12 Surface Area and Volume of Solids

TEXAS

Before

In previous chapters, you learned the following skills, which you'll use in Chapter 12: properties of similar polygons, areas and perimeters of two-dimensional figures, and right triangle trigonometry.

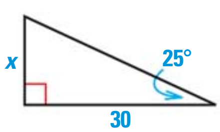
Prerequisite Skills

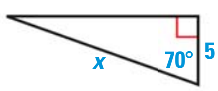
VOCABULARY CHECK

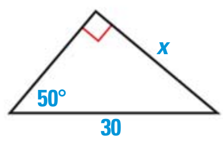
1. Copy and complete: The area of a regular polygon is given by the formula $A =$ _?_.
2. *Explain* what it means for two polygons to be similar.

SKILLS AND ALGEBRA CHECK

Use trigonometry to find the value of *x*. *(Review pp. 466, 473 for 12.2–12.5.)*

3. 

4. 

5. 

Find the circumference and area of the circle with the given dimension. *(Review pp. 746, 755 for 12.2–12.5.)*

6. $r = 2$ m
7. $d = 3$ in.
8. $r = 2\sqrt{5}$ cm

TEXAS @HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 12, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 856. You will also use the key vocabulary listed below.

Big Ideas

1. **Exploring solids and their properties**
2. **Solving problems using surface area and volume**
3. **Connecting similarity to solids**

KEY VOCABULARY

- polyhedron, *p. 794*
  face, edge, vertex
- Platonic solids, *p. 796*
- cross section, *p. 797*
- prism, *p. 803*
- surface area, *p. 803*
- lateral area, *p. 803*
- net, *p. 803*
- right prism, *p. 804*
- oblique prism, *p. 804*
- cylinder, *p. 805*
- right cylinder, *p. 805*
- pyramid, *p. 810*
- regular pyramid, *p. 810*
- cone, *p. 812*
- right cone, *p. 812*
- volume, *p. 819*
- sphere, *p. 838*
- great circle, *p. 839*
- hemisphere, *p. 839*
- similar solids, *p. 847*

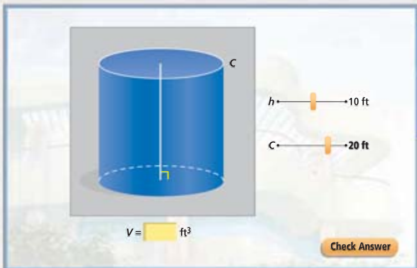
Why?

Knowing how to use surface area and volume formulas can help you solve problems in three dimensions. For example, you can use a formula to find the volume of a column in a building.

Animated Geometry

The animation illustrated below for Exercise 31 on page 825 helps you answer this question: What is the volume of the column?

You can use the height and circumference of a column to find its volume.



Drag the sliders to change the height and circumference of the cylinder.

Animated Geometry at classzone.com

Other animations for Chapter 12: pages 795, 805, 821, 833, 841, and 852

Investigating Geometry **ACTIVITY** *Use before Lesson 12.1*

12.1 Investigate Solids

TEKS G.1.B, G.2.A, G.6.B, G.9.D

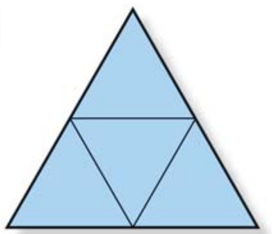
MATERIALS • poster board • scissors • tape • straightedge

QUESTION What solids can be made using congruent regular polygons?

Platonic solids, named after the Greek philosopher Plato (427 B.C.–347 B.C.), are solids that have the same congruent regular polygon as each *face*, or side of the solid.

EXPLORE 1 Make a solid using four equilateral triangles

STEP 1



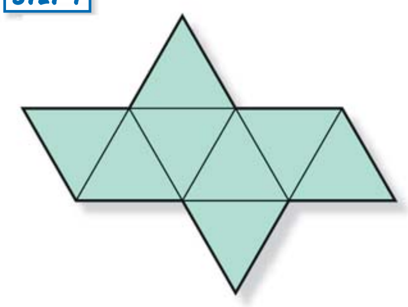
Make a net Copy the full-sized triangle from page 793 on poster board to make a template. Trace the triangle four times to make a *net* like the one shown.

STEP 2

Make a solid Cut out your net. Fold along the lines. Tape the edges together to form a solid. How many faces meet at each *vertex*?

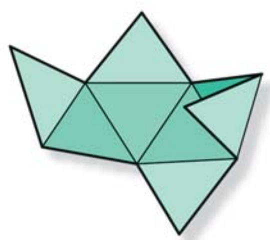
EXPLORE 2 Make a solid using eight equilateral triangles

STEP 1



Make a net Trace your triangle template from Explore 1 eight times to make a net like the one shown.

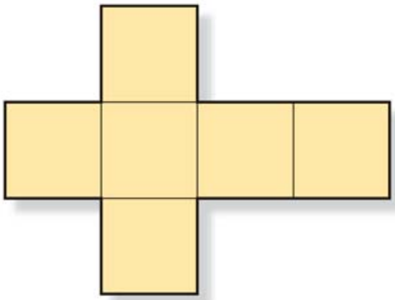
STEP 2



Make a solid Cut out your net. Fold along the lines. Tape the edges together to form a solid. How many faces meet at each vertex?

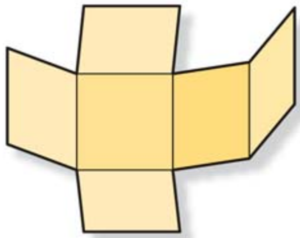
EXPLORE 3 Make a solid using six squares

STEP 1



Make a net Copy the full-sized square from the bottom of the page on poster board to make a template. Trace the square six times to make a net like the one shown.

STEP 2

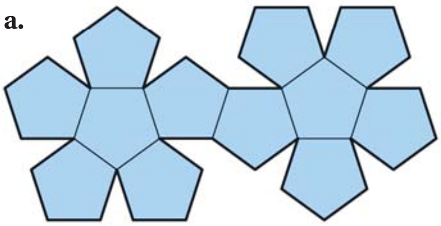


Make a solid Cut out your net. Fold along the lines. Tape the edges together to form a solid. How many faces meet at each vertex?

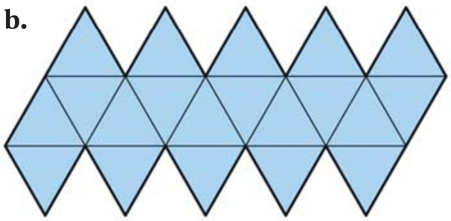
DRAW CONCLUSIONS Use your observations to complete these exercises

1. The two other convex solids that you can make using congruent, regular faces are shown below. For each of these solids, how many faces meet at each vertex?

a.

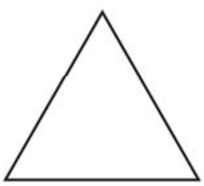


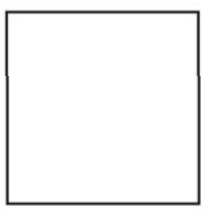
b.



2. *Explain* why it is not possible to make a solid that has six congruent equilateral triangles meeting at each vertex.
3. *Explain* why it is not possible to make a solid that has three congruent regular hexagons meeting at each vertex.
4. Count the number of vertices V, edges E, and faces F for each solid you made. Make a conjecture about the relationship between the sum $F + V$ and the value of E.

Templates:





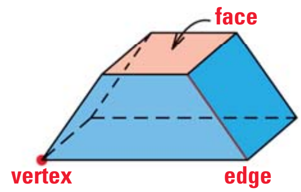
12.1 Explore Solids

TEKS G.1.B, G.6.A, G.9.B, G.9.D

Before	You identified polygons.
Now	You will identify solids.
Why	So you can analyze the frame of a house, as in Example 2.

Key Vocabulary
- **polyhedron**
  face, edge, vertex
- **base**
- **regular polyhedron**
- **convex polyhedron**
- **Platonic solids**
- **cross section**

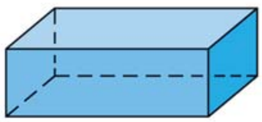
A **polyhedron** is a solid that is bounded by polygons, called **faces**, that enclose a single region of space. An **edge** of a polyhedron is a line segment formed by the intersection of two faces. A **vertex** of a polyhedron is a point where three or more edges meet. The plural of polyhedron is *polyhedra* or *polyhedrons*.



