this reasoning.

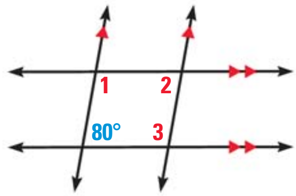


21. **TAKS REASONING** Given $p \parallel q$, *describe* two methods you can use to show that $\angle 1 \cong \angle 4$.

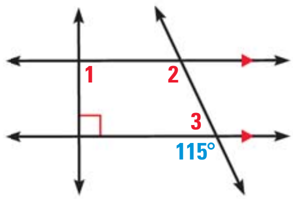


USING PARALLEL LINES **Find $m\angle 1$, $m\angle 2$, and $m\angle 3$. *Explain* your reasoning.**

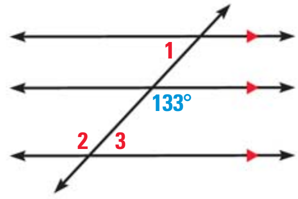
22.



23.



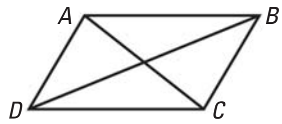
24.



ANGLES **Use the diagram at the right.**

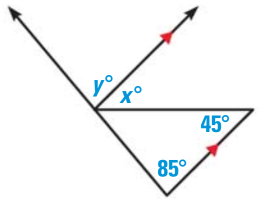
25. Name two pairs of congruent angles if $\overleftrightarrow{AB}$ and $\overleftrightarrow{DC}$ are parallel.

26. Name two pairs of supplementary angles if $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$ are parallel.

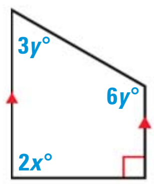


xy ALGEBRA **Find the values of x and y.**

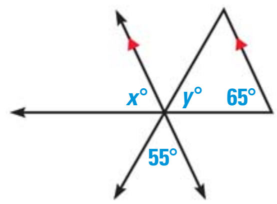
27.



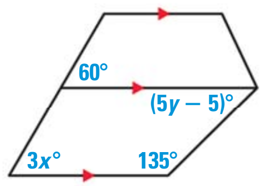
28.



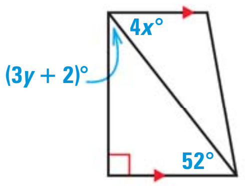
29.



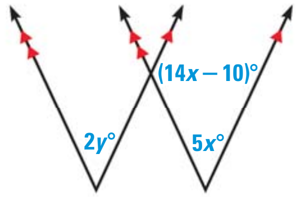
30.



31.



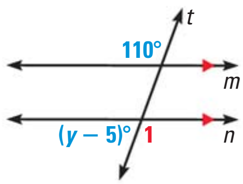
32.



33. **TAKS REASONING** What is the value of y in the diagram?

Ⓐ 70 Ⓑ 75

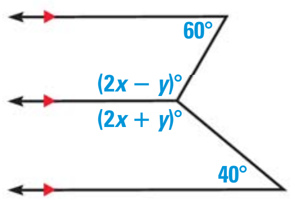
Ⓒ 110 Ⓓ 115



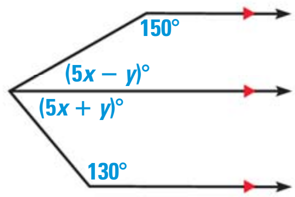
34. **DRAWING** Draw a four-sided figure with sides $\overline{MN}$ and $\overline{PQ}$, such that $\overline{MN} \parallel \overline{PQ}$, $\overline{MP} \parallel \overline{NQ}$, and $\angle MNQ$ is an acute angle. Which angle pairs formed are congruent? *Explain* your reasoning.

CHALLENGE **Find the values of x and y.**

35.



36.



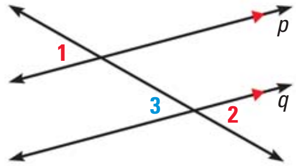
PROBLEM SOLVING

EXAMPLE 3 on p. 156 for Ex. 37

37. PROVING THEOREM 3.2 If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent. Use the steps below to write a proof of the Alternate Exterior Angles Theorem.

GIVEN ▶ $p \parallel q$

PROVE ▶ $\angle 1 \cong \angle 2$



a. Show that $\angle 1 \cong \angle 3$.

b. Then show that $\angle 1 \cong \angle 2$.

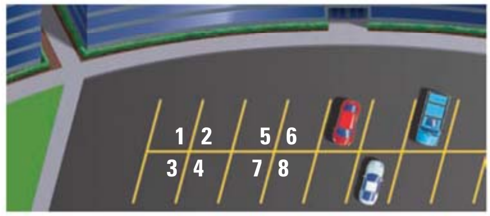
TEXAS @HomeTutor for problem solving help at classzone.com

EXAMPLE 4 on p. 156 for Exs. 38–40

38. PARKING LOT In the diagram, the lines dividing parking spaces are parallel. The measure of $\angle 1$ is 110°.

a. Identify the angle(s) congruent to $\angle 1$.

b. Find $m\angle 6$.



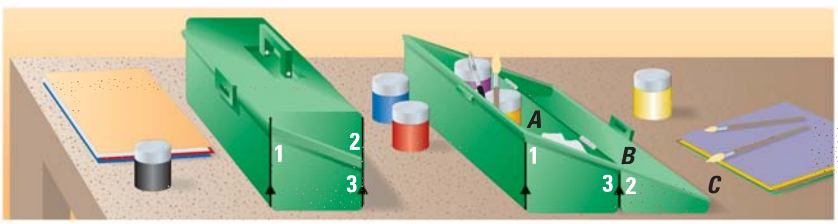
TEXAS @HomeTutor for problem solving help at classzone.com

39. **TAKS REASONING** The *Toddler*™ is a walking robot. Each leg of the robot has two parallel bars and a foot. When the robot walks, the leg bars remain parallel as the foot slides along the surface.

a. As the legs move, are there pairs of angles that are always congruent? always supplementary? If so, which angles?

b. *Explain* how having parallel leg bars allows the robot's foot to stay flat on the floor as it moves.

40. **TAKS REASONING** You are designing a box like the one below.

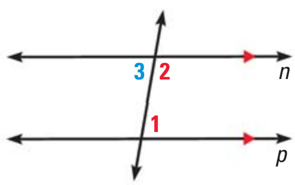


a. The measure of $\angle 1$ is 70°. What is $m\angle 2$? What is $m\angle 3$?

b. *Explain* why $\angle ABC$ is a straight angle.

c. **What If?** If $m\angle 1$ is 60°, will $\angle ABC$ still be a straight angle? Will the opening of the box be *more steep* or *less steep*? *Explain*.

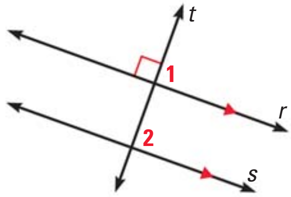
41. PROVING THEOREM 3.3 If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary. Write a proof of the Consecutive Interior Angles Theorem.



GIVEN ▶ $n \parallel p$

PROVE ▶ $\angle 1$ and $\angle 2$ are supplementary.

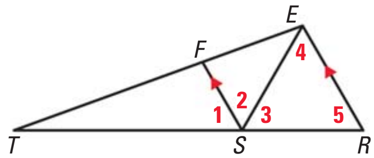
42. PROOF The Perpendicular Transversal Theorem (page 192) states that if a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other. Write a proof of the Perpendicular Transversal Theorem.



GIVEN ▶ $t \perp r$, $r \parallel s$

PROVE ▶ $t \perp s$

43. CHALLENGE In the diagram, $\angle 4 \cong \angle 5$. $\overline{SE}$ bisects $\angle RSF$. Find $m\angle 1$. *Explain* your reasoning.



MIXED REVIEW FOR TAKS

TAKS PRACTICE at classzone.com

REVIEW
Lesson 1.7;
TAKS Workbook

44. TAKS PRACTICE A rectangle has a perimeter of 20 feet. Suppose the length and width of the rectangle are doubled. What is the perimeter of the new rectangle? *TAKS Obj. 8*

Ⓐ 20 ft   Ⓑ 40 ft   Ⓒ 80 ft   Ⓓ 400 ft

REVIEW
Skills Review Handbook p. 871;
TAKS Workbook

45. TAKS PRACTICE Which expression is equivalent to $\frac{18x^3y^{-4}}{15xyz^{-2}}$? *TAKS Obj. 5*

Ⓕ $\frac{6x^2z^2}{5y^5}$ Ⓖ $\frac{6x^2y^5}{5z^2}$ Ⓗ $\frac{6y^3z^2}{5x^2}$ Ⓙ $\frac{6x^2y^3z^2}{5}$

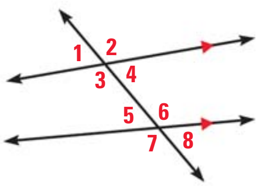
REVIEW
Skills Review Handbook p. 885;
TAKS Workbook

46. TAKS PRACTICE Tom is paying a bill in a restaurant. He wants to leave an 18% tip. Which expression represents the total cost of the meal including the tip, where x is the cost of the food in dollars? *TAKS Obj. 2*

Ⓐ $x + 0.18$ Ⓑ $x + 1.8$ Ⓒ $x + 0.18x$ Ⓓ $x + 1.8x$

QUIZ *for Lessons 3.1–3.2*

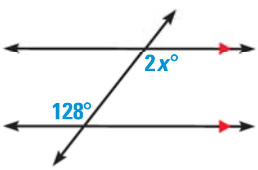
Copy and complete the statement. *(p. 147)*



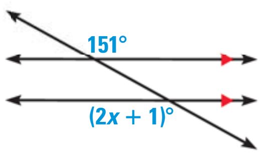
1. $\angle 2$ and __?__ are corresponding angles.
2. $\angle 3$ and __?__ are consecutive interior angles.
3. $\angle 3$ and __?__ are alternate interior angles.
4. $\angle 2$ and __?__ are alternate exterior angles.

Find the value of x. *(p. 154)*

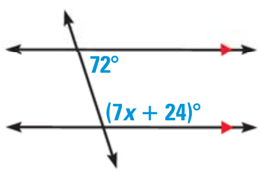
5.



6.



7.



3.3 Prove Lines are Parallel

TEKS G.3.A, G.3.E, G.5.A, G.9.A

Before You used properties of parallel lines to determine angle relationships.

Now You will use angle relationships to prove that lines are parallel.

Why? So you can describe how sports equipment is arranged, as in Ex. 32.

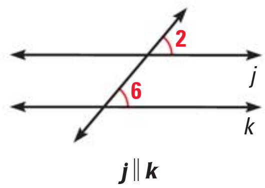
Key Vocabulary
- **paragraph proof**
- **converse,** *p. 80*
- **two-column proof,** *p. 112*

Postulate 16 below is the converse of Postulate 15 in Lesson 3.2. Similarly, the theorems in Lesson 3.2 have true converses. Remember that the converse of a true conditional statement is not necessarily true, so each converse of a theorem must be proved, as in Example 3.

POSTULATE — For Your Notebook

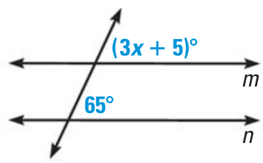
POSTULATE 16 Corresponding Angles Converse

If two lines are cut by a transversal so the corresponding angles are congruent, then the lines are parallel.



EXAMPLE 1 Apply the Corresponding Angles Converse

ALGEBRA Find the value of x that makes $m \parallel n$.



Solution

Lines m and n are parallel if the marked corresponding angles are congruent.

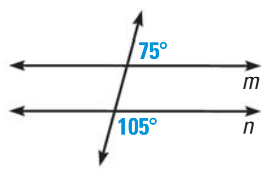
$(3x + 5)° = 65°$ — Use Postulate 16 to write an equation.

$3x = 60$ — Subtract 5 from each side.

$x = 20$ — Divide each side by 3.

▶ The lines m and n are parallel when $x = 20$.

✓ GUIDED PRACTICE for Example 1



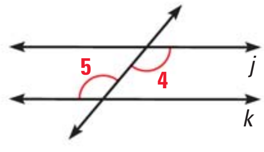
1. Is there enough information in the diagram to conclude that $m \parallel n$? *Explain.*
2. *Explain* why Postulate 16 is the converse of Postulate 15.

THEOREMS — *For Your Notebook*

THEOREM 3.4 Alternate Interior Angles Converse

If two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.

Proof: Example 3, p. 163

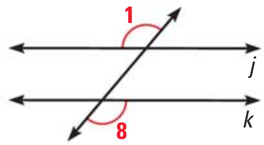


$j \parallel k$

THEOREM 3.5 Alternate Exterior Angles Converse

If two lines are cut by a transversal so the alternate exterior angles are congruent, then the lines are parallel.

Proof: Ex. 36, p. 168

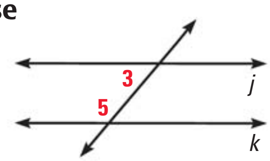


$j \parallel k$

THEOREM 3.6 Consecutive Interior Angles Converse

If two lines are cut by a transversal so the consecutive interior angles are supplementary, then the lines are parallel.

Proof: Ex. 37, p. 168



If $\angle 3$ and $\angle 5$ are supplementary, then $j \parallel k$.

EXAMPLE 2 Solve a real-world problem

SNAKE PATTERNS How can you tell whether the sides of the pattern are parallel in the photo of a diamond-back snake?

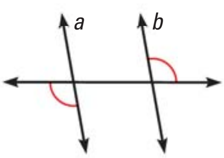
Solution

Because the alternate interior angles are congruent, you know that the sides of the pattern are parallel.

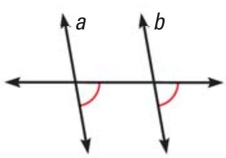
✓ GUIDED PRACTICE for Example 2

Can you prove that lines *a* and *b* are parallel? *Explain* why or why not.

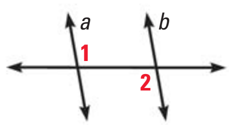
3.



4.



5. $m\angle 1 + m\angle 2 = 180°$

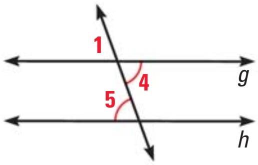


EXAMPLE 3 Prove the Alternate Interior Angles Converse

Prove that if two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.

AVOID ERRORS
Before you write a proof, identify the GIVEN and PROVE statements for the situation described or for any diagram you draw.

Solution



GIVEN ▶ $\angle 4 \cong \angle 5$

PROVE ▶ $g \parallel h$

STATEMENTS	REASONS
1. $\angle 4 \cong \angle 5$	**1.** Given
2. $\angle 1 \cong \angle 4$	**2.** Vertical Angles Congruence Theorem
3. $\angle 1 \cong \angle 5$	**3.** Transitive Property of Congruence
4. $g \parallel h$	**4.** Corresponding Angles Converse

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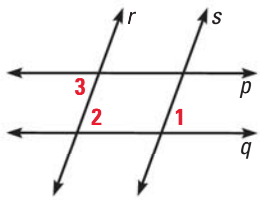
PARAGRAPH PROOFS A proof can also be written in paragraph form, called a **paragraph proof**. The statements and reasons in a paragraph proof are written in sentences, using words to explain the logical flow of the argument.

EXAMPLE 4 Write a paragraph proof

In the figure, $r \parallel s$ and $\angle 1$ is congruent to $\angle 3$. Prove $p \parallel q$.

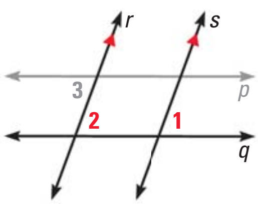


Solution

Look at the diagram to make a plan. The diagram suggests that you look at angles 1, 2, and 3. Also, you may find it helpful to focus on one pair of lines and one transversal at a time.

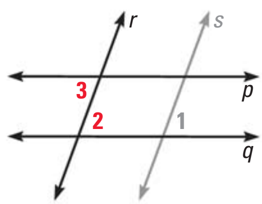
Plan for Proof

a. Look at $\angle 1$ and $\angle 2$.



$\angle 1 \cong \angle 2$ because $r \parallel s$.

b. Look at $\angle 2$ and $\angle 3$.



If $\angle 2 \cong \angle 3$, then $p \parallel q$.

TRANSITIONAL WORDS
In paragraph proofs, **transitional words** such as *so*, *then*, and *therefore* help to make the logic clear.

Plan in Action

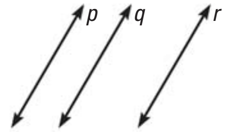
a. It is given that $r \parallel s$, **so** by the Corresponding Angles Postulate, $\angle 1 \cong \angle 2$.

b. It is also given that $\angle 1 \cong \angle 3$. **Then** $\angle 2 \cong \angle 3$ by the Transitive Property of Congruence for angles. **Therefore,** by the Alternate Interior Angles Converse, $p \parallel q$.

THEOREM — *For Your Notebook*

THEOREM 3.7 Transitive Property of Parallel Lines

If two lines are parallel to the same line, then they are parallel to each other.

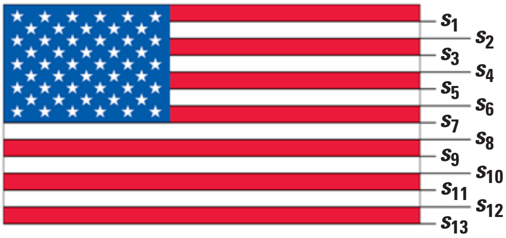


If $p \parallel q$ and $q \parallel r$, then $p \parallel r$.

Proofs: Ex. 38, p. 168; Ex. 38, p. 177

EXAMPLE 5 Use the Transitive Property of Parallel Lines

U.S. FLAG The flag of the United States has 13 alternating red and white stripes. Each stripe is parallel to the stripe immediately below it. Explain why the top stripe is parallel to the bottom stripe.



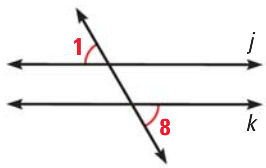
Solution

USE SUBSCRIPTS
When you name several similar items, you can use one variable with subscripts to keep track of the items.

The stripes from top to bottom can be named $s_1, s_2, s_3, \ldots, s_{13}$. Each stripe is parallel to the one below it, so $s_1 \parallel s_2$, $s_2 \parallel s_3$, and so on. Then $s_1 \parallel s_3$ by the Transitive Property of Parallel Lines. Similarly, because $s_3 \parallel s_4$, it follows that $s_1 \parallel s_4$. By continuing this reasoning, $s_1 \parallel s_{13}$. So, the top stripe is parallel to the bottom stripe.

✓ GUIDED PRACTICE for Examples 3, 4, and 5

6. If you use the diagram at the right to prove the Alternate Exterior Angles Converse, what GIVEN and PROVE statements would you use?



7. Copy and complete the following paragraph proof of the Alternate Interior Angles Converse using the diagram in Example 3.

It is given that $\angle 4 \cong \angle 5$. By the __?__, $\angle 1 \cong \angle 4$. Then by the Transitive Property of Congruence, __?__. So, by the __?__, $g \parallel h$.

8. Each step is parallel to the step immediately above it. The bottom step is parallel to the ground. *Explain* why the top step is parallel to the ground.