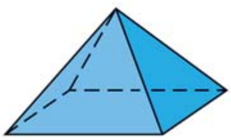
KEY CONCEPT — For Your Notebook

Types of Solids

Polyhedra

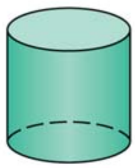


Prism

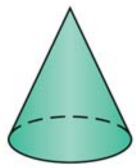


Pyramid

Not Polyhedra



Cylinder

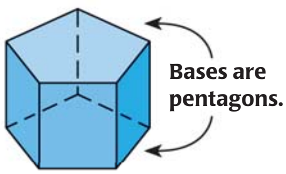


Cone

Sphere

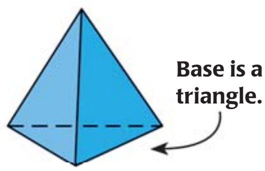
CLASSIFYING SOLIDS Of the five solids above, the prism and the pyramid are polyhedra. To name a prism or a pyramid, use the shape of the *base*.

Pentagonal prism



The two **bases** of a prism are congruent polygons in parallel planes.

Triangular pyramid

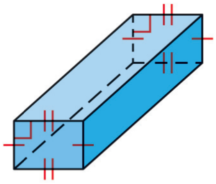


The **base** of a pyramid is a polygon.

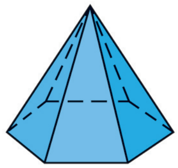
EXAMPLE 1 Identify and name polyhedra

Tell whether the solid is a polyhedron. If it is, name the polyhedron and find the number of faces, vertices, and edges.

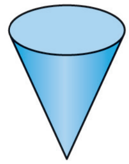
a.



b.



c.



Solution

a. The solid is formed by polygons, so it is a polyhedron. The two bases are congruent rectangles, so it is a rectangular prism. It has 6 faces, 8 vertices, and 12 edges.

b. The solid is formed by polygons, so it is a polyhedron. The base is a hexagon, so it is a hexagonal pyramid. It has 7 faces, consisting of 1 base, 3 visible triangular faces, and 3 non-visible triangular faces. The polyhedron has 7 faces, 7 vertices, and 12 edges.

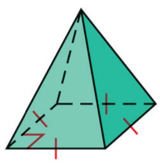
c. The cone has a curved surface, so it is not a polyhedron.

Animated Geometry at classzone.com

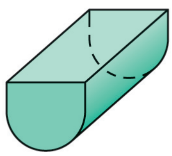
✓ GUIDED PRACTICE for Example 1

Tell whether the solid is a polyhedron. If it is, name the polyhedron and find the number of faces, vertices, and edges.

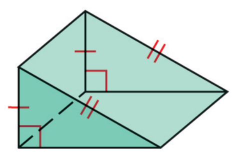
1.



2.



3.

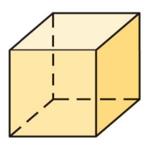


EULER'S THEOREM Notice in Example 1 that the sum of the number of faces and vertices of the polyhedra is two more than the number of edges. This suggests the following theorem, proved by the Swiss mathematician Leonhard Euler (pronounced "oi′-ler"), who lived from 1707 to 1783.

THEOREM *For Your Notebook*

THEOREM 12.1 Euler's Theorem

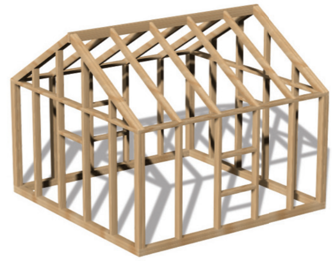
The number of faces (F), vertices (V), and edges (E) of a polyhedron are related by the formula $F + V = E + 2$.



$F = 6, V = 8, E = 12$
$6 + 8 = 12 + 2$

EXAMPLE 2 Use Euler's Theorem in a real-world situation

HOUSE CONSTRUCTION **Find the number of edges on the frame of the house.**



Solution

The frame has one face as its foundation, four that make up its walls, and two that make up its roof, for a total of 7 faces.

To find the number of vertices, notice that there are 5 vertices around each pentagonal wall, and there are no other vertices. So, the frame of the house has 10 vertices.

Use Euler's Theorem to find the number of edges.

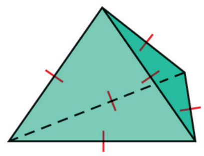
$F + V = E + 2$ **Euler's Theorem**

$7 + 10 = E + 2$ **Substitute known values.**

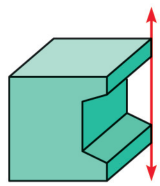
$15 = E$ **Solve for *E*.**

▶ The frame of the house has 15 edges.

REGULAR POLYHEDRA A polyhedron is **regular** if all of its faces are congruent regular polygons. A polyhedron is **convex** if any two points on its surface can be connected by a segment that lies entirely inside or on the polyhedron. If this segment goes outside the polyhedron, then the polyhedron is nonconvex, or *concave*.



regular, convex

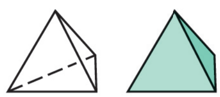


nonregular, concave

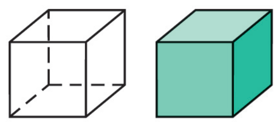
There are five regular polyhedra, called **Platonic solids** after the Greek philosopher Plato (c. 427 B.C.–347 B.C.). The five Platonic solids are shown.

READ VOCABULARY

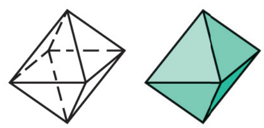
Notice that the names of four of the Platonic solids end in "hedron." *Hedron* is Greek for "side" or "face." Sometimes a cube is called a regular *hexahedron*.



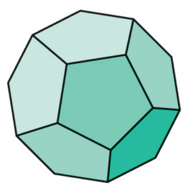
Regular tetrahedron
4 faces



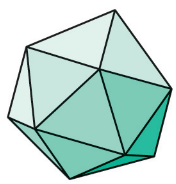
Cube
6 faces



Regular octahedron
8 faces



Regular dodecahedron
12 faces

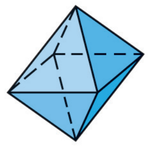


Regular icosahedron
20 faces

There are only five regular polyhedra because the sum of the measures of the angles that meet at a vertex of a convex polyhedron must be less than 360°. This means that the only possible combinations of regular polygons at a vertex that will form a polyhedron are 3, 4, or 5 triangles, 3 squares, and 3 pentagons.

EXAMPLE 3 Use Euler's Theorem with Platonic solids

Find the number of faces, vertices, and edges of the regular octahedron. Check your answer using Euler's Theorem.



ANOTHER WAY

An octahedron has 8 faces, each of which has 3 vertices and 3 edges. Each vertex is shared by 4 faces; each edge is shared by 2 faces. They should only be counted once.

$V = \frac{8 \cdot 3}{4} = 6$

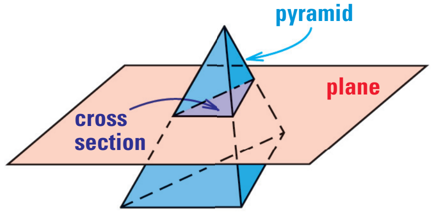
$E = \frac{8 \cdot 3}{2} = 12$

Solution

By counting on the diagram, the octahedron has 8 faces, 6 vertices, and 12 edges. Use Euler's Theorem to check.

$F + V = E + 2$	Euler's Theorem
$8 + 6 = 12 + 2$	Substitute.
$14 = 14$ ✓	This is a true statement. So, the solution checks.

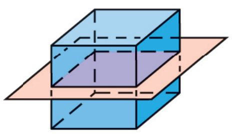
CROSS SECTIONS Imagine a plane slicing through a solid. The intersection of the plane and the solid is called a **cross section**. For example, the diagram shows that an intersection of a plane and a triangular pyramid is a triangle.



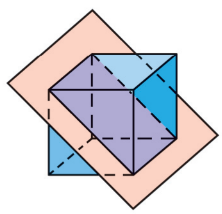
EXAMPLE 4 Describe cross sections

Describe the shape formed by the intersection of the plane and the cube.

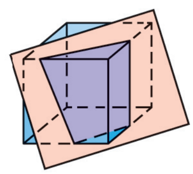
a.



b.



c.