3.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 29, and 37

★ = **TAKS PRACTICE AND REASONING** Exs. 16, 23, 24, 33, 39, 46, and 47

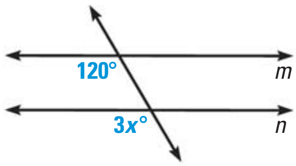
SKILL PRACTICE

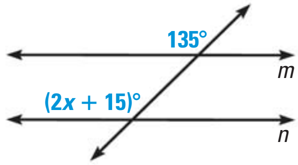
1. **VOCABULARY** Draw a pair of parallel lines with a transversal. Identify all pairs of *alternate exterior angles*.

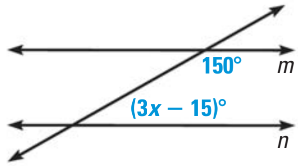
2. **WRITING** Use the theorems from the previous lesson and the converses of those theorems in this lesson. Write three biconditionals about parallel lines and transversals.

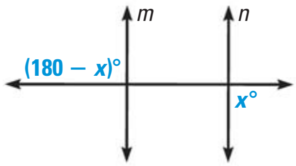
EXAMPLE 1 on p. 161 for Exs. 3–9

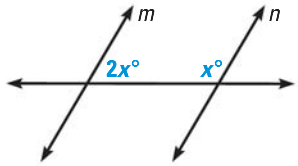
ALGEBRA **Find the value of x that makes $m \parallel n$.**

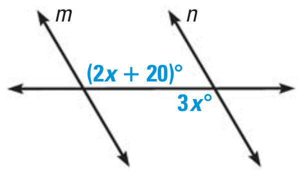
3. 

4. 

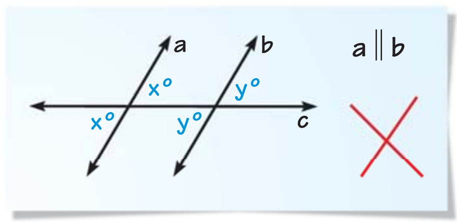
5. 

6. 

7. 

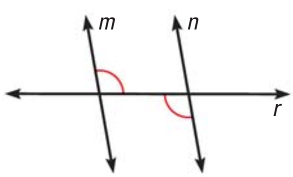
8. 

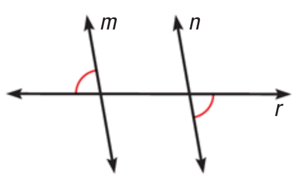
9. **ERROR ANALYSIS** A student concluded that lines a and b are parallel. *Describe* and correct the student's error.

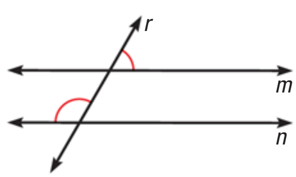


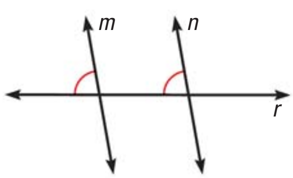
EXAMPLE 2 on p. 162 for Exs. 10–17

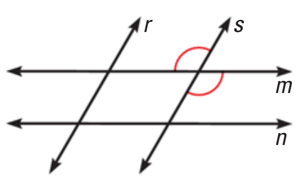
IDENTIFYING PARALLEL LINES **Is there enough information to prove $m \parallel n$? If so, state the postulate or theorem you would use.**

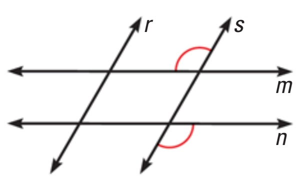
10. 

11. 

12. 

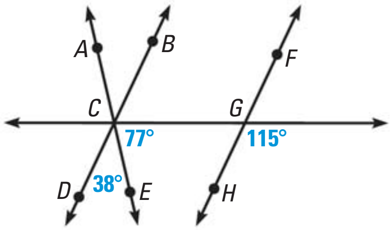
13. 

14. 

15. 

16. ★ **TAKS REASONING** Use lined paper to draw two parallel lines cut by a transversal. Use a protractor to measure one angle. Find the measures of the other seven angles without using the protractor. Give a theorem or postulate you use to find each angle measure.

17. MULTI-STEP PROBLEM Complete the steps below to determine whether $\overleftrightarrow{DB}$ and $\overleftrightarrow{HF}$ are parallel.



a. Find $m\angle DCG$ and $m\angle CGH$.

b. *Describe* the relationship between $\angle DCG$ and $\angle CGH$.

c. Are $\overleftrightarrow{DB}$ and $\overleftrightarrow{HF}$ parallel? *Explain* your reasoning.

EXAMPLE 3
on p. 163
for Ex. 18

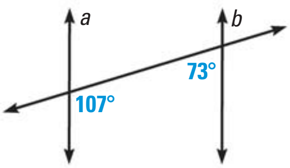
18. PLANNING A PROOF Use these steps to plan a proof of the Consecutive Interior Angles Converse, as stated on page 162.

a. Draw a diagram you can use in a proof of the theorem.

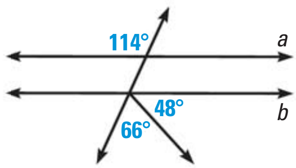
b. Write the GIVEN and PROVE statements.

REASONING **Can you prove that lines *a* and *b* are parallel? If so, *explain* how.**

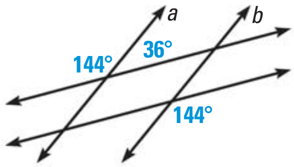
19.



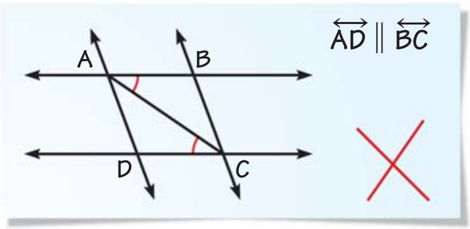
20.



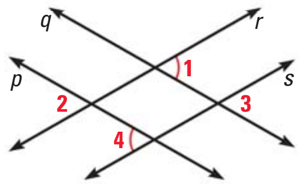
21.



22. ERROR ANALYSIS A student decided that $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$ based on the diagram below. *Describe* and correct the student's error.



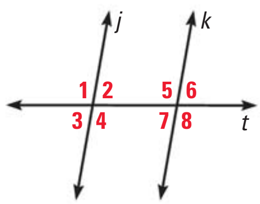
23. TAKS REASONING Use the diagram at the right. You know that $\angle 1 \cong \angle 4$. What can you conclude?



Ⓐ $p \parallel q$ Ⓑ $r \parallel s$

Ⓒ $\angle 2 \cong \angle 3$ Ⓓ None of the above

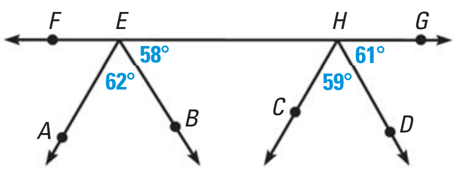
REASONING **Use the diagram at the right for Exercises 24 and 25.**



24. TAKS REASONING In the diagram, assume $j \parallel k$. How many angle measures must be given in order to find the measure of every angle? *Explain* your reasoning.

25. PLANNING A PROOF In the diagram, assume $\angle 1$ and $\angle 7$ are supplementary. Write a plan for a proof showing that lines j and k are parallel.

26. REASONING Use the diagram at the right. Which rays are parallel? Which rays are not parallel? *Justify* your conclusions.



27. **VISUAL REASONING** A point R is not in plane ABC.

a. How many lines through R are perpendicular to plane ABC?

b. How many lines through R are parallel to plane ABC?

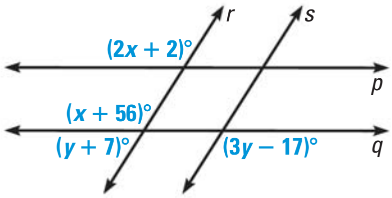
c. How many planes through R are parallel to plane ABC?

28. **CHALLENGE** Use the diagram.

a. Find x so that $p \parallel q$.

b. Find y so that $r \parallel s$.

c. Can r be parallel to s and p be parallel to q at the same time? *Explain.*

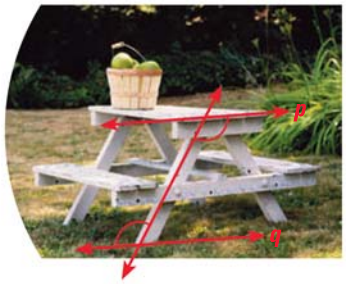


PROBLEM SOLVING

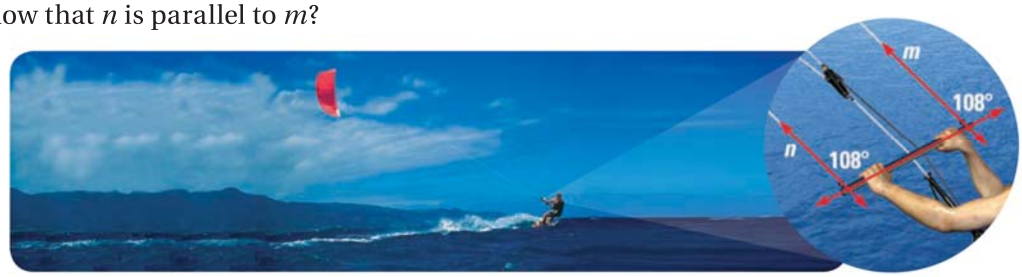
EXAMPLE 2
on p. 162
for Exs. 29–30

29. **PICNIC TABLE** How do you know that the top of the picnic table is parallel to the ground?

TEXAS @HomeTutor for problem solving help at classzone.com



30. **KITEBOARDING** The diagram of the control bar of the kite shows the angles formed between the control bar and the kite lines. How do you know that n is parallel to m?



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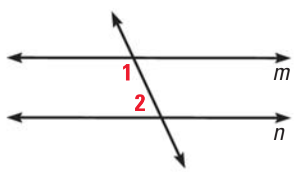
31. **DEVELOPING PROOF** Copy and complete the proof.

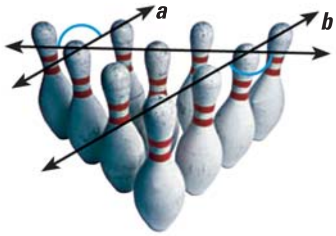
GIVEN ▶ $m\angle 1 = 115°, m\angle 2 = 65°$

PROVE ▶ $m \parallel n$



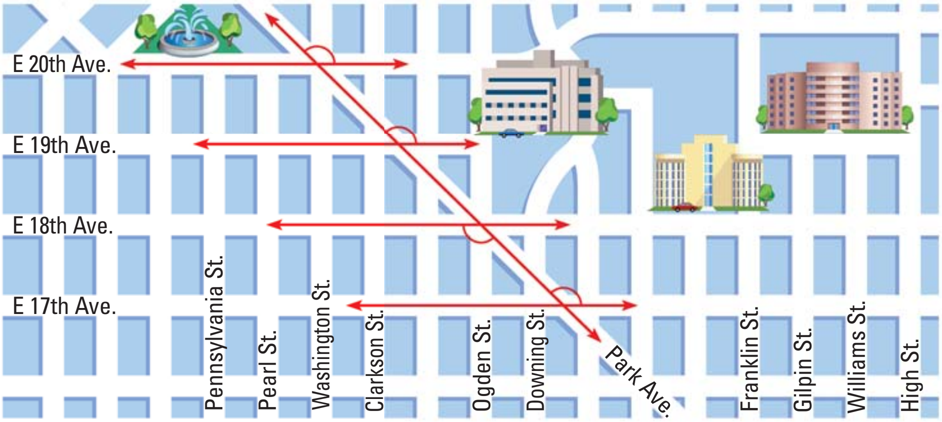
STATEMENTS	REASONS
1. $m\angle 1 = 115°$ and $m\angle 2 = 65°$	1. Given
2. $115° + 65° = 180°$	2. Addition
3. $m\angle 1 + m\angle 2 = 180°$	3. _?_
4. $\angle 1$ and $\angle 2$ are supplementary.	4. _?_
5. $m \parallel n$	5. _?_

32. BOWLING PINS How do you know that the bowling pins are set up in parallel lines?



EXAMPLE 5
on p. 164
for Ex. 33

33. **TAKS REASONING** The map shows part of Denver, Colorado. Use the markings on the map. Are the numbered streets parallel to one another? *Explain* how you can tell.

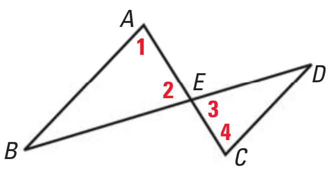


EXAMPLE 3
on p. 163
for Exs. 34–35

PROOF Use the diagram and the given information to write a two-column or paragraph proof.

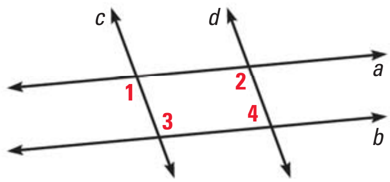
34. **GIVEN** ▶ $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

PROVE ▶ $\overline{AB} \parallel \overline{CD}$



35. **GIVEN** ▶ $a \parallel b$, $\angle 2 \cong \angle 3$

PROVE ▶ $c \parallel d$

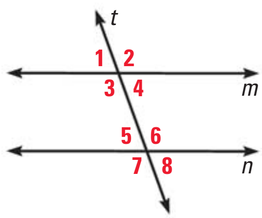


EXAMPLE 4
on p. 163
for Exs. 36–37

PROOF In Exercises 36 and 37, use the diagram to write a paragraph proof.

36. **PROVING THEOREM 3.5** Prove the Alternate Exterior Angles Converse.

37. **PROVING THEOREM 3.6** Prove the Consecutive Interior Angles Converse.



38. **MULTI-STEP PROBLEM** Use these steps to prove Theorem 3.7, the Transitive Property of Parallel Lines.

a. Copy the diagram in the Theorem box on page 164. Draw a transversal through all three lines.

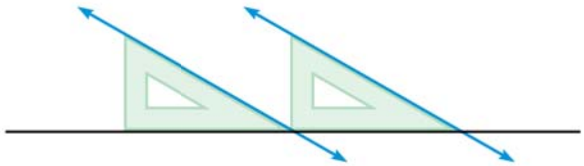
b. Write the GIVEN and PROVE statements.

c. Use the properties of angles formed by parallel lines and transversals to prove the theorem.

= WORKED-OUT SOLUTIONS on p. WS1

= TAKS PRACTICE AND REASONING

39. **TAKS REASONING** Architects and engineers make drawings using a plastic triangle with angle measures 30°, 60°, and 90°. The triangle slides along a fixed horizontal edge.



a. *Explain* why the blue lines shown are parallel.

b. *Explain* how the triangle can be used to draw vertical parallel lines.

REASONING **Use the diagram below in Exercises 40–44. How would you show that the given lines are parallel?**

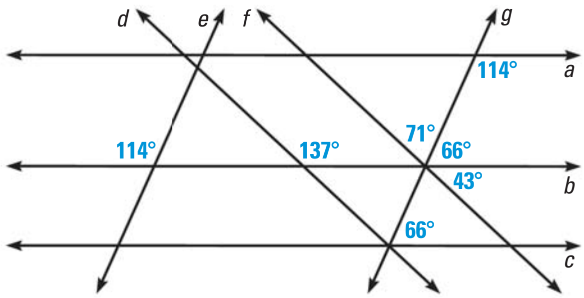
40. a and b

41. b and c

42. d and f

43. e and g

44. a and c



45. **CHALLENGE** Use these steps to investigate the angle bisectors of corresponding angles.

a. Construction Use a compass and straightedge or geometry drawing software to construct line ℓ, point P not on ℓ, and line n through P parallel to ℓ. Construct point Q on ℓ and construct $\overline{PQ}$. Choose a pair of alternate interior angles and construct their angle bisectors.

b. Write a Proof Are the angle bisectors parallel? Make a conjecture. Write a proof of your conjecture.

TAKS PRACTICE at classzone.com

MIXED REVIEW FOR TAKS

REVIEW
Skills Review Handbook p. 894; TAKS Workbook

46. **TAKS PRACTICE** Terry decorates picture frames to sell at a local craft store. She pays \$2.75 for the materials to make each picture frame and sells each frame for \$5. Which equation represents Terry's total earnings, t, when she buys materials for x picture frames and sells y of them? *TAKS Obj. 1*

Ⓐ $t = 2.75x + 5y$　　Ⓑ $t = 2.75y + 5x$

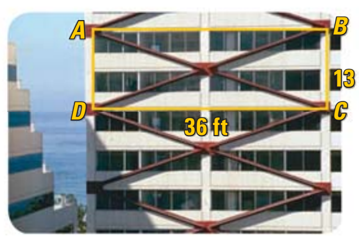
Ⓒ $t = 5y - 2.75x$　　Ⓓ $t = 2.75x - 5y$

REVIEW
Lesson 1.3; TAKS Workbook

47. **TAKS PRACTICE** In the design on the wall of the building, which is closest to the length of $\overline{BD}$? *TAKS Obj. 8*

Ⓕ 29.5 ft　　Ⓖ 38.3 ft

Ⓗ 43.7 ft　　Ⓙ 49.8 ft



MIXED REVIEW FOR TEKS

TAKS PRACTICE
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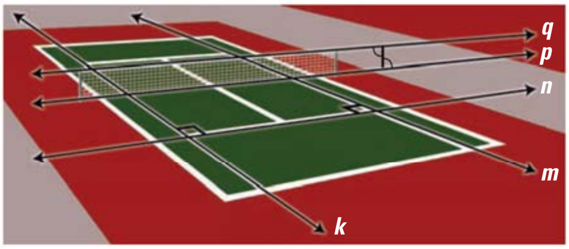
Lessons 3.1–3.3

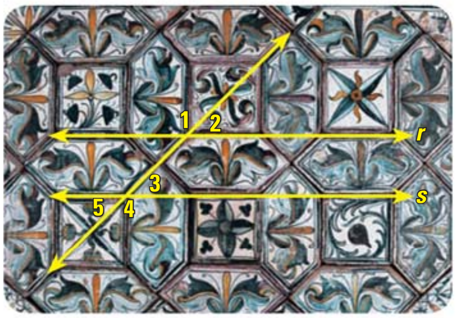
MULTIPLE CHOICE

1. **TENNIS COURT** In the diagram of the tennis court below, which of the pairs of lines are skew? ***TEKS G.9.A***



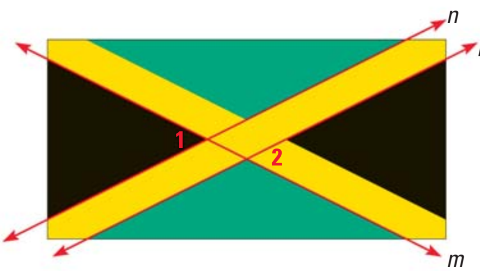
Ⓐ ℓ and m Ⓑ m and q
Ⓒ m and p Ⓓ p and q

2. **TILE FLOOR** In the photo of the tile floor, lines r and s are parallel. Which angle is congruent to $\angle 1$? ***TEKS G.2.B***



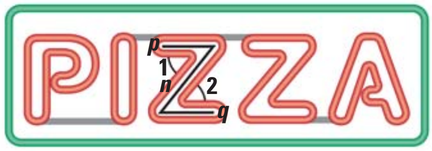
Ⓕ $\angle 2$ Ⓖ $\angle 3$
Ⓗ $\angle 4$ Ⓙ $\angle 5$

3. **JAMAICAN FLAG** In the diagram of the flag of Jamaica, $n \parallel p$ and $m\angle 1 = 53°$. What is the measure of $\angle 2$? ***TEKS G.2.B***



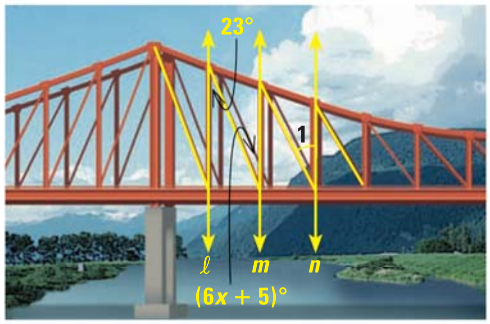
Ⓐ 35° Ⓑ 53°
Ⓒ 70° Ⓓ 145°

4. **NEON SIGN** A neon sign is shown below. How do you know that $p \parallel q$? ***TEKS G.9.A***



Ⓕ $\angle 1$ and $\angle 2$ are complementary.
Ⓖ $\angle 1$ and $\angle 2$ are supplementary.
Ⓗ $\angle 1$ and $\angle 2$ are congruent alternate interior angles.
Ⓙ $\angle 1$ and $\angle 2$ are complementary alternate exterior angles.

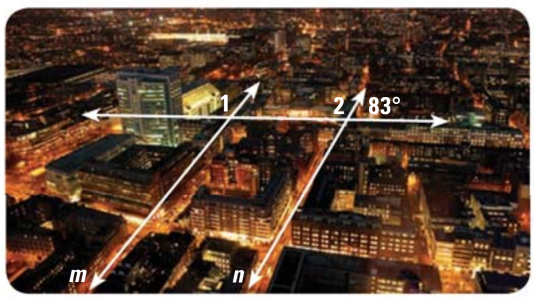
5. **CANTILEVER BRIDGE** In the diagram of the cantilever bridge below, what value of x makes lines ℓ and m parallel? ***TEKS G.9.A***



Ⓐ 3 Ⓑ $\frac{31}{3}$
Ⓒ $\frac{85}{6}$ Ⓓ $\frac{175}{6}$

GRIDDED ANSWER

6. **AERIAL PHOTO** In the aerial view of the city, $m \parallel n$. What is $m\angle 1$ in degrees? ***TEKS G.2.B***



3.4 Find and Use Slopes of Lines

TEKS G.4, G.7.A, G.7.B, G.7.C

Before You used properties of parallel lines to find angle measures.

Now You will find and compare slopes of lines.

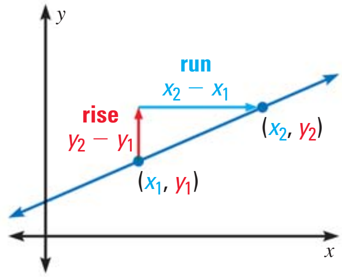
Why So you can compare rates of speed, as in Example 4.

Key Vocabulary
- **slope,** *p. 879*
- **rise,** *p. 879*
- **run,** *p. 879*

The **slope** of a nonvertical line is the ratio of vertical change (*rise*) to horizontal change (*run*) between any two points on the line.

If a line in the coordinate plane passes through points (x_1, y_1) and (x_2, y_2) then the slope m is

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}.$$



KEY CONCEPT — For Your Notebook

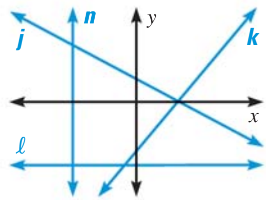
Slope of Lines in the Coordinate Plane

Negative slope: falls from left to right, as in line j

Positive slope: rises from left to right, as in line k

Zero slope (slope of 0): horizontal, as in line ℓ

Undefined slope: vertical, as in line n



EXAMPLE 1 Find slopes of lines in a coordinate plane

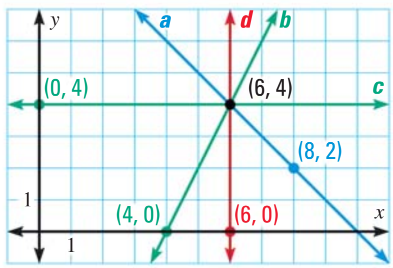
REVIEW SLOPE
For more help with slope, see p. 879.

Find the slope of line *a* and line *d*.

Solution

Slope of line ***a***: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{6 - 8} = \frac{2}{-2} = -1$

Slope of line ***d***: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{6 - 6} = \frac{4}{0}$, which is undefined.



✓ GUIDED PRACTICE for Example 1

Use the graph in Example 1. Find the slope of the line.

1. Line b

2. Line c

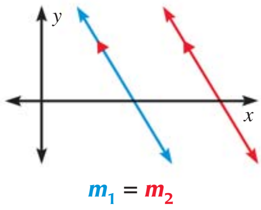
COMPARING SLOPES When two lines intersect in a coordinate plane, the steeper line has the slope with greater absolute value. You can also compare slopes to tell whether two lines are parallel or perpendicular.

POSTULATES — *For Your Notebook*

POSTULATE 17 Slopes of Parallel Lines

In a coordinate plane, two nonvertical lines are parallel if and only if they have the same slope.

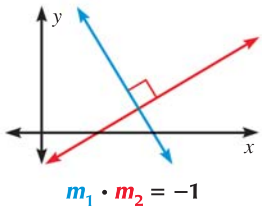
Any two vertical lines are parallel.



POSTULATE 18 Slopes of Perpendicular Lines

In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is −1.

Horizontal lines are perpendicular to vertical lines.

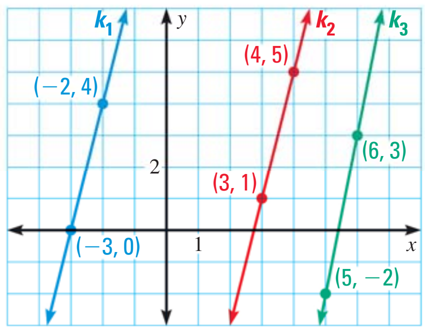


READ VOCABULARY

If the product of two numbers is −1, then the numbers are called *negative reciprocals*.

EXAMPLE 2 Identify parallel lines

Find the slope of each line. Which lines are parallel?



Solution

Find the slope of k_1 through (−2, 4) and (−3, 0).

$$m_1 = \frac{0 - 4}{-3 - (-2)} = \frac{-4}{-1} = 4$$

Find the slope of k_2 through (4, 5) and (3, 1).

$$m_2 = \frac{1 - 5}{3 - 4} = \frac{-4}{-1} = 4$$

Find the slope of k_3 through (6, 3) and (5, −2).

$$m_3 = \frac{-2 - 3}{5 - 6} = \frac{-5}{-1} = 5$$

▶ Compare the slopes. Because k_1 and k_2 have the same slope, they are parallel. The slope of k_3 is different, so k_3 is not parallel to the other lines.

✓ GUIDED PRACTICE for Example 2

3. Line m passes through (−1, 3) and (4, 1). Line t passes through (−2, −1) and (3, −3). Are the two lines parallel? *Explain* how you know.

EXAMPLE 3 Draw a perpendicular line

Line h passes through (3, 0) and (7, 6). Graph the line perpendicular to h that passes through the point (2, 5).

Solution

STEP 1 **Find** the slope m_1 of line h through (3, 0) and (7, 6).

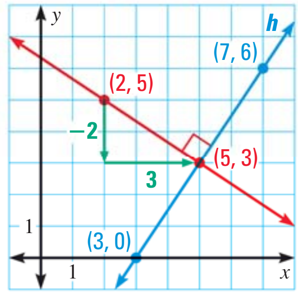
$$m_1 = \frac{6-0}{7-3} = \frac{6}{4} = \frac{3}{2}$$

STEP 2 **Find** the slope m_2 of a line perpendicular to h. Use the fact that the product of the slopes of two perpendicular lines is -1.

$\frac{3}{2} \cdot m_2 = -1$ **Slopes of perpendicular lines**

$m_2 = \frac{-2}{3}$ **Multiply each side by $\frac{2}{3}$.**

STEP 3 **Use** the rise and run to graph the line.

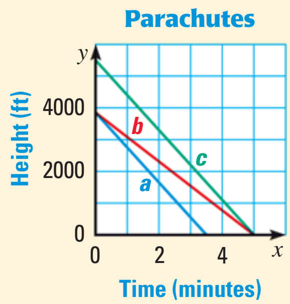


REVIEW GRAPHING

Given a point on a line and the line's slope, you can use the rise and run to find a second point and draw the line.

EXAMPLE 4 TAKS PRACTICE: Multiple Choice

A skydiver made jumps with three parachutes. The graph shows the height of the skydiver from the time the parachute opened to the time of the landing for each jump. Which statement is true?



Ⓐ The parachute was open for the same amount of time in jumps a and b.

Ⓑ The parachute opened at the same height in jumps b and c.

Ⓒ The skydiver descended at the same rate in jumps a and b.

Ⓓ The skydiver descended at the same rate in jumps a and c.

Solution

The rate at which the skydiver descended is represented by the slope of the segments. The segments that have the same slope are a and c.

▶ The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

ELIMINATE CHOICES

The x-intercept represents the amount of time the parachute was open, so the times were not the same in jumps b and c. So, you can eliminate Choice *A*. The y-intercept, which represents the height when the parachute opened, differs in jumps b and c, so you can also eliminate Choice *B*.

✓ GUIDED PRACTICE for Examples 3 and 4

4. Line n passes through (0, 2) and (6, 5). Line m passes through (2, 4) and (4, 0). Is $n \perp m$? *Explain.*

5. In Example 4, which parachute is in the air for the longest time? *Explain.*

6. In Example 4, what do the x-intercepts represent in the situation? How can you use this to eliminate one of the choices?

EXAMPLE 5 Solve a real-world problem

ROLLER COASTERS During the climb on the Magnum XL-200 roller coaster, you move 41 feet upward for every 80 feet you move horizontally. At the crest of the hill, you have moved 400 feet forward.

a. **Making a Table** Make a table showing the height of the Magnum at every 80 feet it moves horizontally. How high is the roller coaster at the top of its climb?

b. **Calculating** Write a fraction that represents the height the Magnum climbs for each foot it moves horizontally. What does the numerator represent?

c. **Using a Graph** Another roller coaster, the Millenium Force, climbs at a slope of 1. At its crest, the horizontal distance from the starting point is 310 feet. Compare this climb to that of the Magnum. Which climb is steeper?

Solution

a.

Horizontal distance (ft)	80	160	240	320	400
Height (ft)	41	82	123	164	205

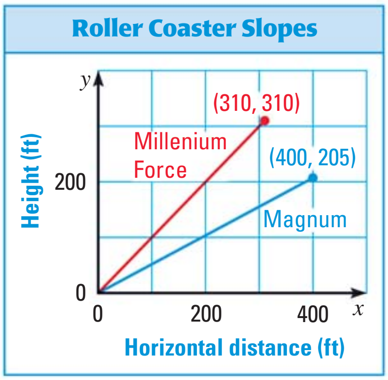
The Magnum XL-200 is 205 feet high at the top of its climb.

b. Slope of the Magnum $= \frac{\text{rise}}{\text{run}} = \frac{41}{80} = \frac{41 \div 80}{80 \div 80} = \frac{0.5125}{1}$

The numerator, 0.5125, represents the slope in decimal form.

c. Use a graph to compare the climbs. Let x be the horizontal distance and let y be the height. Because the slope of the Millenium Force is 1, the rise is equal to the run. So the highest point must be at (310, 310).

▶ The graph shows that the Millenium Force has a steeper climb, because the slope of its line is greater ($1 > 0.5125$).



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✓ GUIDED PRACTICE for Example 5

7. Line q passes through the points (0, 0) and (−4, 5). Line t passes through the points (0, 0) and (−10, 7). Which line is steeper, q or t?

8. **WHAT IF?** Suppose a roller coaster climbed 300 feet upward for every 350 feet it moved horizontally. Is it *more steep* or *less steep* than the Magnum? than the Millenium Force?

3.4 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 13, and 35

★ = **TAKS PRACTICE AND REASONING** Exs. 34, 35, 41, and 43

◆ = **MULTIPLE REPRESENTATIONS** Exs. 37

SKILL PRACTICE

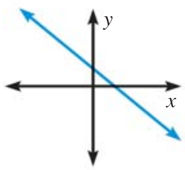
1. **VOCABULARY** *Describe* what is meant by the slope of a nonvertical line.

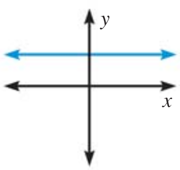
2. **WRITING** What happens when you apply the slope formula to a horizontal line? What happens when you apply it to a vertical line?

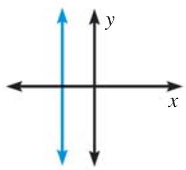
EXAMPLE 1 on p. 171 for Exs. 3–12

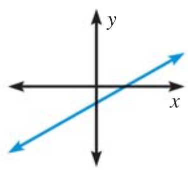
MATCHING **Match the description of the slope of a line with its graph.**

3. m is positive. 4. m is negative. 5. m is zero. 6. m is undefined.

A. 

B. 

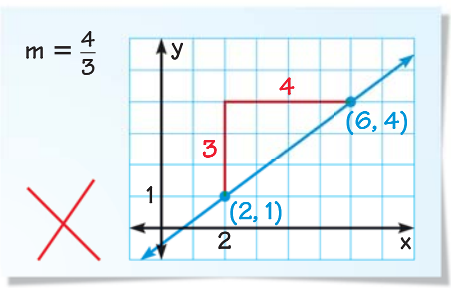
C. 

D. 

FINDING SLOPE **Find the slope of the line that passes through the points.**

7. (3, 5), (5, 6) 8. (−2, 2), (2, −6) 9. (−5, −1), (3, −1) 10. (2, 1), (0, 6)

ERROR ANALYSIS ***Describe*** **and correct the error in finding the slope of the line.**

11. 

12. Slope of the line through (2, 7) and (4, 5)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 5}{4 - 2} = \frac{2}{2} = 1$$

EXAMPLES 2 and 3 on pp. 172–173 for Exs. 13–18

TYPES OF LINES **Tell whether the lines through the given points are** ***parallel***, ***perpendicular***, **or** ***neither***. ***Justify*** **your answer.**

13. **Line 1:** (1, 0), (7, 4)
Line 2: (7, 0), (3, 6)

14. **Line 1:** (−3, 1), (−7, −2)
Line 2: (2, −1), (8, 4)

15. **Line 1:** (−9, 3), (−5, 7)
Line 2: (−11, 6), (−7, 2)

GRAPHING **Graph the line through the given point with the given slope.**

16. $P(3, -2)$, slope $-\frac{1}{6}$ 17. $P(-4, 0)$, slope $\frac{5}{2}$ 18. $P(0, 5)$, slope $\frac{2}{3}$

EXAMPLES 4 and 5 on pp. 173–174 for Exs. 19–22

STEEPNESS OF A LINE **Tell which line through the given points is steeper.**

19. **Line 1:** (−2, 3), (3, 5)
Line 2: (3, 1), (6, 5)

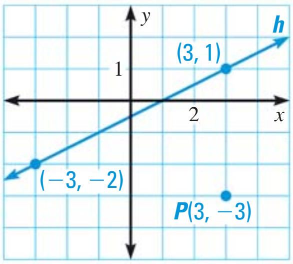
20. **Line 1:** (−2, −1), (1, −2)
Line 2: (−5, −3), (−1, −4)

21. **Line 1:** (−4, 2), (−3, 6)
Line 2: (1, 6), (3, 8)

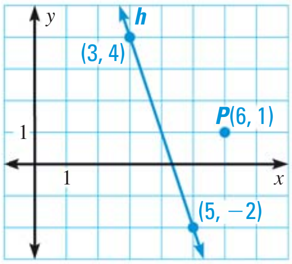
22. **REASONING** Use your results from Exercises 19–21. *Describe* a way to determine which of two lines is steeper without graphing them.

PERPENDICULAR LINES **Find the slope of line *n* perpendicular to line *h* and passing through point *P*. Then copy the graph and graph line *n*.**

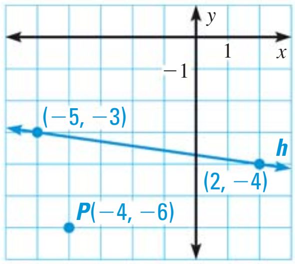
23.



24.



25.



26. **REASONING** Use the concept of slope to decide whether the points $(-3, 3)$, $(1, -2)$, and $(4, 0)$ lie on the same line. *Explain* your reasoning and include a diagram.

GRAPHING **Graph a line with the given description.**

27. Through $(0, 2)$ and parallel to the line through $(-2, 4)$ and $(-5, 1)$

28. Through $(1, 3)$ and perpendicular to the line through $(-1, -1)$ and $(2, 0)$

29. Through $(-2, 1)$ and parallel to the line through $(3, 1)$ and $\left(4, -\frac{1}{2}\right)$

CHALLENGE **Find the unknown coordinate so the line through the points has the given slope.**

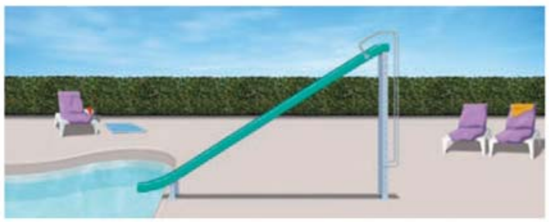
30. $(-3, 2)$, $(0, y)$; slope -2

31. $(-7, -4)$, $(x, 0)$; slope $\frac{1}{3}$

32. $(4, -3)$, $(x, 1)$; slope -4

PROBLEM SOLVING

33. **WATER SLIDE** The water slide is 6 feet tall, and the end of the slide is 9 feet from the base of the ladder. About what slope does the slide have?



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EXAMPLE 5
on p. 174
for Exs. 34–37

34. **TAKS REASONING** Which car has better gas mileage?

Ⓐ A Ⓑ B

Ⓒ Same rate Ⓓ Cannot be determined

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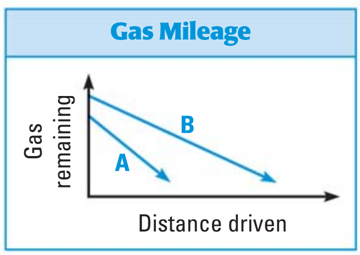
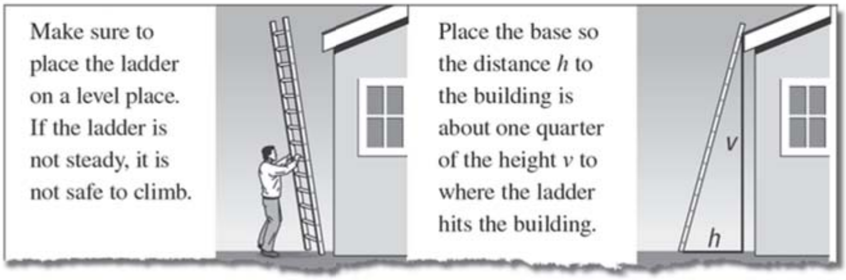


35. **TAKS REASONING** *Compare* the graphs of the three lines described below. Which is most steep? Which is the least steep? Include a sketch in your answer.