Solution

a. The cross section is a square.

b. The cross section is a rectangle.

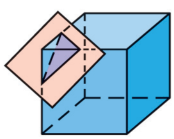
c. The cross section is a trapezoid.

✓ GUIDED PRACTICE for Examples 2, 3, and 4

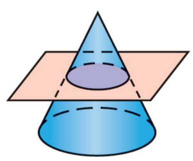
4. Find the number of faces, vertices, and edges of the regular dodecahedron on page 796. Check your answer using Euler's Theorem.

Describe **the shape formed by the intersection of the plane and the solid.**

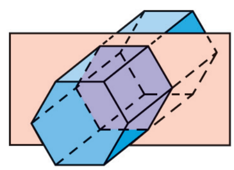
5.



6.



7.



12.1 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 25, and 35

★ = **TAKS PRACTICE AND REASONING** Exs. 28, 30, 31, 39, 41, 52, 53, and 54

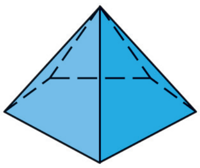
SKILL PRACTICE

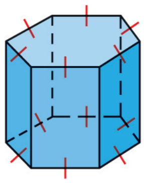
1. **VOCABULARY** Name the five Platonic solids and give the number of faces for each.

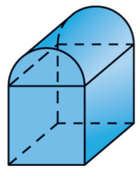
2. **WRITING** State Euler's Theorem in words.

EXAMPLE 1 on p. 795 for Exs. 3–10

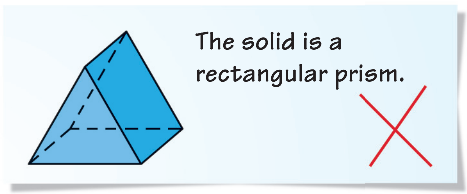
IDENTIFYING POLYHEDRA **Determine whether the solid is a polyhedron. If it is, name the polyhedron. *Explain* your reasoning.**

3. 

4. 

5. 

6. **ERROR ANALYSIS** *Describe* and correct the error in identifying the solid.



SKETCHING POLYHEDRA **Sketch the polyhedron.**

7. Rectangular prism
8. Triangular prism
9. Square pyramid
10. Pentagonal pyramid

EXAMPLES 2 and 3 on pp. 796–797 for Exs. 11–24

APPLYING EULER'S THEOREM **Use Euler's Theorem to find the value of n.**

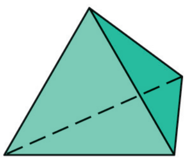
11. Faces: n
Vertices: 12
Edges: 18

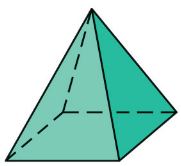
12. Faces: 5
Vertices: n
Edges: 8

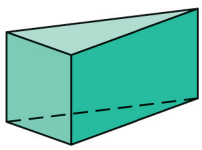
13. Faces: 10
Vertices: 16
Edges: n

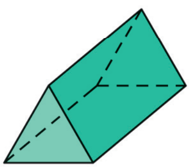
14. Faces: n
Vertices: 12
Edges: 30

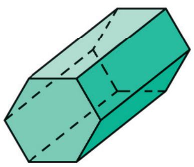
APPLYING EULER'S THEOREM **Find the number of faces, vertices, and edges of the polyhedron. Check your answer using Euler's Theorem.**

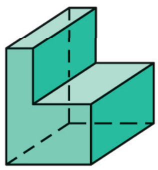
15. 

16. 

17. 

18. 

19. 

20. 

21. **WRITING** *Explain* why a cube is also called a regular hexahedron.

EXAMPLE 4
on p. 797
for Exs. 25–28

PUZZLES **Determine whether the solid puzzle is *convex* or *concave*.**

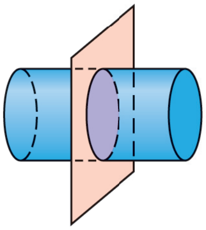
22.

23.

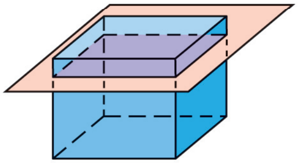
24.

CROSS SECTIONS **Draw and *describe* the cross section formed by the intersection of the plane and the solid.**

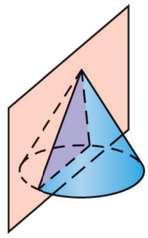
25.



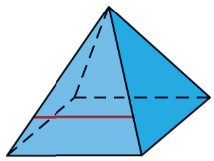
26.



27.



28. **TAKS REASONING** What is the shape of the cross section formed by the plane parallel to the base that intersects the red line drawn on the square pyramid?



Ⓐ Square Ⓑ Triangle

Ⓒ Kite Ⓓ Trapezoid

29. **ERROR ANALYSIS** *Describe* and correct the error in determining that a tetrahedron has 4 faces, 4 edges, and 6 vertices.

30. **TAKS REASONING** Which two solids have the same number of faces?

Ⓐ A triangular prism and a rectangular prism

Ⓑ A triangular pyramid and a rectangular prism

Ⓒ A triangular prism and a square pyramid

Ⓓ A triangular pyramid and a square pyramid

31. **TAKS REASONING** How many faces, vertices, and edges does an octagonal prism have?

Ⓐ 8 faces, 6 vertices, and 12 edges

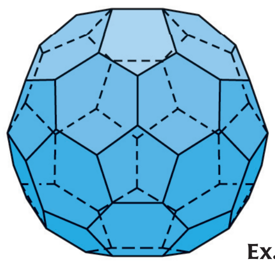
Ⓑ 8 faces, 12 vertices, and 18 edges

Ⓒ 10 faces, 12 vertices, and 20 edges

Ⓓ 10 faces, 16 vertices, and 24 edges

32. **EULER'S THEOREM** The solid shown has 32 faces and 90 edges. How many vertices does the solid have? *Explain* your reasoning.

33. **CHALLENGE** *Describe* how a plane can intersect a cube to form a hexagonal cross section.



Ex. 32

PROBLEM SOLVING

EXAMPLE 2 on p. 796 for Exs. 34–35

34. MUSIC The speaker shown at the right has 7 faces. Two faces are pentagons and 5 faces are rectangles.

a. Find the number of vertices.

b. Use Euler's Theorem to determine how many edges the speaker has.

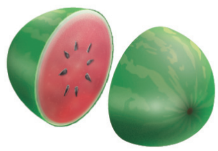
TEXAS @HomeTutor for problem solving help at classzone.com

35. CRAFT BOXES The box shown at the right is a hexagonal prism. It has 8 faces. Two faces are hexagons and 6 faces are squares. Count the edges and vertices. Use Euler's Theorem to check your answer.

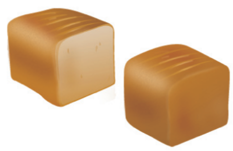
TEXAS @HomeTutor for problem solving help at classzone.com

FOOD ***Describe*** **the shape that is formed by the cut made in the food shown.**

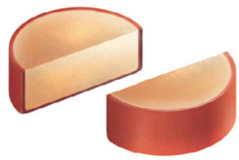
36. Watermelon



37. Bread

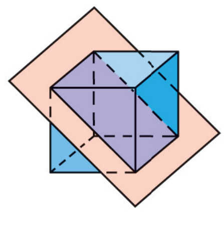


38. Cheese



39. TAKS REASONING Name a polyhedron that has 4 vertices and 6 edges. Can you draw a polyhedron that has 4 vertices, 6 edges, and a different number of faces? *Explain* your reasoning.

40. MULTI-STEP PROBLEM The figure at the right shows a plane intersecting a cube through four of its vertices. An edge length of the cube is 6 inches.



a. *Describe* the shape formed by the cross section.

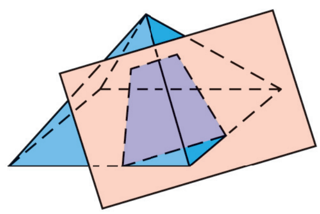
b. What is the perimeter of the cross section?

c. What is the area of the cross section?

41. TAKS REASONING Use the diagram of the square pyramid intersected by a plane.

a. *Describe* the shape of the cross section shown.

b. Can a plane intersect the pyramid at a point? If so, sketch the intersection.



c. *Describe* the shape of the cross section when the pyramid is sliced by a plane parallel to its base.

d. Is it possible to have a pentagon as a cross section of this pyramid? If so, draw the cross section.