Line *a*: through the point $(3, 0)$ with a y-intercept of 4
Line *b*: through the point $(3, 0)$ with a y-intercept greater than 4
Line *c*: through the point $(3, 0)$ with a y-intercept between 0 and 4

36. **MULTI-STEP PROBLEM** Ladder safety guidelines include the following recommendation about ladder placement. The horizontal distance h between the base of the ladder and the object the ladder is resting against should be about one quarter of the vertical distance v between the ground and where the ladder rests against the object.



a. Find the recommended slope for a ladder.

b. Suppose the base of a ladder is 6 feet away from a building. The ladder has the recommended slope. Find v.

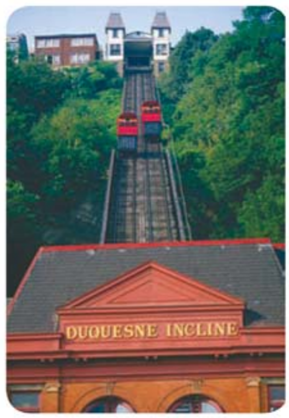
c. Suppose a ladder is 34 feet from the ground where it touches a building. The ladder has the recommended slope. Find h.

37. **MULTIPLE REPRESENTATIONS** The Duquesne (pronounced "du-KAYN") Incline was built in 1888 in Pittsburgh, Pennsylvania, to move people up and down a mountain there. On the incline, you move about 29 feet vertically for every 50 feet you move horizontally. When you reach the top of the hill, you have moved a horizontal distance of about 700 feet.

a. **Making a Table** Make a table showing the vertical distance that the incline moves for each 50 feet of horizontal distance during its climb. How high is the incline at the top?

b. **Drawing a Graph** Write a fraction that represents the slope of the incline's climb path. Draw a graph to show the climb path.

c. **Comparing Slopes** The Burgenstock Incline in Switzerland moves about 144 vertical feet for every 271 horizontal feet. Write a fraction to represent the slope of this incline's path. Which incline is steeper, the *Burgenstock* or the *Duquesne*?



38. **PROVING THEOREM 3.7** Use slopes of lines to write a paragraph proof of the Transitive Property of Parallel Lines on page 164.

AVERAGE RATE OF CHANGE In Exercises 39 and 40, slope can be used to describe an *average rate of change*. To write an average rate of change, rewrite the slope fraction so the denominator is one.

39. **BUSINESS** In 2000, a business made a profit of \$8500. In 2006, the business made a profit of \$15,400. Find the average rate of change in dollars per year from 2000 to 2006.

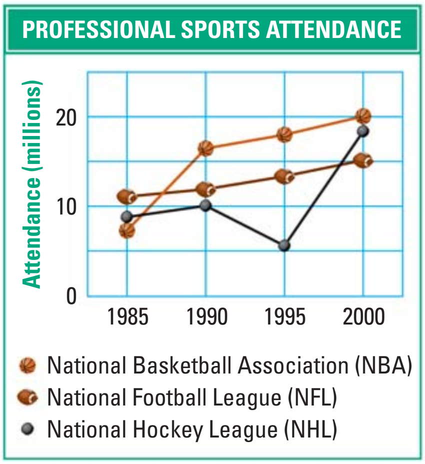
40. **ROCK CLIMBING** A rock climber begins climbing at a point 400 feet above sea level. It takes the climber 45 minutes to climb to the destination, which is 706 feet above sea level. Find the average rate of change in feet per minute for the climber from start to finish.

41. **TAKS REASONING** The line graph shows the regular season attendance (in millions) for three professional sports organizations from 1985 to 2000.

a. During which five-year period did the NBA attendance increase the most? Estimate the rate of change for this five-year period in people per year.

b. During which five-year period did the NHL attendance increase the most? Estimate the rate of change for this five-year period in people per year.

c. **Interpret** The line graph for the NFL seems to be almost linear between 1985 and 2000. Write a sentence about what this means in terms of the real-world situation.



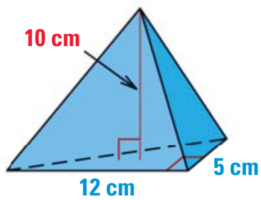
42. **CHALLENGE** Find two values of k such that the points $(-3, 1)$, $(0, k)$, and $(k, 5)$ are collinear. *Explain* your reasoning.

MIXED REVIEW FOR TAKS

TAKS PRACTICE at classzone.com

REVIEW
TAKS Preparation p. 66;
TAKS Workbook

43. **TAKS PRACTICE** What is the volume of the triangular pyramid?
TAKS Obj. 8

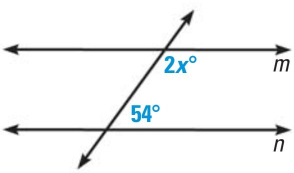


Ⓐ 100 cm^3 Ⓑ 200 cm^3 Ⓒ 260 cm^3 Ⓓ 300 cm^3

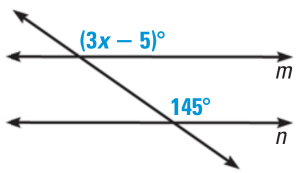
QUIZ for Lessons 3.3–3.4

Find the value of x that makes $m \parallel n$. *(p. 161)*

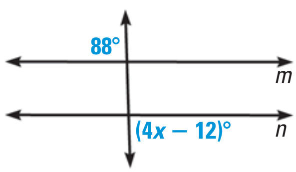
1.



2.



3.



Find the slope of the line that passes through the given points. *(p. 171)*

4. $(1, -1), (3, 3)$ **5.** $(1, 2), (4, 5)$ **6.** $(-3, -2), (-7, -6)$

Technology **ACTIVITY** *Use after Lesson 3.4*

TEXAS @HomeTutor
classzone.com
Keystrokes

3.4 Investigate Slopes

TEKS *a.5, G.2.B, G.7.C, G.9.A*

MATERIALS • graphing calculator or computer

QUESTION **How can you verify the Slopes of Parallel Lines Postulate?**

You can verify the postulates you learned in Lesson 3.4 using geometry drawing software.

EXAMPLE **Verify the Slopes of Parallel Lines Postulate**

STEP 1 ***Show axes*** Show the x-axis and the y-axis by choosing Hide/Show Axes from the F5 menu.

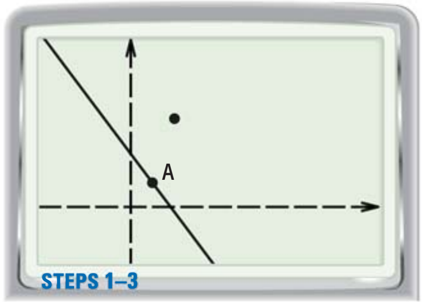
STEP 2 ***Draw line*** Draw a line by choosing Line from the F2 menu. Do not use one of the axes as your line. Choose a point on the line and label it A.

STEP 3 ***Graph point*** Graph a point not on the line by choosing Point from the F2 menu.

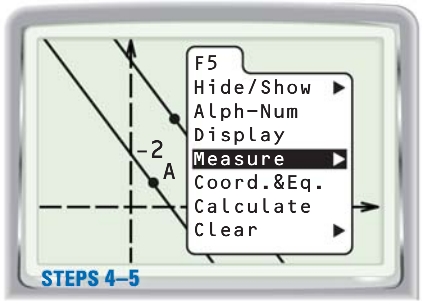
STEP 4 ***Draw parallel line*** Choose Parallel from the F3 menu and select the line. Then select the point not on the line.

STEP 5 ***Measure slopes*** Select one line and choose Measure Slope from the F5 menu. Repeat this step for the second line.

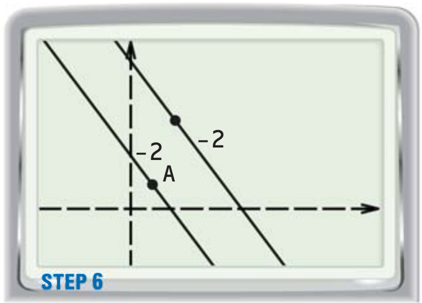
STEP 6 ***Move line*** Drag point A to move the line. What do you expect to happen?



STEPS 1–3



STEPS 4–5



STEP 6

PRACTICE

1. Use geometry drawing software to verify the Slopes of Perpendicular Lines Postulate.
   a. Construct a line and a point not on that line. Use Steps 1–3 from the Example above.
   b. Construct a line that is perpendicular to your original line and passes through the given point.
   c. Measure the slopes of the two lines. Multiply the slopes. What do you expect the product of the slopes to be?

2. **WRITING** Use the arrow keys to move your line from Exercise 1. *Describe* what happens to the product of the slopes when one of the lines is vertical. *Explain* why this happens.

3.5 Write and Graph Equations of Lines

TEKS G.7.A, G.7.B, G.7.C, G.9.A

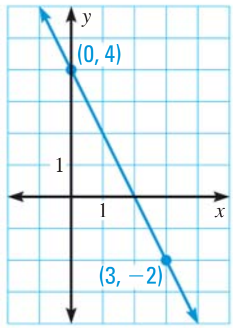
Before You found slopes of lines.
Now You will find equations of lines.
Why? So you can find monthly gym costs, as in Example 4.

Key Vocabulary
- **slope-intercept form**
- **standard form**
- ***x*-intercept,** *p. 879*
- ***y*-intercept,** *p. 879*

Linear equations may be written in different forms. The general form of a linear equation in **slope-intercept form** is $y = mx + b$, where m is the slope and b is the y-intercept.

EXAMPLE 1 Write an equation of a line from a graph

Write an equation of the line in slope-intercept form.



Solution

STEP 1 **Find** the slope. Choose two points on the graph of the line, (0, 4) and (3, −2).

$$m = \frac{4 - (-2)}{0 - 3} = \frac{6}{-3} = -2$$

STEP 2 **Find** the y-intercept. The line intersects the y-axis at the point (0, 4), so the y-intercept is 4.

STEP 3 **Write** the equation.

$y = mx + b$ **Use slope-intercept form.**

$y = -2x + 4$ **Substitute −2 for *m* and 4 for *b*.**

EXAMPLE 2 Write an equation of a parallel line

Write an equation of the line passing through the point (−1, 1) that is parallel to the line with the equation $y = 2x - 3$.

Solution

STEP 1 **Find** the slope m. The slope of a line parallel to $y = 2x - 3$ is the same as the given line, so the slope is 2.

STEP 2 **Find** the y-intercept b by using $m = 2$ and $(x, y) = (-1, 1)$.

$y = mx + b$ **Use slope-intercept form.**

$1 = 2(-1) + b$ **Substitute for *x*, *y*, and *m*.**

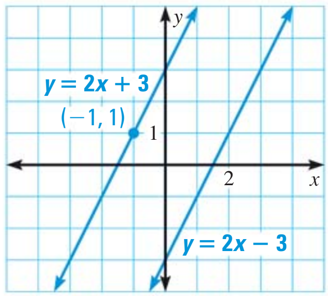
$3 = b$ **Solve for *b*.**

▶ Because $m = 2$ and $b = 3$, an equation of the line is $y = 2x + 3$.

LINEAR EQUATIONS
The graph of a linear equation represents all the solutions of the equation. So, the given point must be a solution of the equation.

CHECKING BY GRAPHING You can check that equations are correct by graphing. In Example 2, you can use a graph to check that $y = 2x - 3$ is parallel to $y = 2x + 3$.

AnimatedGeometry at classzone.com



EXAMPLE 3 Write an equation of a perpendicular line

Write an equation of the line *j* passing through the point (2, 3) that is perpendicular to the line *k* with the equation $y = -2x + 2$.

Solution

STEP 1 **Find** the slope m of line j. Line k has a slope of -2.

$-2 \cdot m = -1$ The product of the slopes of $\perp$ lines is -1.

$m = \frac{1}{2}$ Divide each side by -2.

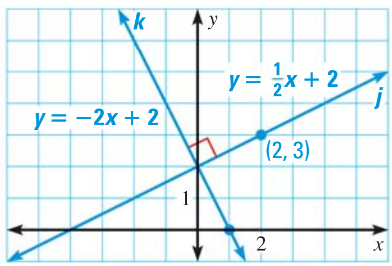
STEP 2 **Find** the y-intercept b by using $m = \frac{1}{2}$ and $(x, y) = (2, 3)$.

$y = mx + b$ Use slope-intercept form.

$3 = \frac{1}{2}(2) + b$ Substitute for x, y, and m.

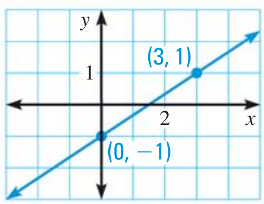
$2 = b$ Solve for b.

▶ Because $m = \frac{1}{2}$ and $b = 2$, an equation of line j is $y = \frac{1}{2}x + 2$. You can check that the lines j and k are perpendicular by graphing, then using a protractor to measure one of the angles formed by the lines.



✓ GUIDED PRACTICE for Examples 1, 2, and 3

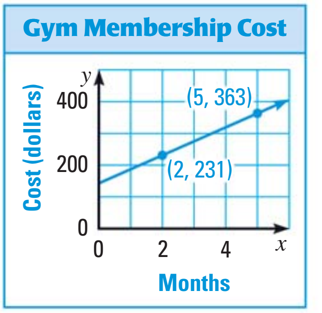
1. Write an equation of the line in the graph at the right.



2. Write an equation of the line that passes through $(-2, 5)$ and $(1, 2)$.
3. Write an equation of the line that passes through the point $(1, 5)$ and is parallel to the line with the equation $y = 3x - 5$. Graph the lines to check that they are parallel.
4. How do you know the lines $x = 4$ and $y = 2$ are perpendicular?

EXAMPLE 4 Write an equation of a line from a graph

GYM MEMBERSHIP The graph models the total cost of joining a gym. Write an equation of the line. Explain the meaning of the slope and the y-intercept of the line.



Solution

STEP 1 **Find** the slope.

$$m = \frac{363 - 231}{5 - 2} = \frac{132}{3} = 44$$

STEP 2 **Find** the y-intercept. Use the slope and one of the points on the graph.

$y = mx + b$ — Use slope-intercept form.

$231 = 44 \cdot 2 + b$ — Substitute for x, y, and m.

$143 = b$ — Simplify.

STEP 3 **Write** the equation. Because $m = 44$ and $b = 143$, an equation of the line is $y = 44x + 143$.

▶ The equation $y = 44x + 143$ models the cost. The slope is the monthly fee, \$44, and the y-intercept is the initial cost to join the gym, \$143.

STANDARD FORM Another form of a linear equation is *standard form*. In **standard form**, the equation is written as $Ax + By = C$, where A and B are not both zero.

EXAMPLE 5 Graph a line with equation in standard form

Graph $3x + 4y = 12$.

Solution

CHOOSE A METHOD
Another way you could graph the equation is to solve the equation for y. Then the equation will be in slope-intercept form. Use rise and run from the point where the line crosses the y-axis to find a second point. Then graph the line.

The equation is in standard form, so you can use the intercepts.

STEP 1 **Find** the intercepts.

To find the x-intercept, let $y = 0$.

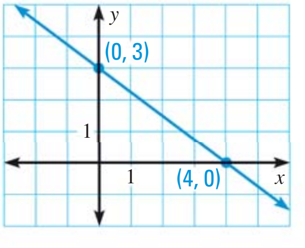
$$3x + 4y = 12$$
$$3x + 4(0) = 12$$
$$x = 4$$

To find the y-intercept, let $x = 0$.

$$3x + 4y = 12$$
$$3(0) + 4y = 12$$
$$y = 3$$

STEP 2 **Graph** the line.

The intercepts are (4, 0) and (0, 3). Graph these points, then draw a line through the points.



✓ **GUIDED PRACTICE** for Examples 4 and 5

5. The equation $y = 50x + 125$ models the total cost of joining a climbing gym. What are the meaning of the slope and the y-intercept of the line?

Graph the equation.

6. $2x - 3y = 6$
7. $y = 4$
8. $x = -3$

WRITING EQUATIONS You can write linear equations to model real-world situations, such as comparing costs to find a better buy.

EXAMPLE 6 Solve a real-world problem

DVD RENTAL You can rent DVDs at a local store for \$4.00 each. An Internet company offers a flat fee of \$15.00 per month for as many rentals as you want. How many DVDs do you need to rent to make the online rental a better buy?

ANOTHER WAY
For alternative methods for solving the problem in Example 6, turn to page 188 for the **Problem Solving Workshop**.

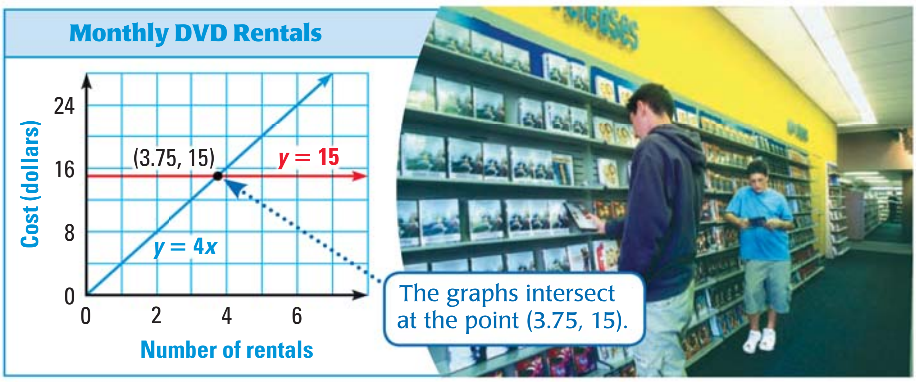
Solution

STEP 1 **Model** each rental with an equation.

Cost of one month's rental online: $y = 15$

Cost of one month's rental locally: $y = 4x$, where x represents the number of DVDs rented

STEP 2 **Graph** each equation.



READ VOCABULARY
The point at which the costs are the same is sometimes called the *break-even point*.

▶ The point of intersection is (3.75, 15). Using the graph, you can see that it is cheaper to rent locally if you rent 3 or fewer DVDs per month. If you rent 4 or more DVDs per month, it is cheaper to rent online.

✓ **GUIDED PRACTICE** for Example 6

9. **WHAT IF?** In Example 6, suppose the online rental is \$16.50 per month and the local rental is \$4 each. How many DVDs do you need to rent to make the online rental a better buy?

10. How would your answer to Exercise 9 change if you had a 2-for-1 coupon that you could use once at the local store?

3.5 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 17, 23, and 61
★ = **TAKS PRACTICE AND REASONING** Exs. 9, 29, 64, 65, 67, and 68

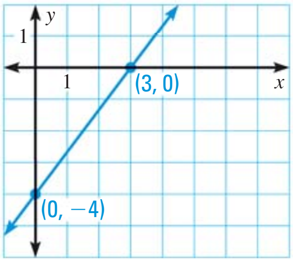
SKILL PRACTICE

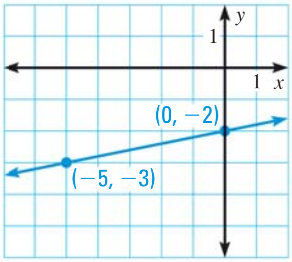
1. **VOCABULARY** What does *intercept* mean in the expression *slope-intercept form*?

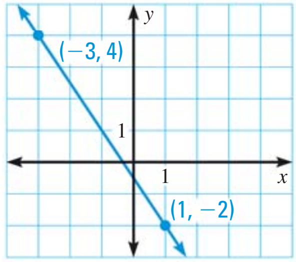
2. **WRITING** Explain how you can use the standard form of a linear equation to find the intercepts of a line.

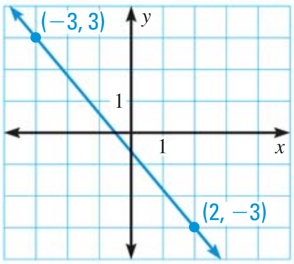
EXAMPLE 1 on p. 180 for Exs. 3–22

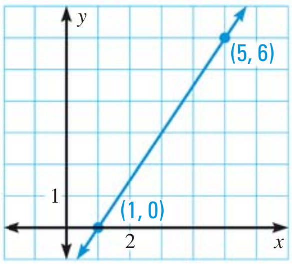
WRITING EQUATIONS Write an equation of the line shown.

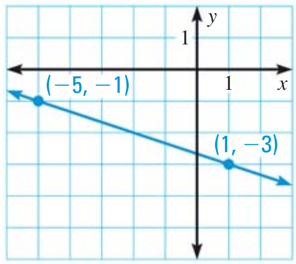
3. 

4. 

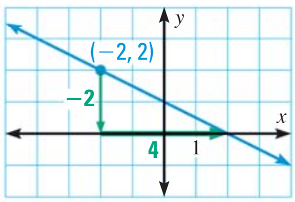
5. 

6. 

7. 

8. 

★ **TAKS REASONING** Which equation is an equation of the line in the graph?



Ⓐ $y = -\frac{1}{2}x$ Ⓑ $y = -\frac{1}{2}x + 1$

Ⓒ $y = -2x$ Ⓓ $y = -2x + 1$

WRITING EQUATIONS **Write an equation of the line with the given slope *m* and *y*-intercept *b*.**

10. $m = -5, b = -12$
11. $m = 3, b = 2$
12. $m = 4, b = -6$
13. $m = -\frac{5}{2}, b = 0$
14. $m = \frac{4}{9}, b = -\frac{2}{9}$
15. $m = -\frac{11}{5}, b = -12$

WRITING EQUATIONS **Write an equation of the line that passes through the given point *P* and has the given slope *m*.**

16. $P(-1, 0), m = -1$
17. $P(5, 4), m = 4$
18. $P(6, -2), m = 3$
19. $P(-8, -2), m = -\frac{2}{3}$
20. $P(0, -3), m = -\frac{1}{6}$
21. $P(-13, 7), m = 0$

22. **WRITING EQUATIONS** Write an equation of a line with undefined slope that passes through the point $(3, -2)$.

EXAMPLE 2
on p. 180
for Exs. 23–29

PARALLEL LINES **Write an equation of the line that passes through point P and is parallel to the line with the given equation.**

23. $P(0, -1), y = -2x + 3$ **24.** $P(-7, -4), y = 16$ **25.** $P(3, 8), y - 1 = \frac{1}{5}(x + 4)$

26. $P(-2, 6), x = -5$ **27.** $P(-2, 1), 10x + 4y = -8$ **28.** $P(4, 0), -x + 2y = 12$

29. **TAKS REASONING** Line a passes through points $(-2, 1)$ and $(2, 9)$. Which equation is an equation of a line parallel to line a?

Ⓐ $y = -2x + 5$ Ⓑ $y = -\frac{1}{2}x + 5$ Ⓒ $y = \frac{1}{2}x - 5$ Ⓓ $y = 2x - 5$

EXAMPLE 3
on p. 181
for Exs. 30–35

PERPENDICULAR LINES **Write an equation of the line that passes through point P and is perpendicular to the line with the given equation.**

30. $P(0, 0), y = -9x - 1$ **31.** $P(-1, 1), y = \frac{7}{3}x + 10$ **32.** $P(4, -6), y = -3$

33. $P(2, 3), y - 4 = -2(x + 3)$ **34.** $P(0, -5), x = 20$ **35.** $P(-8, 0), 3x - 5y = 6$

EXAMPLE 5
on p. 182
for Exs. 36–45

GRAPHING EQUATIONS **Graph the equation.**

36. $8x + 2y = -10$ **37.** $x + y = 1$ **38.** $4x - y = -8$

39. $-x + 3y = -9$ **40.** $y - 2 = -1$ **41.** $y + 2 = x - 1$

42. $x + 3 = -4$ **43.** $2y - 4 = -x + 1$ **44.** $3(x - 2) = -y - 4$

45. **ERROR ANALYSIS** *Describe* and correct the error in finding the x- and y-intercepts of the graph of $5x - 3y = -15$.

To find the x-intercept, let x = 0:	To find the y-intercept, let y = 0:
$5x - 3y = -15$	$5x - 3y = -15$
$5(0) - 3y = -15$	$5x - 3(0) = -15$
$y = 5$ ✗	$x = -3$ ✗

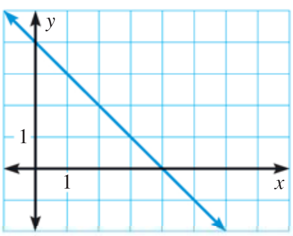
IDENTIFYING PARALLEL LINES **Which lines are parallel, if any?**

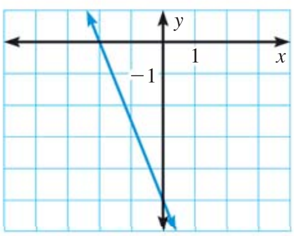
46. $y = 3x - 4$
$x + 3y = 6$
$3(x + 1) = y - 2$

47. $x + 2y = 9$
$y = 0.5x + 7$
$-x + 2y = -5$

48. $x - 6y = 10$
$6x - y = 11$
$x + 6y = 12$

USING INTERCEPTS **Identify the x- and y-intercepts of the line. Use the intercepts to write an equation of the line.**

49.


50.


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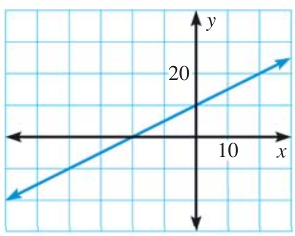


52. **INTERCEPTS** A line passes through the points $(-10, -3)$ and $(6, 1)$. Where does the line intersect the x-axis? Where does the line intersect the y-axis?

SOLUTIONS TO EQUATIONS **Graph the linear equations. Then use the graph to estimate how many solutions the equations share.**

53. $y = 4x + 9$
$4x - y = 1$

54. $3y + 4x = 16$
$2x - y = 18$

55. $y = -5x + 6$
$10x + 2y = 12$

56. **ALGEBRA** Solve Exercises 53–55 algebraically. (For help, see Skills Review Handbook, p. 880.) Make a conjecture about how the solution(s) can tell you whether the lines intersect, are parallel, or are the same line.

57. **ALGEBRA** Find a value for k so that the line through $(-1, k)$ and $(-7, -2)$ is parallel to the line with equation $y = x + 1$.

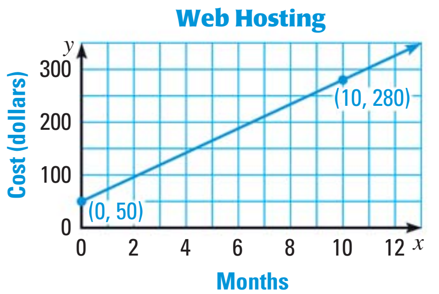
58. **ALGEBRA** Find a value for k so that the line through $(k, 2)$ and $(7, 0)$ is perpendicular to the line with equation $y = x - \frac{28}{5}$.

59. **CHALLENGE** Graph the points $R(-7, -3)$, $S(-2, 3)$, and $T(10, -7)$. Connect them to make $\triangle RST$. Write an equation of the line containing each side. *Explain* how you can use slopes to show that $\triangle RST$ has one right angle.

PROBLEM SOLVING

EXAMPLE 4 on p. 182 for Exs. 60–61

60. **WEB HOSTING** The graph models the total cost of using a web hosting service for several months. Write an equation of the line. Tell what the slope and y-intercept mean in this situation. Then find the total cost of using the web hosting service for one year.



61. **SCIENCE** Scientists believe that a Tyrannosaurus Rex weighed about 2000 kilograms by age 14. It then had a growth spurt for four years, gaining 2.1 kilograms per day. Write an equation to model this situation. What are the slope and y-intercept? Tell what the slope and y-intercept mean in this situation.

Field Museum, Chicago, Illinois

EXAMPLE 6 on p. 183 for Exs. 62–65

62. **MULTI-STEP PROBLEM** A national park has two options: a \$50 pass for all admissions during the year, or a \$4 entrance fee each time you enter.

a. Model Write an equation to model the cost of going to the park for a year using a pass and another equation for paying a fee each time.

b. Graph Graph both equations you wrote in part (a).

c. Interpret How many visits do you need to make for the pass to be cheaper? *Explain*.

= WORKED-OUT SOLUTIONS on p. WS1

= TAKS PRACTICE AND REASONING

63. PIZZA COSTS You are buying slices of pizza for you and your friends. A small slice costs $2 and a large slice costs $3. You have $24 to spend. Write an equation in standard form $Ax + By = C$ that models this situation. What do the values of A, B, and C mean in this situation?

64. TAKS REASONING You run at a rate of 4 miles per hour and your friend runs at a rate of 3.5 miles per hour. Your friend starts running 10 minutes before you, and you run for a half hour on the same path. Will you catch up to your friend? Use a graph to support your answer.

65. TAKS REASONING Audrey and Sara are making jewelry. Audrey buys 2 bags of beads and 1 package of clasps for a total of $13. Sara buys 5 bags of beads and 2 packages of clasps for a total of $27.50.

a. Let b be the price of one bag of beads and let c be the price of one package of clasps. Write equations to represent the total cost for Audrey and the total cost for Sara.

b. Graph the equations from part (a).

c. *Explain* the meaning of the intersection of the two lines in terms of the real-world situation.

66. CHALLENGE Michael is deciding which gym membership to buy. Points (2, 112) and (4, 174) give the cost of gym membership at one gym after two and four months. Points (1, 62) and (3, 102) give the cost of gym membership at a second gym after one and three months. Write equations to model the cost of each gym membership. At what point do the graphs intersect, if they intersect? Which gym is cheaper? *Explain*.

MIXED REVIEW FOR TAKS

TAKS **PRACTICE** at classzone.com

REVIEW
Skills Review Handbook p. 888; TAKS Workbook

67. TAKS PRACTICE The table shows the number of cellular phone subscribers and the number of home phone lines in the U.S. from 1998–2002. Which statement is true? *TAKS Obj. 9*

Year	Cellular Phone Subscribers	Home Phone Lines
1998	69,209,000	110,000,000
1999	86,047,000	115,000,000
2000	109,478,000	115,000,000
2001	128,375,000	112,000,000
2002	140,766,000	103,000,000

Ⓐ As the number of cellular phone subscribers increases, the number of home phone lines increases.

Ⓑ As the number of cellular phone subscribers increases, the number of home phone lines decreases.

Ⓒ The increase in the number of cellular phone subcribers from 1998–2002 is linear.

Ⓓ No relationship can be determined from the data in the table.

REVIEW
Skills Review Handbook p. 872; TAKS Workbook

68. TAKS PRACTICE Which expression is equivalent to $-2(3x - x^2) + 4(x + 8)$? *TAKS Obj. 2*

Ⓕ $-4x^2 + 4x + 32$ Ⓖ $-2x^2 - 2x + 32$

Ⓗ $-x^2 - 2x + 8$ Ⓙ $2x^2 - 2x + 32$

PROBLEM SOLVING WORKSHOP
LESSON 3.5

TEKS a.1, a.4, a.6, G.4

Using ALTERNATIVE METHODS

Another Way to Solve Example 6, page 183

MULTIPLE REPRESENTATIONS In Example 6 on page 183, you saw how to graph equations to solve a problem about renting DVDs. Another way you can solve the problem is *using a table*. Alternatively, you can use the equations to solve the problem *algebraically*.

PROBLEM

DVD RENTAL You can rent DVDs at a local store for $4.00 each. An Internet company offers a flat fee of $15.00 per month for as many rentals as you want. How many DVDs do you need to rent to make the online rental a better buy?

METHOD 1

Using a Table You can make a table to answer the question.

STEP 1 **Make** a table representing each rental option.

DVDs rented	Renting locally	Renting online
1	$4	$15
2	$8	$15

STEP 2 **Add** rows to your table until you see a pattern.

DVDs rented	Renting locally	Renting online
1	$4	$15
2	$8	$15
3	$12	$15
4	$16	$15
5	$20	$15
6	$24	$15

STEP 3 **Analyze** the table. Notice that the values in the second column (the cost of renting locally) are less than the values in the third column (the cost of renting online) for three or fewer DVDs. However, the values in the second column are greater than those in the third column for four or more DVDs.

▶ It is cheaper to rent locally if you rent 3 or fewer DVDs per month. If you rent 4 or more DVDs per month, it is cheaper to rent online.

METHOD 2

Using Algebra You can solve one of the equations for one of its variables. Then substitute that expression for the variable in the other equation.

STEP 1 **Write** an equation for each rental option.

Cost of one month's rental online: $y = 15$

Cost of one month's rental locally: $y = 4x$, where x represents the number of DVDs rented

STEP 2 **Substitute** the value of y from one equation into the other equation.

$y = 4x$

$15 = 4x$ — **Substitute 15 for *y*.**

$3.75 = x$ — **Divide each side by 4.**

STEP 3 **Analyze** the solution of the equation. If you could rent 3.75 DVDs, your cost for local and online rentals would be the same. However, you can only rent a whole number of DVDs. Look at what happens when you rent 3 DVDs and when you rent 4 DVDs, the whole numbers just less than and just greater than 3.75.

▶ It is cheaper to rent locally if you rent 3 or fewer DVDs per month. If you rent 4 or more DVDs per month, it is cheaper to rent online.

PRACTICE

1. **IN-LINE SKATES** You can rent in-line skates for \$5 per hour, or buy a pair of skates for \$130. How many hours do you need to skate for the cost of buying skates to be cheaper than renting them?

2. **WHAT IF?** Suppose the in-line skates in Exercise 1 also rent for \$12 per day. How many days do you need to skate for the cost of buying skates to be cheaper than renting them?

3. **BUTTONS** You buy a button machine for \$200 and supplies to make one hundred fifty buttons for \$30. Suppose you charge \$2 for a button. How many buttons do you need to sell to earn back what you spent?

4. **MANUFACTURING** A company buys a new widget machine for \$1200. It costs \$5 to make each widget. The company sells each widget for \$15. How many widgets do they need to sell to earn back the money they spent on the machine?

5. **WRITING** Which method(s) did you use to solve Exercises 1–4? *Explain* your choice(s).

6. **MONEY** You saved \$1000. If you put this money in a savings account, it will earn 1.5% annual interest. If you put the \$1000 in a certificate of deposit (CD), it will earn 3% annual interest. To earn the most money, does it ever make sense to put your money in the savings account? *Explain*.

3.6 Prove Theorems About Perpendicular Lines

TEKS G.2.A, G.2.B, G.3.E, G.9.A

Before You found the distance between points in the coordinate plane.

Now You will find the distance between a point and a line.

Why? So you can determine lengths in art, as in Example 4.

Key Vocabulary
- distance from a point to a line

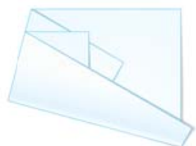
ACTIVITY FOLD PERPENDICULAR LINES

Materials: paper, protractor

STEP 1

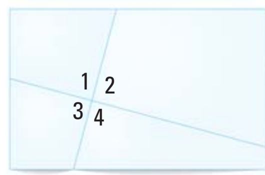
Fold a piece of paper.

STEP 2



Fold the paper again, so that the original fold lines up on itself.

STEP 3



Unfold the paper.

DRAW CONCLUSIONS

1. What type of angles appear to be formed where the fold lines intersect?
2. Measure the angles with a protractor. Which angles are congruent? Which angles are right angles?

The activity above suggests several properties of perpendicular lines.

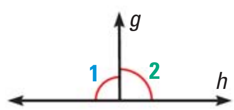
THEOREMS *For Your Notebook*

THEOREM 3.8

If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.

If $\angle 1 \cong \angle 2$, then $g \perp h$.

Proof: Ex. 31, p. 196

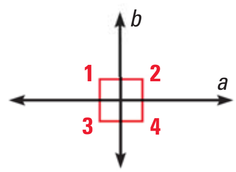


THEOREM 3.9

If two lines are perpendicular, then they intersect to form four right angles.

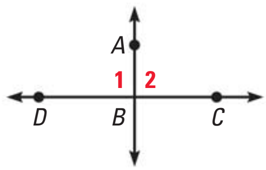
If $a \perp b$, then $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$ are right angles.

Proof: Ex. 32, p. 196



EXAMPLE 1 Draw conclusions

In the diagram at the right, $\overleftrightarrow{AB} \perp \overleftrightarrow{BC}$. What can you conclude about $\angle 1$ and $\angle 2$?



Solution

$\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$ are perpendicular, so by Theorem 3.9, they form four right angles. You can conclude that $\angle 1$ and $\angle 2$ are right angles, so $\angle 1 \cong \angle 2$.

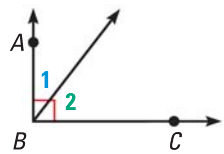
THEOREM *For Your Notebook*

THEOREM 3.10

If two sides of two adjacent acute angles are perpendicular, then the angles are complementary.

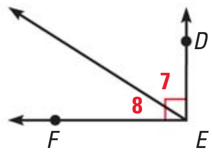
If $\overleftrightarrow{BA} \perp \overleftrightarrow{BC}$, then $\angle 1$ and $\angle 2$ are complementary.

Proof: Example 2, below



EXAMPLE 2 Prove Theorem 3.10

Prove that if two sides of two adjacent acute angles are perpendicular, then the angles are complementary.



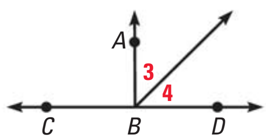
GIVEN ▶ $\overrightarrow{ED} \perp \overrightarrow{EF}$

PROVE ▶ $\angle 7$ and $\angle 8$ are complementary.

STATEMENTS	REASONS
1. $\overrightarrow{ED} \perp \overrightarrow{EF}$	**1.** Given
2. $\angle DEF$ is a right angle.	**2.** $\perp$ lines intersect to form 4 rt. $\angle$s. (Theorem 3.9)
3. $m\angle DEF = 90°$	**3.** Definition of a right angle
4. $m\angle 7 + m\angle 8 = m\angle DEF$	**4.** Angle Addition Postulate
5. $m\angle 7 + m\angle 8 = 90°$	**5.** Substitution Property of Equality
6. $\angle 7$ and $\angle 8$ are complementary.	**6.** Definition of complementary angles

✓ GUIDED PRACTICE for Examples 1 and 2

1. Given that $\angle ABC \cong \angle ABD$, what can you conclude about $\angle 3$ and $\angle 4$? *Explain* how you know.