42. PLATONIC SOLIDS Make a table of the number of faces, vertices, and edges for the five Platonic solids. Use Euler's Theorem to check each answer.

REASONING Is it possible for a cross section of a cube to have the given shape? If yes, *describe* or sketch how the plane intersects the cube.

43. Circle **44.** Pentagon **45.** Rhombus

46. Isosceles triangle **47.** Regular hexagon **48.** Scalene triangle

49. **CUBE** *Explain* how the numbers of faces, vertices, and edges of a cube change when you cut off each feature.

a. A corner **b.** An edge **c.** A face **d.** 3 corners

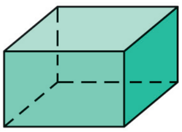
50. **TETRAHEDRON** *Explain* how the numbers of faces, vertices, and edges of a regular tetrahedron change when you cut off each feature.

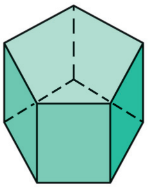
a. A corner **b.** An edge **c.** A face **d.** 2 edges

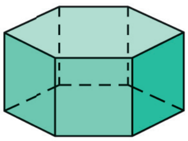
51. **CHALLENGE** The *angle defect D* at a vertex of a polyhedron is defined as follows:

$D = 360° - $ (sum of all angle measures at the vertex)

Verify that for the figures with regular bases below, $DV = 720$ where V is the number of vertices.





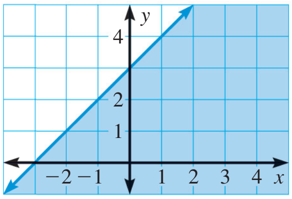


MIXED REVIEW FOR TAKS

TAKS PRACTICE at classzone.com

REVIEW
Skills Review Handbook p. 881; TAKS Workbook

52. **TAKS PRACTICE** Which inequality best describes the graph at the right? *TAKS Obj. 1*



Ⓐ $y \le x + 3$ Ⓑ $y \le 3x$

Ⓒ $y \le x - 3$ Ⓓ $y \le 3 - x$

REVIEW
TAKS Preparation p. 500; TAKS Workbook

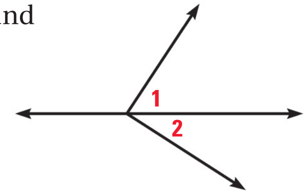
53. **TAKS PRACTICE** Zach spent \$240 on a stereo and compact discs. The stereo cost \$150. The compact discs cost \$15 each. Which equation can you use to determine c, the number of compact discs that Zach bought? *TAKS Obj. 10*

Ⓕ $15c - 150 = 240$ Ⓖ $15c + 150 = 240$

Ⓗ $15 - 150c = 240$ Ⓙ $15 + 150c = 240$

REVIEW
Lesson 1.5; TAKS Workbook

54. **TAKS PRACTICE** In the figure below, $m\angle 1 = 57°$ and $m\angle 2 = 33°$. Which statement is true? *TAKS Obj. 10*



Ⓐ $\angle 1$ and $\angle 2$ are congruent angles.

Ⓑ $\angle 1$ and $\angle 2$ are complementary angles.

Ⓒ $\angle 1$ and $\angle 2$ are supplementary angles.

Ⓓ $\angle 1$ and $\angle 2$ are vertical angles.

12.2 Investigate Surface Area

TEKS G.2.A, G.6.B, G.8.A. G.8.D

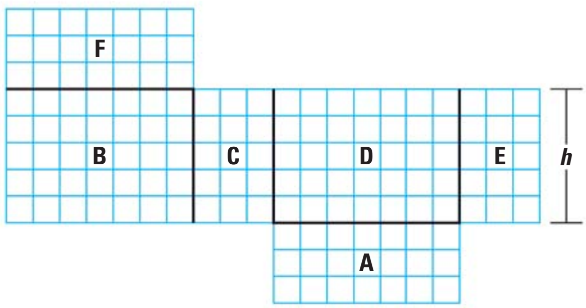
MATERIALS • graph paper • scissors • tape

QUESTION How can you find the surface area of a polyhedron?

A *net* is a pattern that can be folded to form a polyhedron. To find the *surface area* of a polyhedron, you can find the area of its net.

EXPLORE Create a polyhedron using a net

STEP 1 ***Draw a net*** Copy the net below on graph paper. Be sure to label the sections of the net.



STEP 2 ***Create a polyhedron*** Cut out the net and fold it along the black lines to form a polyhedron. Tape the edges together. Describe the polyhedron. Is it regular? Is it convex?

STEP 3 ***Find surface area*** The *surface area* of a polyhedron is the sum of the areas of its faces. Find the surface area of the polyhedron you just made. (Each square on the graph paper measures 1 unit by 1 unit.)

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Lay the net flat again and find the following measures.

   A: the area of Rectangle A

   P: the perimeter of Rectangle A

   h: the height of Rectangles B, C, D, and E

2. Use the values from Exercise 1 to find $2A + Ph$. *Compare* this value to the surface area you found in Step 3 above. What do you notice?

3. Make a conjecture about the surface area of a rectangular prism.

4. Use graph paper to draw the net of another rectangular prism. Fold the net to make sure that it forms a rectangular prism. Use your conjecture from Exercise 3 to calculate the surface area of the prism.

12.2 Surface Area of Prisms and Cylinders

TEKS G.6.B, G.8.C, G.8.D, G.9.D

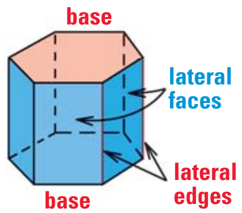
Before You found areas of polygons.

Now You will find the surface areas of prisms and cylinders.

Why? So you can find the surface area of a drum, as in Ex. 22.

Key Vocabulary
- **prism**
  lateral faces, lateral edges
- **surface area**
- **lateral area**
- **net**
- **right prism**
- **oblique prism**
- **cylinder**
- **right cylinder**

A **prism** is a polyhedron with two congruent faces, called *bases*, that lie in parallel planes. The other faces, called **lateral faces**, are parallelograms formed by connecting the corresponding vertices of the bases. The segments connecting these vertices are **lateral edges**. Prisms are classified by the shapes of their bases.



The **surface area** of a polyhedron is the sum of the areas of its faces. The **lateral area** of a polyhedron is the sum of the areas of its lateral faces.

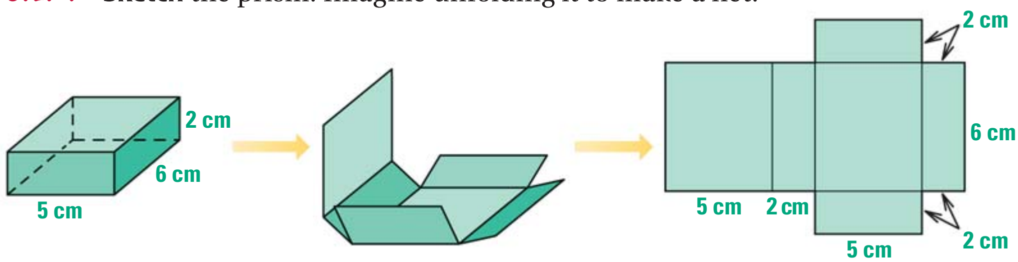
Imagine that you cut some edges of a polyhedron and unfold it. The two-dimensional representation of the faces is called a **net**. As you saw in the Activity on page 802, the surface area of a prism is equal to the area of its net.

EXAMPLE 1 Use the net of a prism

Find the surface area of a rectangular prism with height 2 centimeters, length 5 centimeters, and width 6 centimeters.

Solution

STEP 1 **Sketch** the prism. Imagine unfolding it to make a net.



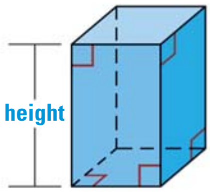
STEP 2 **Find** the areas of the rectangles that form the faces of the prism.

Congruent faces	Dimensions	Area of each face
Left and right faces	6 cm by 2 cm	$6 \cdot 2 = 12 \text{ cm}^2$
Front and back faces	5 cm by 2 cm	$5 \cdot 2 = 10 \text{ cm}^2$
Top and bottom faces	6 cm by 5 cm	$6 \cdot 5 = 30 \text{ cm}^2$

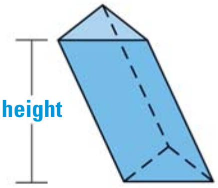
STEP 3 **Add** the areas of all the faces to find the surface area.