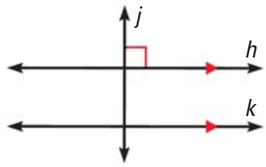
2. Write a plan for proof for Theorem 3.9, that if two lines are perpendicular, then they intersect to form four right angles.

THEOREMS — *For Your Notebook*

THEOREM 3.11 Perpendicular Transversal Theorem

If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other.

If $h \parallel k$ and $j \perp h$, then $j \perp k$.

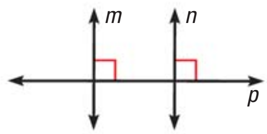


Proof: Ex. 42, p. 160; Ex. 33, p. 196

THEOREM 3.12 Lines Perpendicular to a Transversal Theorem

In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

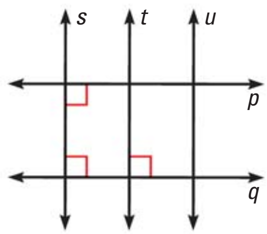
If $m \perp p$ and $n \perp p$, then $m \parallel n$.



Proof: Ex. 34, p. 196

EXAMPLE 3 Draw conclusions

Determine which lines, if any, must be parallel in the diagram. Explain your reasoning.

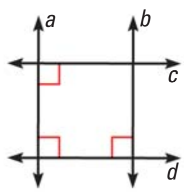


Solution

Lines p and q are both perpendicular to s, so by Theorem 3.12, $p \parallel q$. Also, lines s and t are both perpendicular to q, so by Theorem 3.12, $s \parallel t$.

✓ GUIDED PRACTICE for Example 3

Use the diagram at the right.

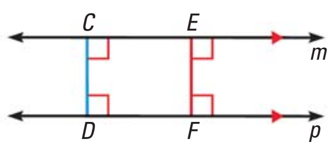


3. Is $b \parallel a$? *Explain* your reasoning.

4. Is $b \perp c$? *Explain* your reasoning.

DISTANCE FROM A LINE The **distance from a point to a line** is the length of the perpendicular segment from the point to the line. This perpendicular segment is the shortest distance between the point and the line. For example, the distance between point A and line k is AB. You will prove this in Chapter 5.

Distance from a point to a line

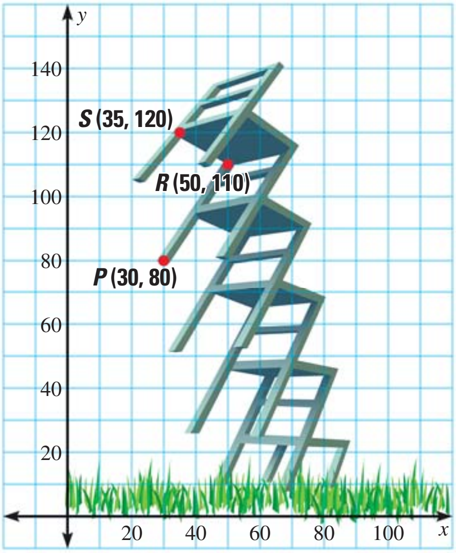


Distance between two parallel lines

The *distance between two parallel lines* is the length of any perpendicular segment joining the two lines. For example, the distance between line p and line m above is CD or EF.

EXAMPLE 4 Find the distance between two parallel lines

SCULPTURE The sculpture below is drawn on a graph where units are measured in inches. What is the approximate length of $\overline{SR}$, the depth of a seat?



Solution

You need to find the length of a perpendicular segment from a back leg to a front leg on one side of the chair.

Using the points $P(30, 80)$ and $R(50, 110)$, the slope of each leg is

$$\frac{110 - 80}{50 - 30} = \frac{30}{20} = \frac{3}{2}.$$

The segment SR has a slope of

$$\frac{120 - 110}{35 - 50} = -\frac{10}{15} = -\frac{2}{3}.$$

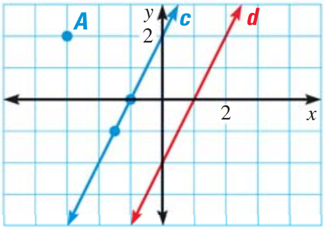
The segment $\overline{SR}$ is perpendicular to the leg so the distance SR is

$$d = \sqrt{(35 - 50)^2 + (120 - 110)^2} \approx 18.0 \text{ inches.}$$

▶ The length of $\overline{SR}$ is about 18.0 inches.

✓ GUIDED PRACTICE for Example 4

Use the graph at the right for Exercises 5 and 6.



5. What is the distance from point A to line c?

6. What is the distance from line c to line d?

7. Graph the line $y = x + 1$. What point on the line is the shortest distance from the point $(4, 1)$? What is the distance? Round to the nearest tenth.

3.6 EXERCISES

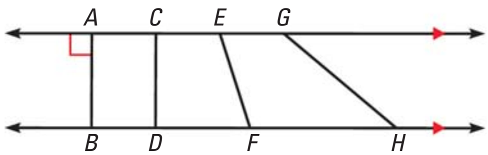
HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 19, 23, and 29

★ = **TAKS PRACTICE AND REASONING** Exs. 11, 12, 21, 30, 39, 40, and 41

SKILL PRACTICE

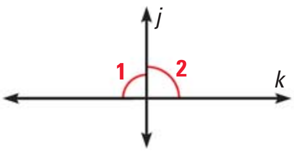
1. **VOCABULARY** The length of which segment shown is called the distance between the two parallel lines? *Explain*.



EXAMPLES 1 and 2 on p. 191 for Exs. 2–7

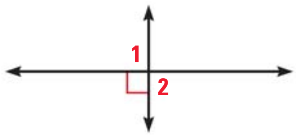
JUSTIFYING STATEMENTS **Write the theorem that justifies the statement.**

2. $j \perp k$



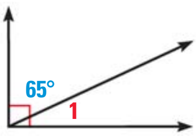
3. $\angle 4$ and $\angle 5$ are complementary.

4. $\angle 1$ and $\angle 2$ are right angles.

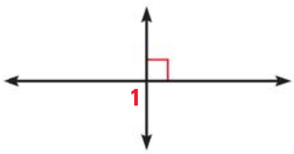


APPLYING THEOREMS **Find $m\angle 1$.**

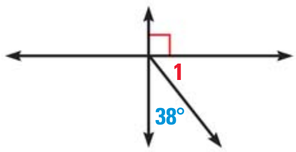
5.



6.



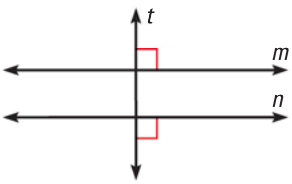
7.



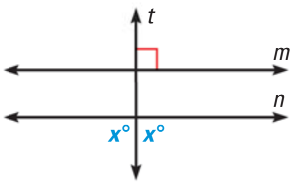
EXAMPLE 3 on p. 192 for Exs. 8–12

SHOWING LINES PARALLEL ***Explain*** **how you would show that $m \parallel n$.**

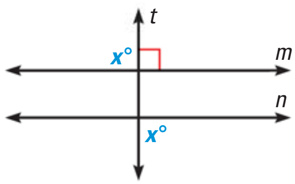
8.



9.



10.



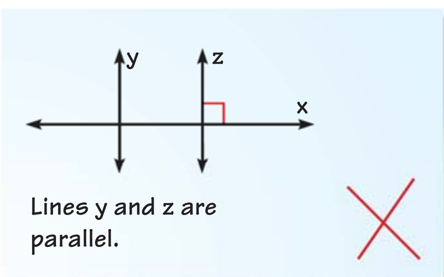
11. ★ **TAKS REASONING** *Explain* how to draw two parallel lines using only a straightedge and a protractor.

12. ★ **TAKS REASONING** *Describe* how you can fold a sheet of paper to create two parallel lines that are perpendicular to the same line.

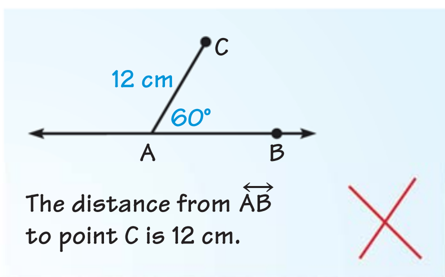
EXAMPLES 3 and 4 on pp. 192–193 for Exs. 13–14

ERROR ANALYSIS ***Explain*** **why the statement about the figure is incorrect.**

13.

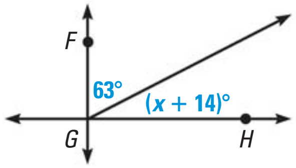


14.

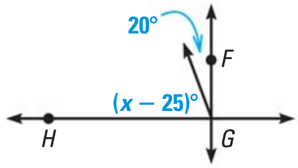


FINDING ANGLE MEASURES **In the diagram, $\overleftrightarrow{FG} \perp \overleftrightarrow{GH}$. Find the value of *x*.**

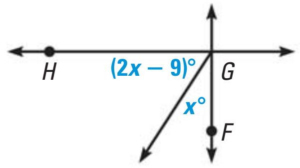
15.



16.

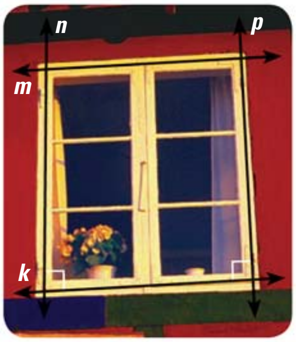


17.



DRAWING CONCLUSIONS **Determine which lines, if any, must be parallel. *Explain* your reasoning.**

18.



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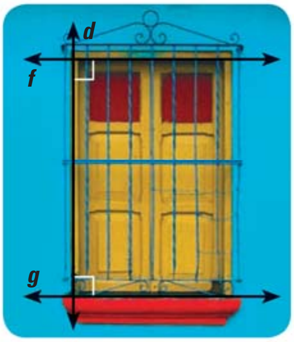
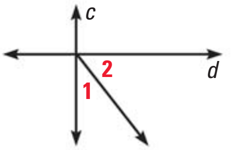


20.

21. **TAKS REASONING** Which statement must be true if $c \perp d$?

Ⓐ $m\angle 1 + m\angle 2 = 90°$ Ⓑ $m\angle 1 + m\angle 2 < 90°$

Ⓒ $m\angle 1 + m\angle 2 > 90°$ Ⓓ Cannot be determined

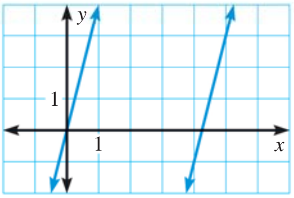


22. **WRITING** *Explain* why the distance between two lines is only defined for parallel lines.

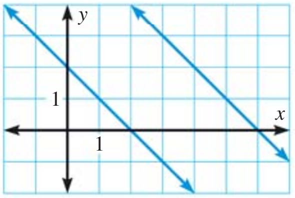
EXAMPLE 4
on p. 193
for Exs. 23–24

FINDING DISTANCES **Use the Distance Formula to find the distance between the two parallel lines. Round to the nearest tenth, if necessary.**

23.

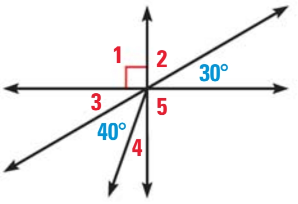


24.



25. **CONSTRUCTION** You are given a line *n* and a point *P* not on *n*. Use a compass to find two points on *n* equidistant from *P*. Then use the steps for the construction of a segment bisector (page 33) to construct a line perpendicular to *n* through *P*.

26. **FINDING ANGLES** Find all the unknown angle measures in the diagram at the right. *Justify* your reasoning for each angle measure.

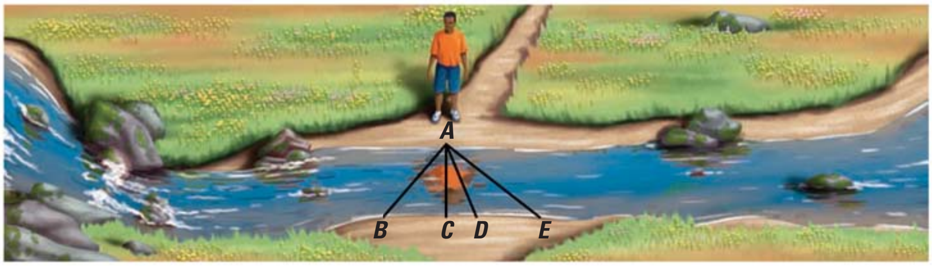


27. **FINDING DISTANCES** Find the distance between the lines with the equations $y = \frac{3}{2}x + 4$ and $-3x + 2y = -1$.

28. **CHALLENGE** *Describe* how you would find the distance from a point to a plane. Can you find the distance from a line to a plane? *Explain*.

PROBLEM SOLVING

29. **STREAMS** You are trying to cross a stream from point A. Which point should you jump to in order to jump the shortest distance? *Explain*.



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30. **TAKS REASONING** The segments that form the path of a crosswalk are usually perpendicular to the crosswalk. Sketch what the segments would look like if they were perpendicular to the crosswalk. Which method requires less paint? *Explain*.

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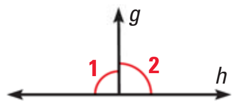
EXAMPLE 2
on p. 191
for Exs. 31–34

31. **PROVING THEOREM 3.8** Copy and complete the proof that if two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.

GIVEN ▶ $\angle 1$ and $\angle 2$ are a linear pair.
$\angle 1 \cong \angle 2$

PROVE ▶ $g \perp h$



STATEMENTS	REASONS
1. $\angle 1$ and $\angle 2$ are a linear pair.	1. Given
2. $\angle 1$ and $\angle 2$ are supplementary.	2. ?
3. ?	3. Definition of supplementary angles
4. $\angle 1 \cong \angle 2$	4. Given
5. $m\angle 1 = m\angle 2$	5. ?
6. $m\angle 1 + m\angle 1 = 180°$	6. Substitution Property of Equality
7. $2(m\angle 1) = 180°$	7. Combine like terms.
8. $m\angle 1 = 90°$	8. ?
9. ?	9. Definition of a right angle
10. $g \perp h$	10. ?

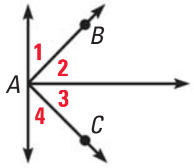
PROVING THEOREMS **Write a proof of the given theorem.**

32. Theorem 3.9

33. Theorem 3.11, Perpendicular Transversal Theorem

34. Theorem 3.12, Lines Perpendicular to a Transversal Theorem

CHALLENGE **Suppose the given statement is true. Determine whether $\overrightarrow{AB} \perp \overrightarrow{AC}$.**



35. $\angle 1$ and $\angle 2$ are congruent.

36. $\angle 3$ and $\angle 4$ are complementary.

37. $m\angle 1 = m\angle 3$ and $m\angle 2 = m\angle 4$

38. $m\angle 1 = 40°$ and $m\angle 4 = 50°$

MIXED REVIEW FOR TAKS

TAKS PRACTICE at classzone.com

REVIEW
Skills Review Handbook p. 881;
TAKS Workbook

39. **TAKS PRACTICE** A packing company ships bunches of bananas in cartons. An empty carton weighs 2.8 pounds. Each bunch of bananas weighs at least 1.5 pounds. Which inequality represents the weight, w, (in pounds) of a carton of bananas in terms of b, the number of bunches of bananas in a carton? *TAKS Obj. 1*

Ⓐ $w \geq 1.5b + 2.8$ Ⓑ $w \geq 2.8b + 1.5$

Ⓒ $w \leq 1.5b + 2.8$ Ⓓ $w \leq 2.8b + 1.5$

REVIEW
Lesson 3.5;
TAKS Workbook

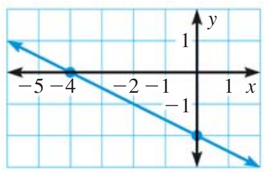
40. **TAKS PRACTICE** What are the x- and y-intercepts, respectively, of the graph? *TAKS Obj. 3*

Ⓕ (0, −4) and (−2, 0)

Ⓖ (−4, 0) and (0, −2)

Ⓗ (−2, 0) and (−4, 0)

Ⓙ (0, −2) and (0, −4)



REVIEW
TAKS Preparation p. 66;
TAKS Workbook

41. **TAKS PRACTICE** A sphere has a radius r. Which ratio compares the volume of the sphere to the surface area of the sphere? *TAKS Obj. 10*

Ⓐ $\frac{r}{3}$ Ⓑ $\frac{3}{r}$ Ⓒ $\frac{\pi r}{3}$ Ⓓ $\frac{3r}{1}$

QUIZ *for Lessons 3.5–3.6*

Write an equation of the line that passes through point P and is parallel to the line with the given equation. *(p. 180)*

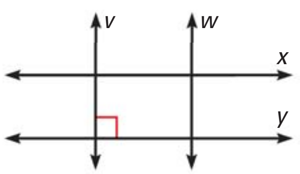
1. $P(0, 0)$, $y = -3x + 1$ **2.** $P(-5, -6)$, $y - 8 = 2x + 10$ **3.** $P(1, -2)$, $x = 15$

Write an equation of the line that passes through point P and is perpendicular to the line with the given equation. *(p. 180)*

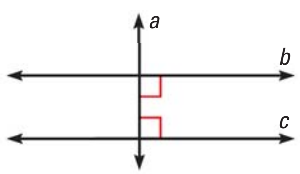
4. $P(3, 4)$, $y = 2x - 1$ **5.** $P(2, 5)$, $y = -6$ **6.** $P(4, 0)$, $12x + 3y = 9$

Determine which lines, if any, must be parallel. *Explain.* *(p. 190)*

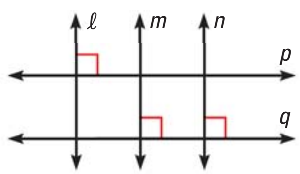
7.



8.



9.



Extension

Use after Lesson 3.6

Taxicab Geometry

TEKS a.4, G.1.C, G.7.A, G.7.C

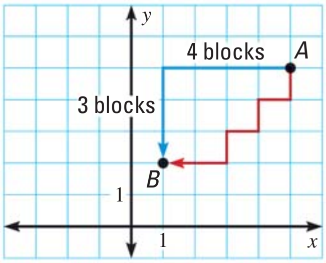
GOAL Find distances in a non-Euclidean geometry.

Key Vocabulary
- taxicab geometry

You have learned that the shortest distance between two points is the length of the straight line segment between them. This is true in the *Euclidean* geometry that you are studying. But think about what happens when you are in a city and want to get from point A to point B. You cannot walk through the buildings, so you have to go along the streets.

Taxicab geometry is the non-Euclidean geometry that a taxicab or a pedestrian must obey.

In taxicab geometry, you can travel either horizontally or vertically parallel to the axes. In this geometry, the distance between two points is the shortest number of *blocks* between them.



HISTORY NOTE

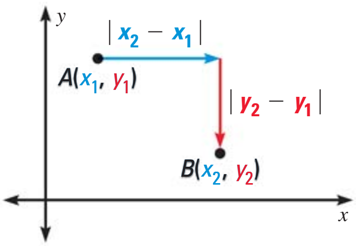
Euclidean geometry is named after a Greek mathematician. Euclid (circa third century B.C.) used postulates and deductive reasoning to prove the theorems you are studying in this book. **Non-Euclidean geometries** start by assuming different postulates, so they result in different theorems.

KEY CONCEPT — *For Your Notebook*

Taxicab Distance

The distance between two points is the sum of the differences in their coordinates.

$$AB = |x_2 - x_1| + |y_2 - y_1|$$



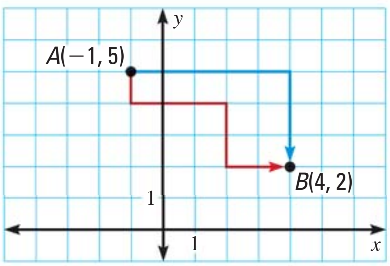
EXAMPLE 1 Find a taxicab distance

Find the taxicab distance from $A(-1, 5)$ to $B(4, 2)$. Draw two different shortest paths from A to B.

Solution

$$AB = |x_2 - x_1| + |y_2 - y_1|$$
$$= |4 - (-1)| + |2 - 5|$$
$$= |5| + |-3|$$
$$= 8$$

▶ The shortest path is 8 blocks. Two possible paths are shown.



REVIEW ABSOLUTE VALUE

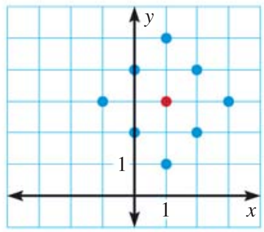
For help with absolute value, see p. 870.

CIRCLES In Euclidean geometry, a *circle* is all points that are the same distance from a fixed point, called the *center*. That distance is the *radius*. Taxicab geometry uses the same definition for a circle, but taxicab circles are not round.

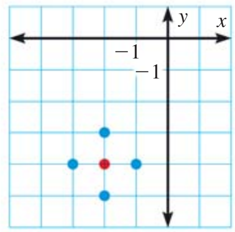
EXAMPLE 2 Draw a taxicab circle

Draw the taxicab circle with the given radius r and center C.

a. $r = 2$, $C(1, 3)$



b. $r = 1$, $C(-2, -4)$



PRACTICE

EXAMPLE 1 on p. 198 for Exs. 1–6

FINDING DISTANCE **Find the taxicab distance between the points.**

1. (4, 2), (0, 0) **2.** (3, 5), (6, 2) **3.** (−6, 3), (8, 5)

4. (−1, −3), (5, −2) **5.** (−3, 5), (−1, 5) **6.** (−7, 3), (−7, −4)

EXAMPLE 2 on p. 199 for Exs. 7–9

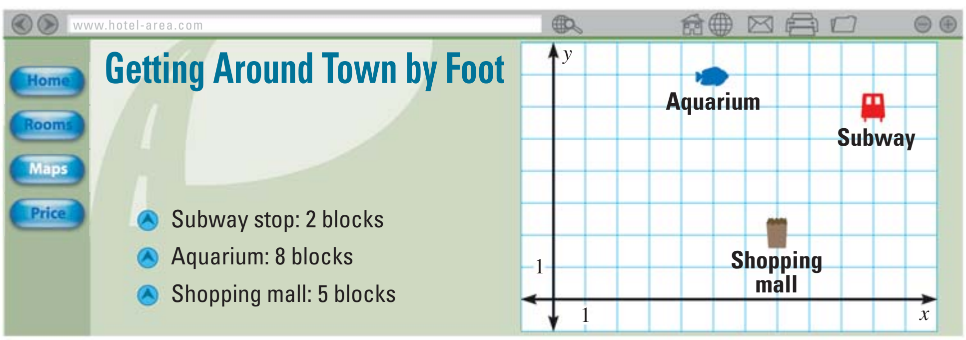
DRAWING CIRCLES **Draw the taxicab circle with radius r and center C.**

7. $r = 2$, $C(3, 4)$ **8.** $r = 4$, $C(0, 0)$ **9.** $r = 5$, $C(-1, 3)$

FINDING MIDPOINTS **A *midpoint* in taxicab geometry is a point where the distance to the endpoints are equal. Find all the midpoints of $\overline{AB}$.**

10. $A(2, 4)$, $B(-2, -2)$ **11.** $A(1, -3)$, $B(1, 3)$ **12.** $A(2, 2)$, $B(-3, 0)$

13. TRAVEL PLANNING A hotel's website claims that the hotel is an easy walk to a number of sites of interest. What are the coordinates of the hotel?



14. REASONING The taxicab distance between two points is always greater than or equal to the Euclidean distance between the two points. *Explain* what must be true about the points for both distances to be equal.

MIXED REVIEW FOR TEKS

TAKS PRACTICE
classzone.com

Lessons 3.4–3.6

MULTIPLE CHOICE

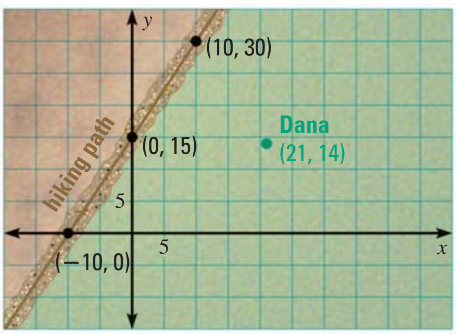
1. **PARTY** Marcos is planning a party. He would like to have the party at a roller skating rink or bowling alley. Which equations represent the total cost, R, to rent the roller rink and the total cost, B, to rent the bowling alley, where x is the number of hours he rents the facility? *TEKS A.6.E*

Cost to Rent ($)		
Hours	**Roller skating rink**	**Bowling alley**
1	35	20
2	70	40
3	105	60
4	140	80
5	175	100

Ⓐ $R = 20x$, $B = 35x$　　Ⓑ $R = 35x$, $B = 20x$

Ⓒ $R = 140x$, $B = 80x$　　Ⓓ $R = 140x + 35$, $B = 80x + 20$

2. **HIKING** Dana is walking across a meadow to get to a hiking path, as shown. The units represented in the coordinate grid are feet. What is the shortest distance she can walk to reach the path? *TEKS G.7.C*



Ⓕ $\sqrt{442}$ feet　　Ⓖ $\sqrt{327}$ feet

Ⓗ $5\sqrt{13}$ feet　　Ⓙ $5\sqrt{327}$ feet

3. **EQUATION OF A LINE** Which equation of a line is parallel to the line $y = -3x$ and perpendicular to the line $y = \frac{1}{3}x + \frac{5}{3}$? *TEKS G.7.B*

Ⓐ $y = 3x + 5$　　Ⓑ $y = -3x + 5$

Ⓒ $y = \frac{1}{3}x + 5$　　Ⓓ $y = -\frac{1}{3}x + 5$

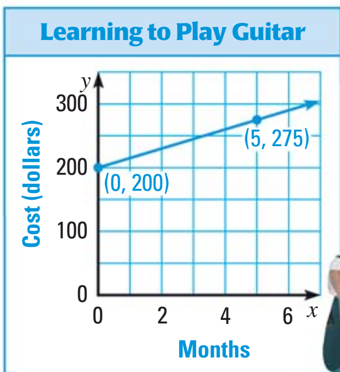
4. **CABLE CAR** The Johnstown Inclined Plane in Johnstown, Pennsylvania, is a cable car that transports people up and down the side of Yoder Hill. During the cable car's climb, you move about 17 feet upward for every 25 feet you go forward. At the top of the incline, the horizontal distance from where you started is about 500 feet. What is the vertical distance from where you started? *TEKS G.7.B*

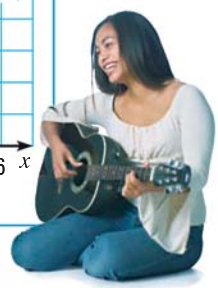
Ⓕ 340 feet　　Ⓖ 517 feet

Ⓗ 705 feet　　Ⓙ 735 feet

GRIDDED ANSWER

5. **GUITAR LESSONS** The graph models the accumulated cost of buying a used guitar and taking lessons over the first several months. Find the slope of the line in decimal form. *TEKS G.7.B*





3 CHAPTER SUMMARY

BIG IDEAS

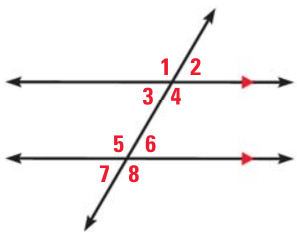
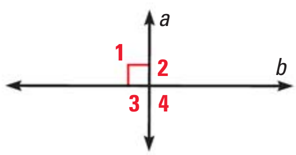
For Your Notebook

Big Idea 1

TEKS G.9.A

Using Properties of Parallel and Perpendicular Lines

When parallel lines are cut by a transversal, angle pairs are formed. Perpendicular lines form congruent right angles.

	$\angle 2$ and $\angle 6$ are corresponding angles, and they are congruent. $\angle 3$ and $\angle 6$ are alternate interior angles, and they are congruent. $\angle 1$ and $\angle 8$ are alternate exterior angles, and they are congruent. $\angle 3$ and $\angle 5$ are consecutive interior angles, and they are supplementary.
	If $a \perp b$, then $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are all right angles.

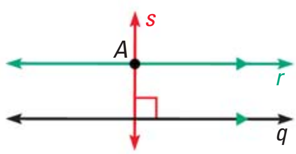
Big Idea 2

TEKS G.3.E

Proving Relationships Using Angle Measures

You can use the angle pairs formed by lines and a transversal to show that the lines are parallel. Also, if lines intersect to form a right angle, you know that the lines are perpendicular.

Through point A not on line q, there is only one line r parallel to q and one line s perpendicular to q.



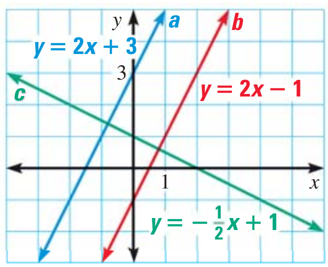
Big Idea 3

TEKS G.5.A, G.7.A, G.7.B, G.7.C

Making Connections to Lines in Algebra

In Algebra 1, you studied slope as a rate of change and linear equations as a way of modeling situations.

Slope and equations of lines are also a useful way to represent the lines and segments that you study in Geometry. For example, the slopes of parallel lines are the same ($a \parallel b$), and the product of the slopes of perpendicular lines is -1 ($a \perp c$, and $b \perp c$).



3 CHAPTER REVIEW

REVIEW KEY VOCABULARY

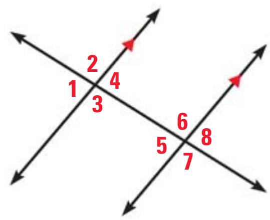
For a list of postulates and theorems, see pp. 926–931.

- parallel lines, *p. 147*
- skew lines, *p. 147*
- parallel planes, *p. 147*
- transversal, *p. 149*
- corresponding angles, *p. 149*
- alternate interior angles, *p. 149*
- alternate exterior angles, *p. 149*
- consecutive interior angles, *p. 149*
- paragraph proof, *p. 163*
- slope, *p. 171*
- slope-intercept form, *p. 180*
- standard form, *p. 182*
- distance from a point to a line, *p. 192*

VOCABULARY EXERCISES

1. Copy and complete: Two lines that do not intersect and are not coplanar are called _?_.
2. **WRITING** *Compare* alternate interior angle pairs and consecutive interior angle pairs.

Copy and complete the statement using the figure at the right.



3. ∠1 and _?_ are corresponding angles.
4. ∠3 and _?_ are alternate interior angles.
5. ∠4 and _?_ are consecutive interior angles.
6. ∠7 and _?_ are alternate exterior angles.

Identify the form of the equation as *slope-intercept form* or *standard form*.

7. $14x - 2y = 26$
8. $y = 7x - 13$

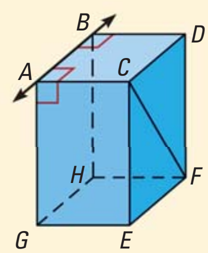
REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 3.

3.1 Identify Pairs of Lines and Angles *pp. 147–152*

EXAMPLE

Think of each segment in the rectangular box at the right as part of a line.



a. $\overleftrightarrow{BD}$, $\overleftrightarrow{AC}$, $\overleftrightarrow{BH}$, and $\overleftrightarrow{AG}$ appear perpendicular to $\overleftrightarrow{AB}$.

b. $\overleftrightarrow{CD}$, $\overleftrightarrow{GH}$, and $\overleftrightarrow{EF}$ appear parallel to $\overleftrightarrow{AB}$.

c. $\overleftrightarrow{CF}$ and $\overleftrightarrow{EG}$ appear skew to $\overleftrightarrow{AB}$.

d. Plane *EFG* appear parallel to plane *ABC*.

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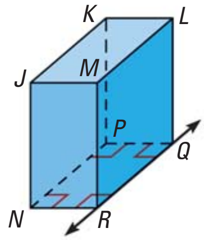
Chapter Review Practice

EXERCISES

EXAMPLE 1
on p. 147
for Exs. 9–12

Think of each segment in the diagram of a rectangular box as part of a line. Which line(s) or plane(s) contain point *N* and appear to fit the description?



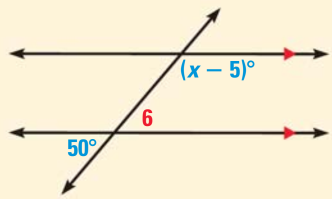
9. Line(s) perpendicular to $\overleftrightarrow{QR}$
10. Line(s) parallel to $\overleftrightarrow{QR}$
11. Line(s) skew to $\overleftrightarrow{QR}$
12. Plane(s) parallel to plane *LMQ*

3.2 Use Parallel Lines and Transversals

pp. 154–160

EXAMPLE

Use properties of parallel lines to find the value of *x*.



By the Vertical Angles Congruence Theorem, $m\angle 6 = 50°$.

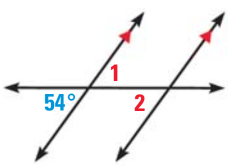
$(x - 5)° + m\angle 6 = 180°$	Consecutive Interior Angles Theorem
$(x - 5)° + 50° = 180°$	Substitute 50° for $m\angle 6$.
$x = 135$	Solve for *x*.

EXERCISES

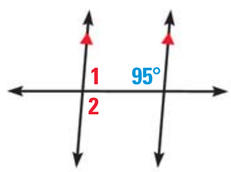
EXAMPLES 1 and 2
on pp. 154–155
for Exs. 13–19

Find $m\angle 1$ and $m\angle 2$. *Explain* your reasoning.

13.



14.



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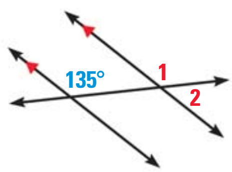
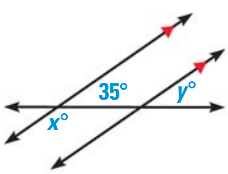
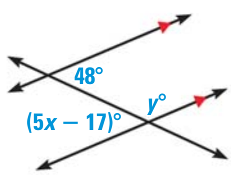


Find the values of *x* and *y*.

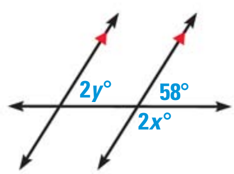
16.



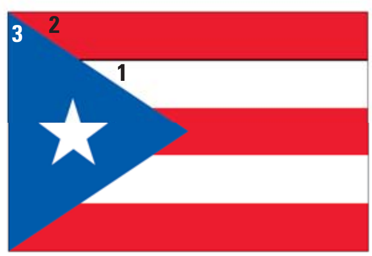
17.



18.



19. **FLAG OF PUERTO RICO** Sketch the rectangular flag of Puerto Rico as shown at the right. Find the measure of $\angle 1$ if $m\angle 3 = 55°$. *Justify* each step in your argument.



3 CHAPTER REVIEW

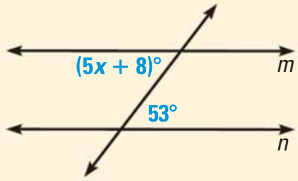
3.3 Prove Lines are Parallel

pp. 161–169

EXAMPLE

Find the value of *x* that makes $m \parallel n$.

Lines *m* and *n* are parallel when the marked corresponding angles are congruent.



$$(5x + 8)° = 53°$$

$$5x = 45$$

$$x = 9$$

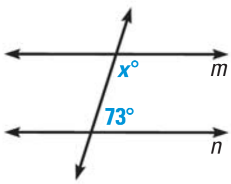
▶ The lines *m* and *n* are parallel when $x = 9$.

EXERCISES

EXAMPLE 1
on p. 161
for Exs. 20–22

Find the value of *x* that makes $m \parallel n$.

20.



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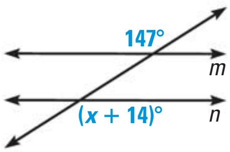

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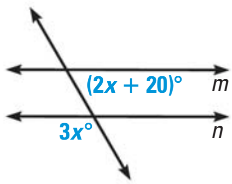
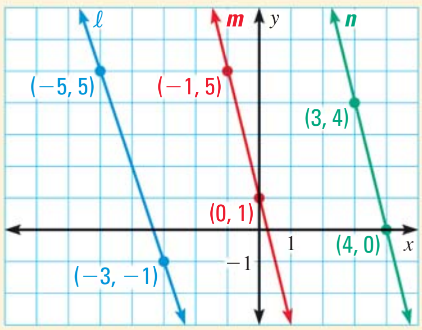


3.4 Find and Use Slopes of Lines

pp. 171–178

EXAMPLE

Find the slope of each line. Which lines are parallel?



$$\text{Slope of } \ell = \frac{-1-5}{-3-(-5)} = \frac{-6}{2} = -3$$

$$\text{Slope of } m = \frac{1-5}{0-(-1)} = \frac{-4}{1} = -4$$

$$\text{Slope of } n = \frac{0-4}{4-3} = \frac{-4}{1} = -4$$

▶ Because *m* and *n* have the same slope, they are parallel. The slope of ℓ is different, so ℓ is not parallel to the other lines.

EXERCISES

EXAMPLES 2 and 3
on pp. 172–173
for Exs. 23–24

Tell whether the lines through the given points are *parallel*, *perpendicular*, or *neither*.

23. Line 1: (8, 12), (7, −5)
Line 2: (−9, 3), (8, 2)

24. Line 1: (3, −4), (−1, 4)
Line 2: (2, 7), (5, 1)

3.5 Write and Graph Equations of Lines *pp. 180–187*

EXAMPLE

Write an equation of the line *k* passing through the point (−4, 1) that is perpendicular to the line *n* with the equation $y = 2x - 3$.

First, find the slope of line *k*. Line *n* has a slope of 2.

$$2 \cdot m = -1$$

$$m = -\frac{1}{2}$$

Then, use the given point and the slope in the slope-intercept form to find the *y*-intercept.

$$y = mx + b$$

$$1 = -\frac{1}{2}(-4) + b$$

$$-1 = b$$

▶ An equation of line *k* is $y = -\frac{1}{2}x - 1$.

EXERCISES

EXAMPLES 2 and 3 on pp. 180–181 for Exs. 25–26

Write equations of the lines that pass through point *P* and are (a) parallel and (b) perpendicular to the line with the given equation.