▶ The surface area of the prism is $S = 2(12) + 2(10) + 2(30) = 104 \text{ cm}^2$.

RIGHT PRISMS The height of a prism is the perpendicular distance between its bases. In a **right prism**, each lateral edge is perpendicular to both bases. A prism with lateral edges that are not perpendicular to the bases is an **oblique prism**.



Right rectangular prism



Oblique triangular prism

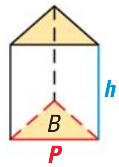
THEOREM — *For Your Notebook*

THEOREM 12.2 Surface Area of a Right Prism

The surface area S of a right prism is

$$S = 2B + Ph = aP + Ph,$$

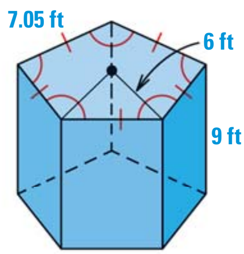
where a is the apothem of the base, B is the area of a base, P is the perimeter of a base, and h is the height.



$S = 2B + Ph = aP + Ph$

EXAMPLE 2 Find the surface area of a right prism

Find the surface area of the right pentagonal prism.



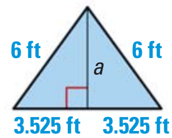
Solution

STEP 1 **Find** the perimeter and area of a base of the prism.

Each base is a regular pentagon.

Perimeter $P = 5(7.05) = 35.25$

Apothem $a = \sqrt{6^2 - 3.525^2} \approx 4.86$



REVIEW APOTHEM
For help with finding the apothem, see p. 762.

STEP 2 **Use** the formula for the surface area that uses the apothem.

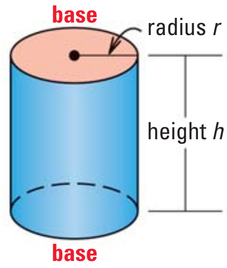
$S = aP + Ph$	**Surface area of a right prism**
$\approx (4.86)(35.25) + (35.25)(9)$	**Substitute known values.**
≈ 488.57	**Simplify.**

▶ The surface area of the right pentagonal prism is about 488.57 square feet.

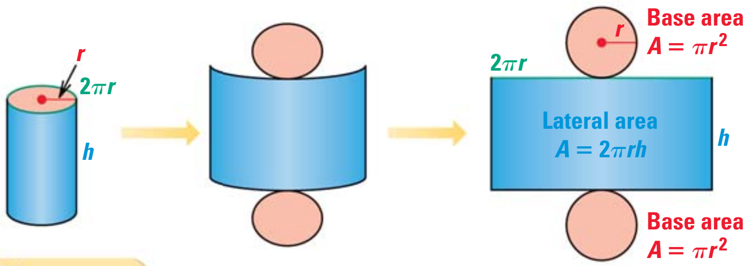
✓ GUIDED PRACTICE for Examples 1 and 2

1. Draw a net of a triangular prism.
2. Find the surface area of a right rectangular prism with height 7 inches, length 3 inches, and width 4 inches using (a) a net and (b) the formula for the surface area of a right prism.

CYLINDERS A **cylinder** is a solid with congruent circular bases that lie in parallel planes. The height of a cylinder is the perpendicular distance between its bases. The radius of a base is the *radius* of the cylinder. In a **right cylinder**, the segment joining the centers of the bases is perpendicular to the bases.



The lateral area of a cylinder is the area of its curved surface. It is equal to the product of the circumference and the height, or $2\pi rh$. The surface area of a cylinder is equal to the sum of the lateral area and the areas of the two bases.



Animated Geometry at classzone.com

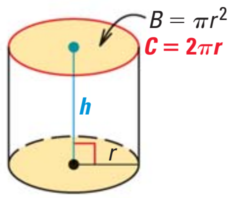
THEOREM — For Your Notebook

THEOREM 12.3 Surface Area of a Right Cylinder

The surface area S of a right cylinder is

$$S = 2B + Ch = 2\pi r^2 + 2\pi rh,$$

where B is the area of a base, C is the circumference of a base, r is the radius of a base, and h is the height.



$S = 2B + Ch = 2\pi r^2 + 2\pi rh$

EXAMPLE 3 Find the surface area of a cylinder

COMPACT DISCS You are wrapping a stack of 20 compact discs using a shrink wrap. Each disc is cylindrical with height 1.2 millimeters and radius 60 millimeters. What is the minimum amount of shrink wrap needed to cover the stack of 20 discs?

Solution

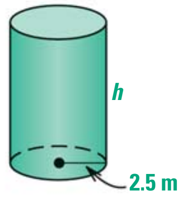
The 20 discs are stacked, so the height of the stack will be $20(1.2) = 24$ mm. The radius is 60 millimeters. The minimum amount of shrink wrap needed will be equal to the surface area of the stack of discs.

$S = 2\pi r^2 + 2\pi rh$	Surface area of a cylinder
$= 2\pi(60)^2 + 2\pi(60)(24)$	Substitute known values.
$\approx 31{,}667$	Use a calculator.

▸ You will need at least 31,667 square millimeters, or about 317 square centimeters of shrink wrap.

EXAMPLE 4 Find the height of a cylinder

Find the height of the right cylinder shown, which has a surface area of 157.08 square meters.



Solution

Substitute known values in the formula for the surface area of a right cylinder and solve for the height h.

$S = 2\pi r^2 + 2\pi rh$	Surface area of a cylinder
$157.08 = 2\pi(2.5)^2 + 2\pi(2.5)h$	Substitute known values.
$157.08 = 12.5\pi + 5\pi h$	Simplify.
$157.08 - 12.5\pi = 5\pi h$	Subtract 12.5π from each side.
$117.81 \approx 5\pi h$	Simplify. Use a calculator.
$7.5 \approx h$	Divide each side by 5π.

▶ The height of the cylinder is about 7.5 meters.

✓ GUIDED PRACTICE for Examples 3 and 4

3. Find the surface area of a right cylinder with height 18 centimeters and radius 10 centimeters. Round your answer to two decimal places.

4. Find the radius of a right cylinder with height 5 feet and surface area 208π square feet.

12.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 9, and 23

★ = **TAKS PRACTICE AND REASONING** Exs. 17, 24, 25, 26, and 31

SKILL PRACTICE

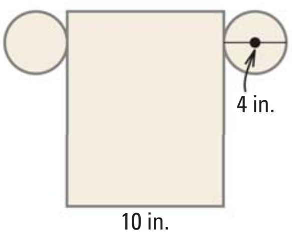
1. VOCABULARY Sketch a triangular prism. Identify its *bases*, *lateral faces*, and *lateral edges*.

2. WRITING *Explain* how the formula $S = 2B + Ph$ applies to finding the surface area of both a right prism and a right cylinder.

EXAMPLE 1 on p. 803 for Exs. 3–5

USING NETS Find the surface area of the solid formed by the net. Round your answer to two decimal places.

3.



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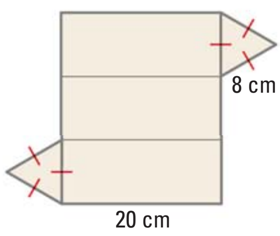

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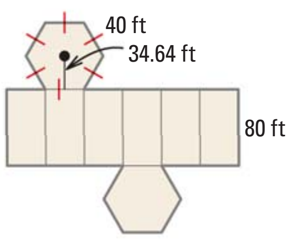
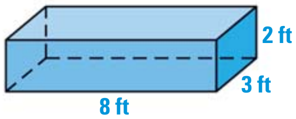


EXAMPLE 2 on p. 804 for Exs. 6–8

SURFACE AREA OF A PRISM **Find the surface area of the right prism. Round your answer to two decimal places.**

6.



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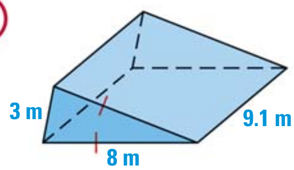
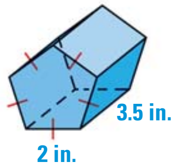


8.



EXAMPLE 3 on p. 805 for Exs. 9–12

SURFACE AREA OF A CYLINDER **Find the surface area of the right cylinder using the given radius *r* and height *h*. Round your answer to two decimal places.**

9.

$r = 0.8$ in.
$h = 2$ in.

10.

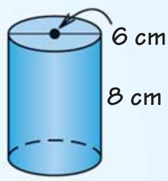
$r = 12$ mm
$h = 40$ mm

11.

$r = 8$ in.
$h = 8$ in.

12. **ERROR ANALYSIS** *Describe* and correct the error in finding the surface area of the right cylinder.

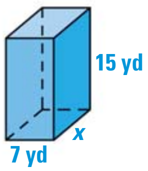
$$S = 2\pi(6^2) + 2\pi(6)(8)$$
$$= 2\pi(36) + 2\pi(48)$$
$$= 168\pi$$
$$\approx 528 \text{ cm}^2$$



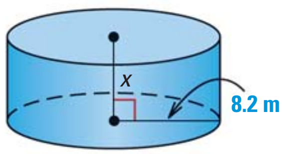
EXAMPLE 4 on p. 806 for Exs. 13–15

ALGEBRA **Solve for *x* given the surface area *S* of the right prism or right cylinder. Round your answer to two decimal places.**

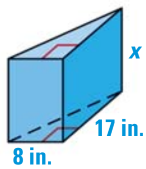
13. $S = 606 \text{ yd}^2$



14. $S = 1097 \text{ m}^2$



15. $S = 616 \text{ in.}^2$



16. **SURFACE AREA OF A PRISM** A triangular prism with a right triangular base has leg length 9 units and hypotenuse length 15 units. The height of the prism is 8 units. Sketch the prism and find its surface area.

17. **TAKS REASONING** The length of each side of a cube is multiplied by 3. What is the change in the surface area of the cube?

Ⓐ The surface area is 3 times the original surface area.

Ⓑ The surface area is 6 times the original surface area.

Ⓒ The surface area is 9 times the original surface area.

Ⓓ The surface area is 27 times the original surface area.

18. **SURFACE AREA OF A CYLINDER** The radius and height of a right cylinder are each divided by $\sqrt{5}$. What is the change in surface area of the cylinder?

19. **SURFACE AREA OF A PRISM** Find the surface area of a right hexagonal prism with all edges measuring 10 inches.

20. **HEIGHT OF A CYLINDER** Find the height of a cylinder with a surface area of 108π square meters. The radius of the cylinder is twice the height.

21. **CHALLENGE** The *diagonal* of a cube is a segment whose endpoints are vertices that are not on the same face. Find the surface area of a cube with diagonal length 8 units. Round your answer to two decimal places.

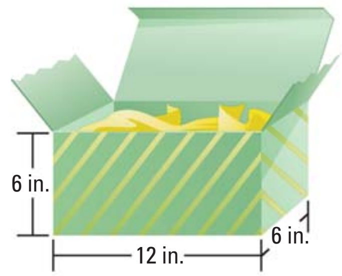
PROBLEM SOLVING

EXAMPLE 3 on p. 805 for Ex. 22

22. **BASS DRUM** A bass drum has a diameter of 20 inches and a depth of 8 inches. Find the surface area of the drum.

TEXAS @HomeTutor for problem solving help at classzone.com

23. **GIFT BOX** An open gift box is shown at the right. When the gift box is closed, it has a length of 12 inches, a width of 6 inches, and a height of 6 inches.



   a. What is the minimum amount of wrapping paper needed to cover the closed gift box?
   b. Why is the area of the net of the box larger than the amount of paper found in part (a)?
   c. When wrapping the box, why would you want more paper than the amount found in part (a)?

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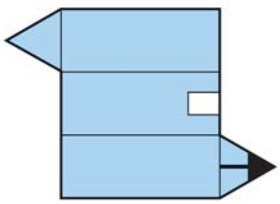
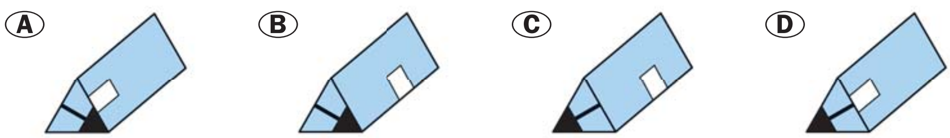
24. **TAKS REASONING** A right cylinder has a radius of 4 feet and height of 10 feet.
   a. Find the surface area of the cylinder.
   b. Suppose you can either *double the radius* or *double the height*. Which do you think will create a greater surface area?
   c. Check your answer in part (b) by calculating the new surface areas.

25. **TAKS REASONING** Which three-dimensional figure does the net represent?

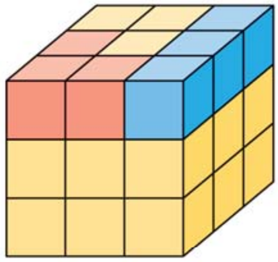




○ = WORKED-OUT SOLUTIONS on p. WS1
= TAKS PRACTICE AND REASONING

26. **TAKS REASONING** A company makes two types of recycling bins. One type is a right rectangular prism with length 14 inches, width 12 inches, and height 36 inches. The other type is a right cylinder with radius 6 inches and height 36 inches. Both types of bins are missing a base, so the bins have one open end. Which recycle bin requires more material to make? *Explain*.

27. **MULTI-STEP PROBLEM** Consider a cube that is built using 27 unit cubes as shown at the right.

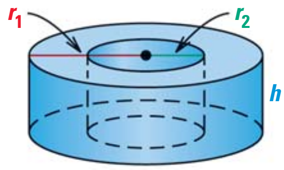


a. Find the surface area of the solid formed when the red unit cubes are removed from the solid shown.

b. Find the surface area of the solid formed when the blue unit cubes are removed from the solid shown.

c. Why are your answers different in parts (a) and (b)? *Explain*.

28. **SURFACE AREA OF A RING** The ring shown is a right cylinder of radius r_1 with a cylindrical hole of r_2. The ring has height h.

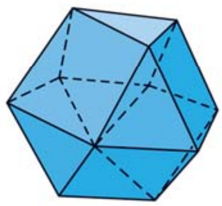


a. Find the surface area of the ring if r_1 is 12 meters, r_2 is 6 meters, and h is 8 meters. Round your answer to two decimal places.

b. Write a formula that can be used to find the surface area S of any cylindrical ring where $0 < r_2 < r_1$.

29. **DRAWING SOLIDS** A cube with edges 1 foot long has a cylindrical hole with diameter 4 inches drilled through one of its faces. The hole is drilled perpendicular to the face and goes completely through to the other side. Draw the figure and find its surface area.

30. **CHALLENGE** A cuboctahedron has 6 square faces and 8 equilateral triangle faces, as shown. A cuboctahedron can be made by slicing off the corners of a cube.



a. Sketch a net for the cuboctahedron.

b. Each edge of a cuboctahedron has a length of 5 millimeters. Find its surface area.

MIXED REVIEW FOR TAKS

TAKS PRACTICE at classzone.com

REVIEW
Skills Review Handbook p. 885; TAKS Workbook

31. **TAKS PRACTICE** Catherine and her friends are having dinner at a restaurant. The total cost of their dinner is \$59.82 including tax. Catherine and her friends have a total of \$70. They want to leave a tip that is 20% of the total bill. Is \$70 enough to cover the bill and the 20% tip?
TAKS Obj. 10

Ⓐ No, they need \$0.25 more.

Ⓑ No, they need \$1.78 more.

Ⓒ Yes, and they have \$4.20 left over.

Ⓓ Yes, they have the exact amount.

12.3 Surface Area of Pyramids and Cones

TEKS G.6.B, G.8.B, G.8.D, G.9.D

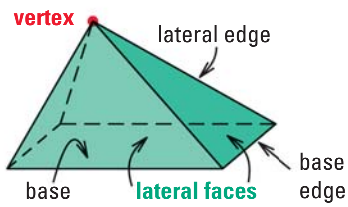
Before You found surface areas of prisms and cylinders.

Now You will find surface areas of pyramids and cones.

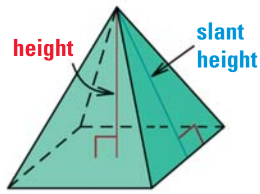
Why? So you can find the surface area of a volcano, as in Ex. 33.