25. $P(3, -1)$, $y = 6x - 4$

26. $P(-6, 5)$, $7y + 4x = 2$

3.6 Prove Theorems About Perpendicular Lines *pp. 190–197*

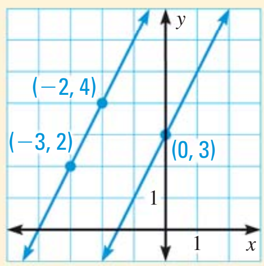
EXAMPLE

Find the distance between $y = 2x + 3$ and $y = 2x + 8$.

Find the length of a perpendicular segment from one line to the other. Both lines have a slope of 2, so the slope of a perpendicular segment to each line is $-\frac{1}{2}$.

The segment from (0, 3) to (−2, 4) has a slope of $\frac{4-3}{-2-0} = -\frac{1}{2}$. So, the distance between the lines is

$d = \sqrt{(-2-0)^2 + (4-3)^2} = \sqrt{5} \approx 2.2$ units.

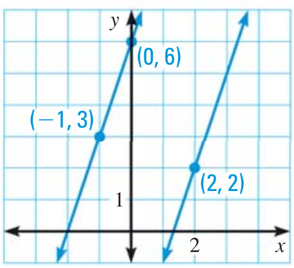


EXERCISES

EXAMPLE 4 on p. 193 for Exs. 27–28

Use the Distance Formula to find the distance between the two parallel lines. Round to the nearest tenth, if necessary.

27.



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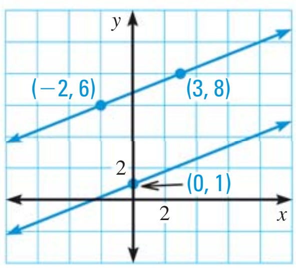
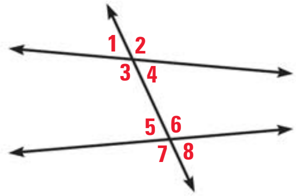


3 CHAPTER TEST

Classify the pairs of angles as *corresponding, alternate interior, alternate exterior,* or *consecutive interior.*

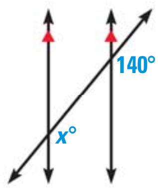


1. $\angle 1$ and $\angle 8$ **2.** $\angle 2$ and $\angle 6$ **3.** $\angle 3$ and $\angle 5$

4. $\angle 4$ and $\angle 5$ **5.** $\angle 3$ and $\angle 7$ **6.** $\angle 3$ and $\angle 6$

Find the value of *x*.

7.



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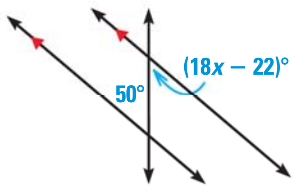

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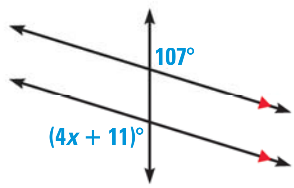
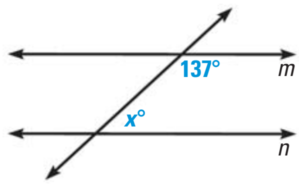


Find the value of *x* that makes $m \parallel n$.

10.



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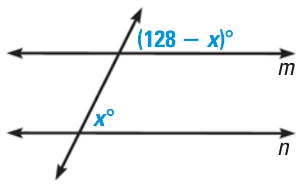

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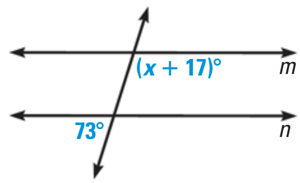


Find the slope of the line that passes through the points.

13. (3, −1), (3, 4) **14.** (2, 7), (−1, −3) **15.** (0, 5), (−6, 12)

Write an equation of the line that passes through the given point *P* and has the given slope *m*.

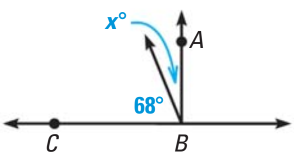
16. $P(-2, 4)$, $m = 3$ **17.** $P(7, 12)$, $m = -0.2$ **18.** $P(3, 5)$, $m = -8$

Write an equation of the line that passes through point *P* and is perpendicular to the line with the given equation.

19. $P(1, 3)$, $y = 2x - 1$ **20.** $P(0, 2)$, $y = -x + 3$ **21.** $P(2, -3)$, $x - y = 4$

In Exercises 22–24, $\overline{AB} \perp \overline{BC}$. Find the value of *x*.

22.



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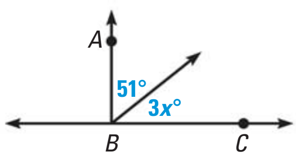

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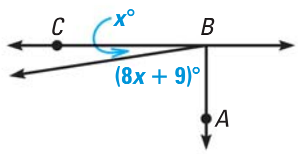
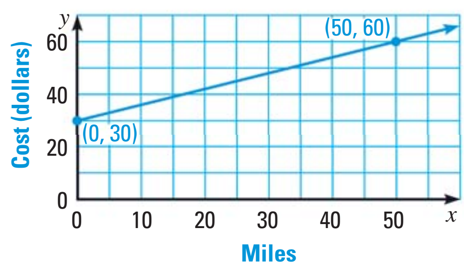


25. RENTAL COSTS The graph at the right models the cost of renting a moving van. Write an equation of the line. Then find the cost of renting the van for a 100 mile trip.



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GRAPH AND SOLVE LINEAR INEQUALITIES

EXAMPLE 1 Graph a linear inequality in two variables

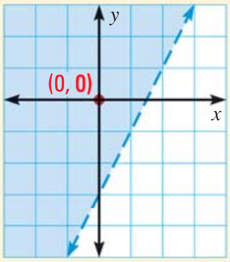
Graph the inequality $0 > 2x - 3 - y$.

Solution

Rewrite the inequality in slope-intercept form, $y > 2x - 3$.

The boundary line $y = 2x - 3$ is not part of the solution, so use a dashed line.

To decide where to shade, use a point not on the line, such as (0, 0), as a test point. Because $0 > 2 \cdot 0 - 3$, (0, 0) is a solution. Shade the half-plane that includes (0, 0).



EXAMPLE 2 Use an inequality to solve a real-world problem

SAVINGS Lily has saved \$49. She plans to save \$12 per week to buy a camera that costs \$124. In how many weeks will she be able to buy the camera?

Solution

Let w represent the number of weeks needed.

$49 + 12w \geq 124$ Write an algebraic model.

$12w \geq 75$ Subtract 49 from each side.

$w \geq 6.25$ Divide each side by 12.

▶ She must save for 7 weeks to be able to buy the camera.

EXERCISES

EXAMPLE 1 for Exs. 1–8

Graph the linear inequality.

1. $y > -2x + 3$
2. $y \leq 0.5x - 4$
3. $-2.5x + y \geq 1.5$
4. $x < 3$
5. $y < -2$
6. $5x - y > -5$
7. $2x + 3y \geq -18$
8. $3x - 4y \leq 6$

EXAMPLE 2 for Exs. 9–11

Solve.

9. **LOANS** Eric borrowed \$46 from his mother. He will pay her back at least \$8 each month. At most, how many months will it take him?

10. **GRADES** Manuel's quiz scores in history are 76, 81, and 77. What score must he get on his fourth quiz to have an average of at least 80?

11. **PHONE CALLS** Company A charges a monthly fee of \$5 and \$.07 per minute for phone calls. Company B charges no monthly fee, but charges \$.12 per minute. After how many minutes of calls is the cost of using Company A less than the cost of using Company B?

3 TAKS PREPARATION

TEXAS

TAKS Obj. 3
TEKS A.6.A, A.6.C, A.6.E, A.6.F

REVIEWING PROPERTIES OF LINEAR EQUATIONS PROBLEMS

Recall from Algebra that linear equations can be written in the following forms.

KEY CONCEPT

Forms of Linear Equations

Slope-intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Standard form: $Ax + By = C$

Note that in real-life problems, the slope of a line is sometimes known as the *rate of change*, because it compares two different quantities that are changing. Some examples of rates of change are miles per hour or dollars per day.

FUNCTION NOTATION The symbol $f(x)$ can replace y in the slope-intercept form of the equation of a line. This is known as *function notation*. Because function notation allows you to write $f(x)$ in place of y, the graph of a function f is the set of all points $(x, f(x))$, where x is in the domain of the function.

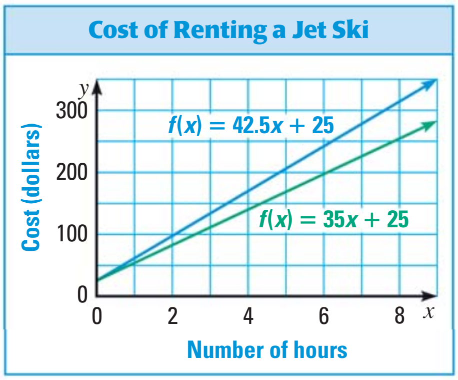
EXAMPLE

JET SKI RENTAL The equation $f(x) = 35x + 25$ models the total cost of renting a jet ski. Suppose the slope of the corresponding graph increases to 42.5. What does this mean in the context of the real-life situation?

Solution

The slope represents the cost in dollars over time, or the hourly rate, for renting a jet ski.

If the slope increases to 42.5, the rate for renting a jet ski increases from \$35.00 per hour to \$42.50 per hour. As you can see in the graph, the slope is steeper. Notice that the y-intercept remains the same.



LINEAR EQUATIONS PROBLEMS ON TAKS

Below are examples of linear equations problems in multiple choice format. Try solving the problems before looking at the solutions. (Cover the solutions with a piece of paper.) Then check your solutions against the ones given.

1. What is the y-intercept of the function $f(x) = -4(x + 2)$?

A 2

B −2

C −4

D −8

Solution

$f(x) = -4(x + 2)$ Function notation

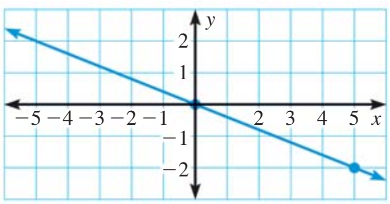
$y = -4(x + 2)$ Replace $f(x)$ with y.

$y = -4x - 8$ Slope-intercept form

The y-intercept is −8. So, the correct answer is D.

Ⓐ Ⓑ Ⓒ Ⓓ

2. What is the rate of change of the graph below?



F 0

G −0.4

H −1.4

J −2.5

Solution

The rate of change is the slope. Choose two points on the graph, for example (0, 0) and (5, −2).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Equation for slope

$$= \frac{-2 - 0}{5 - 0} = -0.4$$ Substitute and simplify.

So, the correct answer is G.

Ⓕ Ⓖ Ⓗ Ⓙ

3. Which best describes the effect on the graph of $f(x) = 2x - 4$ if the y-intercept is changed to 3?

A The slope increases.

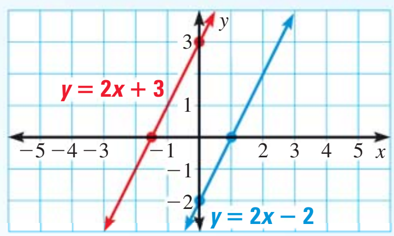
B The new line passes through the origin.

C The x-intercept decreases.

D The y-intercept decreases.

Solution

You can see from the graph that the x-intercept decreases from 2 to −1.5, while the y-intercept increases and the slope remains the same.



So, the correct answer is C.

Ⓐ Ⓑ Ⓒ Ⓓ

3 TAKS PRACTICE

PRACTICE FOR TAKS OBJECTIVE 3

1. What is the y-intercept of the function $f(x) = 0.8(x - 2) + 3.2(x + 1.5)$?

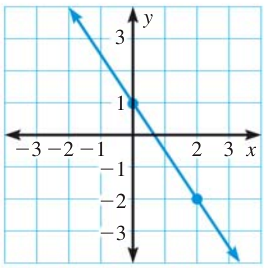
   A -1.6

   B -0.5

   C 3.2

   D 4.8

2. What is the rate of change of the graph?



   F $\frac{2}{3}$

   G $\frac{3}{2}$

   H $-\frac{2}{3}$

   J $-\frac{3}{2}$

3. Which best describes the effect on the graph of $f(x) = -0.25(x - 8)$ if the y-intercept is changed to -6?

   A The y-intercept increases.

   B The new line passes through $(-6, 0)$.

   C The x-intercept decreases.

   D The x-intercept increases.

4. What is m, the slope of the line that contains the points $(1, -1)$, $(0, 5)$, and $(-1, 11)$?

   F 5

   G -6

   H $\frac{1}{6}$

   J $-\frac{1}{5}$

5. Which of the following describes the line containing the points $(0, 6)$ and $(3, -6)$?

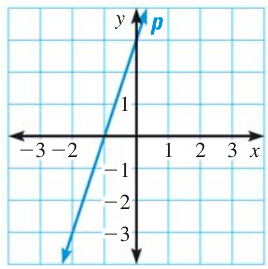
   A $y = -4x + 6$

   B $y = \frac{1}{4}x + 6$

   C $y = 4x + 6$

   D $y = -\frac{1}{4}x + 8$

6. Suppose line p is shifted so that the x-intercept decreases and the y-intercept remains the same. What can you say about the slope?



   F The slope increases and is positive.

   G The slope increases and is negative.

   H The slope decreases and is positive.

   J The slope decreases and is negative.

MIXED TAKS PRACTICE

7. Which expression can be used to find the values of f(x) in the table below? ***TAKS Obj. 2***

x	−2	−1	0	1	2	3
$f(x)$	10	7	4	1	−2	−5

   A $5x$

   B $x + 4$

   C $-2x + 12$

   D $-3x + 4$

MIXED TAKS PRACTICE

8. Jay earns \$5 per hour for mowing and gets a \$10 tip. Which equation describes his total earnings, t, for h hours of work? ***TAKS Obj. 1***

F $t = 10h + 5$

G $t = 5h + 10$

H $t = 10h - 5$

J $t = 5h - 10$

9. Kate is 4 years older than Sue. Their ages sum to 30. Which pair of equations gives Kate's age, k, and Sue's age, s? ***TAKS Obj. 4***

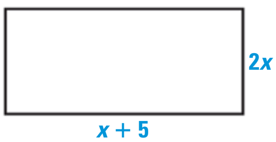
A $k + 4 = s$
$k + s = 30$

B $k = s + 4$
$k + s = 30$

C $k = s + 4$
$k + 30 = s$

D $k + s = 4$
$k + s = 30$

10. Which equation represents the perimeter P of the rectangle below? ***TAKS Obj. 2***



F $P = 6x + 5$

G $P = 2x^2 + 10x$

H $P = 3x + 5$

J $P = 6x + 10$

11. Which equation describes the line that passes through the point (2, 6) and is parallel to the line represented by the equation $-2x + y = 1$? ***TAKS Obj. 3***

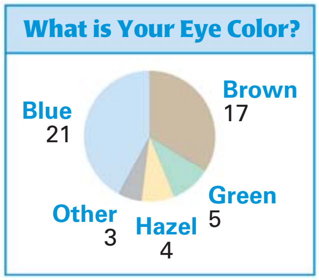
A $y = -2x + 10$

B $y = 2x + 2$

C $y = 0.5x + 5$

D $y = -0.5x + 7$

12. The results of a survey are displayed in the circle graph below. What information cannot be determined from the graph? ***TAKS Obj. 9***



F The percentage of people surveyed who have blue eyes

G The eye colors of the people who responded "Other"

H The number of people surveyed who have green or hazel eyes

J The total number of people surveyed

13. Which expression is equivalent to $\frac{6x^5y^{-2}}{10x^3y^4z}$? ***TAKS Obj. 5***

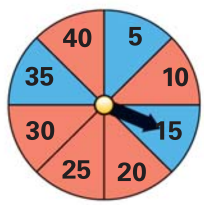
A $\frac{3x^2y^2}{5z}$

B $\frac{3x^2}{5y^6z}$

C $\frac{3x^2}{5y^2z}$

D $\frac{3z}{5x^2y^6}$

14. **GRIDDED ANSWER** Jenny spins the spinner twice. What is the probability that the spinner lands on red on the first spin and an even number on the second spin? Round your answer to the thousandths place. ***TAKS Obj. 9***

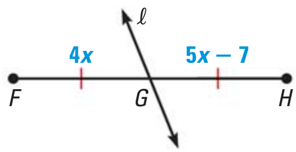


Record your answer and fill in the bubbles on your answer document. Be sure to use the correct place value.

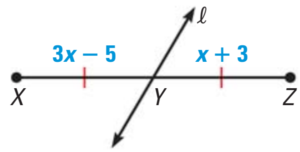
CUMULATIVE REVIEW Chapters 1–3

Line ℓ bisects the segment. Find the indicated lengths. *(p. 15)*

1. GH and FH



2. XY and XZ

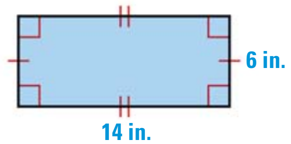


Classify the angle with the given measure as *acute*, *obtuse*, *right*, or *straight*. *(p. 24)*

3. $m\angle A = 28°$
4. $m\angle A = 113°$
5. $m\angle A = 79°$
6. $m\angle A = 90°$

Find the perimeter and area of the figure. *(p. 49)*

7.



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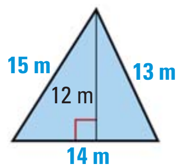

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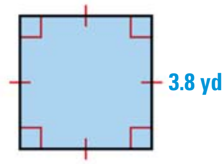


Describe **the pattern in the numbers. Write the next number in the pattern.** *(p. 72)*

10. 1, 8, 27, 64, . . .
11. 128, 32, 8, 2, . . .
12. 2, −6, 18, −54, . . .

Use the Law of Detachment to make a valid conclusion. *(p. 87)*

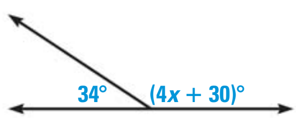
13. If $6x < 42$, then $x < 7$. The value of $6x$ is 24.
14. If an angle measure is greater than 90°, then it is an obtuse angle. The measure of $\angle A$ is 103°.
15. If a musician plays a violin, then the musician plays a stringed instrument. The musician is playing a violin.

Solve the equation. Write a reason for each step. *(p. 105)*

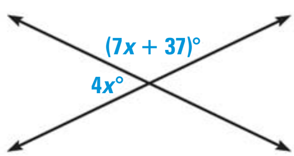
16. $3x - 14 = 34$
17. $-4(x + 3) = -28$
18. $43 - 9(x - 7) = -x - 6$

Find the value of the variable(s). *(pp. 124, 154)*

19.



20.



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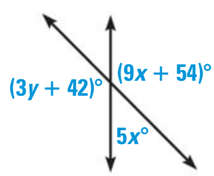

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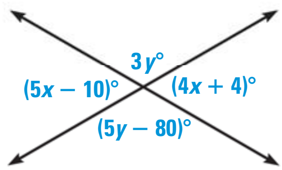

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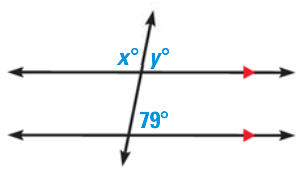

24.