Key Vocabulary
- **pyramid**
- **vertex of a pyramid**
- **regular pyramid**
- **slant height**
- **cone**
- **vertex of a cone**
- **right cone**
- **lateral surface**

A **pyramid** is a polyhedron in which the base is a polygon and the lateral faces are triangles with a common vertex, called the **vertex of the pyramid**. The intersection of two lateral faces is a *lateral edge*. The intersection of the base and a lateral face is a *base edge*. The height of the pyramid is the perpendicular distance between the base and the vertex.



Pyramid



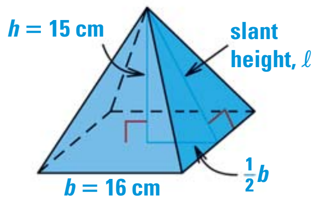
Regular pyramid

NAME PYRAMIDS
Pyramids are classified by the shapes of their bases.

A **regular pyramid** has a regular polygon for a base and the segment joining the vertex and the center of the base is perpendicular to the base. The lateral faces of a regular pyramid are congruent isosceles triangles. The **slant height** of a regular pyramid is the height of a lateral face of the regular pyramid. A nonregular pyramid does not have a slant height.

EXAMPLE 1 Find the area of a lateral face of a pyramid

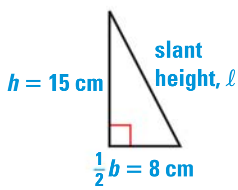
A regular square pyramid has a height of 15 centimeters and a base edge length of 16 centimeters. Find the area of each lateral face of the pyramid.



Solution

Use the Pythagorean Theorem to find the slant height ℓ.

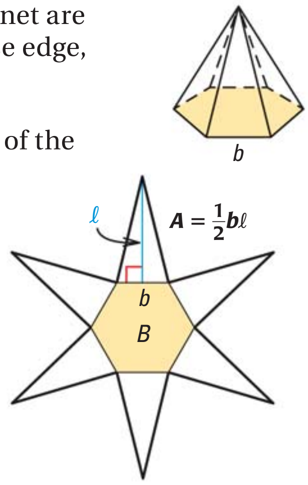
$\ell^2 = h^2 + \left(\frac{1}{2}b\right)^2$	**Write formula.**
$\ell^2 = 15^2 + 8^2$	**Substitute for h and $\frac{1}{2}b$.**
$\ell^2 = 289$	**Simplify.**
$\ell = 17$	**Find the positive square root.**



▶ The area of each triangular face is $A = \frac{1}{2}b\ell = \frac{1}{2}(16)(17) = 136$ square centimeters.

SURFACE AREA A regular hexagonal pyramid and its net are shown at the right. Let b represent the length of a base edge, and let ℓ represent the slant height of the pyramid.

The area of each lateral face is $\frac{1}{2}b\ell$ and the perimeter of the base is $P = 6b$. So, the surface area S is as follows.



$S =$ (Area of base) $+ 6$(Area of lateral face)

$S = B + 6\left(\frac{1}{2}b\ell\right)$ **Substitute.**

$S = B + \frac{1}{2}(6b)\ell$ **Rewrite $6\left(\frac{1}{2}b\ell\right)$ as $\frac{1}{2}(6b)\ell$.**

$S = B + \frac{1}{2}P\ell$ **Substitute P for $6b$.**

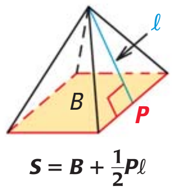
THEOREM *For Your Notebook*

THEOREM 12.4 Surface Area of a Regular Pyramid

The surface area S of a regular pyramid is

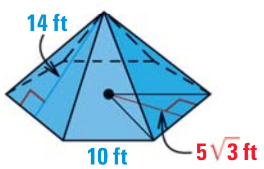
$$S = B + \frac{1}{2}P\ell,$$

where B is the area of the base, P is the perimeter of the base, and ℓ is the slant height.



EXAMPLE 2 Find the surface area of a pyramid

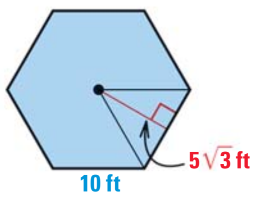
Find the surface area of the regular hexagonal pyramid.



Solution

REVIEW AREA
For help with finding the area of regular polygons, see p. 762.

First, find the area of the base using the formula for the area of a regular polygon, $\frac{1}{2}aP$. The apothem a of the hexagon is $5\sqrt{3}$ feet and the perimeter P is $6 \cdot 10 = 60$ feet. So, the area of the base B is $\frac{1}{2}(5\sqrt{3})(60) = 150\sqrt{3}$ square feet. Then, find the surface area.



$S = B + \frac{1}{2}P\ell$ **Formula for surface area of regular pyramid**

$= 150\sqrt{3} + \frac{1}{2}(60)(14)$ **Substitute known values.**

$= 150\sqrt{3} + 420$ **Simplify.**

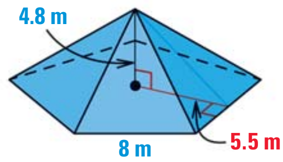
≈ 679.81 **Use a calculator.**

▶ The surface area of the regular hexagonal pyramid is about 679.81 ft^2.

✓ GUIDED PRACTICE for Examples 1 and 2

1. Find the area of each lateral face of the regular pentagonal pyramid shown.

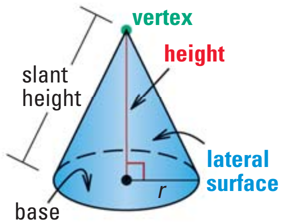
2. Find the surface area of the regular pentagonal pyramid shown.



CONES A **cone** has a circular base and a **vertex** that is not in the same plane as the base. The radius of the base is the *radius* of the cone. The height is the perpendicular distance between the vertex and the base.

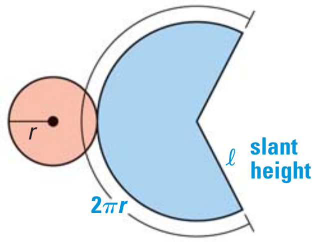
In a **right cone**, the segment joining the vertex and the center of the base is perpendicular to the base and the slant height is the distance between the vertex and a point on the base edge.

The **lateral surface** of a cone consists of all segments that connect the vertex with points on the base edge.



Right cone

SURFACE AREA When you cut along the slant height and base edge and lay a right cone flat, you get the net shown at the right.



The circular base has an area of πr^2 and the lateral surface is the sector of a circle. You can use a proportion to find the area of the sector, as shown below.

$$\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\text{Arc length}}{\text{Circumference of circle}}$$ **Set up proportion.**

$$\frac{\text{Area of sector}}{\pi \ell^2} = \frac{2\pi r}{2\pi \ell}$$ **Substitute.**

$$\text{Area of sector} = \pi \ell^2 \cdot \frac{2\pi r}{2\pi \ell}$$ **Multiply each side by $\pi \ell^2$.**

$$\text{Area of sector} = \pi r \ell$$ **Simplify.**

The surface area of a cone is the sum of the base area, πr^2, and the lateral area, $\pi r \ell$. Notice that the quantity $\pi r \ell$ can be written as $\frac{1}{2}(2\pi r)\ell$, or $\frac{1}{2}C\ell$.

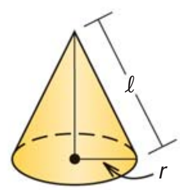
THEOREM *For Your Notebook*

THEOREM 12.5 Surface Area of a Right Cone

The surface area S of a right cone is

$$S = B + \frac{1}{2}C\ell = \pi r^2 + \pi r \ell,$$

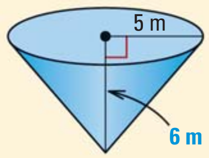
where B is the area of the base, C is the circumference of the base, r is the radius of the base, and ℓ is the slant height.



$S = B + \frac{1}{2}C\ell = \pi r^2 + \pi r \ell$

EXAMPLE 3 TAKS PRACTICE: Multiple Choice

A right cone is shown. What is the approximate surface area of the cone?

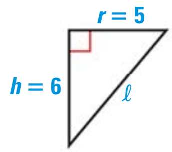


(A) 122.7 m^2 (B) 172.8 m^2

(C) 201.2 m^2 (D) 314.3 m^2

Solution

To find the slant height ℓ of the right cone, use the Pythagorean Theorem.



$\ell^2 = h^2 + r^2$ **Write formula.**

$\ell^2 = 6^2 + 5^2$ **Substitute.**

$\ell \approx 7.81$ **Find positive square root.**

Use the formula for the surface area of a right cone.

$S = \pi r^2 + \pi r\ell$ **Theorem 12.5**

$5 = (5^2) + \pi(5)(7.81)$ **Substitute.**

≈ 201.2 **Simplify and use a calculator.**

▶ So, the correct answer is C. (A) (B) (C) (D)

EXAMPLE 4 Find the lateral area of a cone

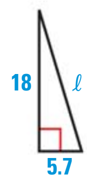
TRAFFIC CONE The traffic cone can be approximated by a right cone with radius 5.7 inches and height 18 inches. Find the approximate lateral area of the traffic cone.

Solution

To find the slant height ℓ, use the Pythagorean Theorem.

$\ell^2 = 18^2 + (5.7)^2$, so $\ell \approx 18.9$ inches.

Find the lateral area.



Lateral area $= \pi r\ell$ **Write formula.**

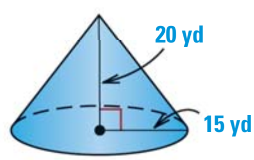
$= \pi(5.7)(18.9)$ **Substitute known values.**

≈ 338.4 **Simplify and use a calculator.**

▶ The lateral area of the traffic cone is about 338.4 square inches.

✓ GUIDED PRACTICE for Examples 3 and 4

3. Find the lateral area of the right cone shown.
4. Find the surface area of the right cone shown.



12.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 11, and 29

★ = **TAKS PRACTICE AND REASONING** Exs. 17, 31, 35, and 36

SKILL PRACTICE

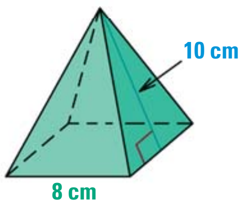
1. **VOCABULARY** Draw a regular square pyramid. Label its *height*, *slant height*, and *base*.

2. **WRITING** *Compare* the height and slant height of a right cone.

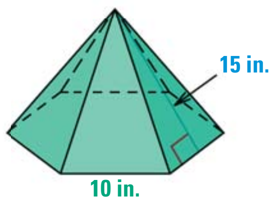
EXAMPLE 1 on p. 810 for Exs. 3–5

AREA OF A LATERAL FACE **Find the area of each lateral face of the regular pyramid.**

3.



4.



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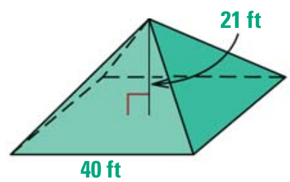
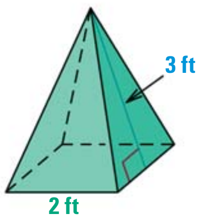


EXAMPLE 2 on p. 811 for Exs. 6–9

SURFACE AREA OF A PYRAMID **Find the surface area of the regular pyramid. Round your answer to two decimal places.**

6.



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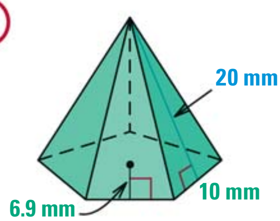

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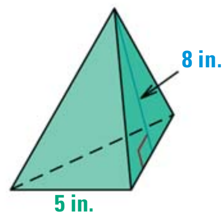
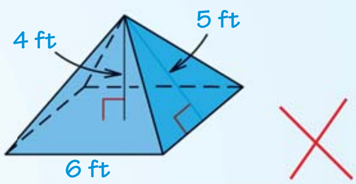


9. **ERROR ANALYSIS** *Describe* and correct the error in finding the surface area of the regular pyramid.

$$S = B + \frac{1}{2}P\ell$$
$$= 6^2 + \frac{1}{2}(24)(4)$$
$$= 84 \text{ ft}^2$$



EXAMPLES 3 and 4 on p. 813 for Exs. 10–17

LATERAL AREA OF A CONE **Find the lateral area of the right cone. Round your answer to two decimal places.**

10.

$r = 7.5$ cm
$h = 25$ cm

11.

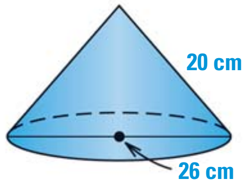
$r = 1$ in.
$h = 4$ in.

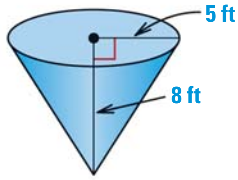
12.

$d = 7$ in.
$h = 1$ ft

SURFACE AREA OF A CONE **Find the surface area of the right cone. Round your answer to two decimal places.**

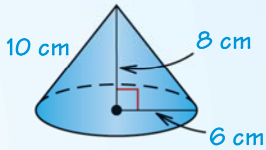
13.


14.


15.


16. **ERROR ANALYSIS** *Describe* and correct the error in finding the surface area of the right cone.

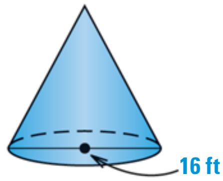
$$S = \pi(r^2) + \pi r^2 \ell$$
$$= \pi(36) + \pi(36)(10)$$
$$= 396\pi \text{ cm}^2$$



17. **TAKS REASONING** The surface area of the right cone is 200π square feet. What is the slant height of the cone?

Ⓐ 10.5 ft Ⓑ 17 ft

Ⓒ 23 ft Ⓓ 24 ft



VISUAL REASONING **In Exercises 18–21, sketch the described solid and find its surface area. Round your answer to two decimal places.**

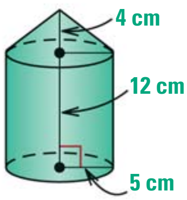
18. A right cone has a radius of 15 feet and a slant height of 20 feet.

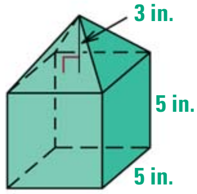
19. A right cone has a diameter of 16 meters and a height of 30 meters.

20. A regular pyramid has a slant height of 24 inches. Its base is an equilateral triangle with a base edge length of 10 inches.

21. A regular pyramid has a hexagonal base with a base edge length of 6 centimeters and a slant height of 9 centimeters.

COMPOSITE SOLIDS **Find the surface area of the solid. The pyramids are regular and the cones are right. Round your answers to two decimal places, if necessary.**

22.


23.


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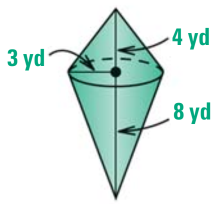


25. **TETRAHEDRON** Find the surface area of a regular tetrahedron with edge length 4 centimeters.

26. **CHALLENGE** A right cone with a base of radius 4 inches and a regular pyramid with a square base both have a slant height of 5 inches. Both solids have the same surface area. Find the length of a base edge of the pyramid. Round your answer to the nearest hundredth of an inch.

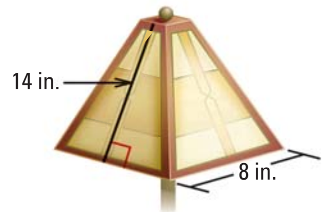
PROBLEM SOLVING

EXAMPLE 2
on p. 811
for Ex. 27

27. CANDLES A candle is in the shape of a regular square pyramid with base edge length 6 inches. Its height is 4 inches. Find its surface area.

TEXAS @HomeTutor for problem solving help at classzone.com

28. LAMPSHADE A glass lampshade is shaped like a regular square pyramid.



a. Approximate the lateral area of the lampshade shown.

b. *Explain* why your answer to part (a) is not the exact lateral area.

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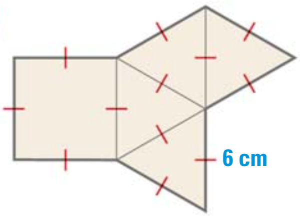
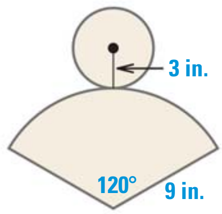
USING NETS **Name the figure that is represented by the net. Then find its surface area. Round your answer to two decimal places.**

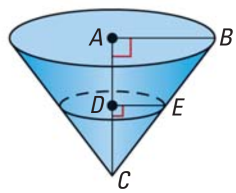
29.



30.



31. **TAKS REASONING** In the figure, $AC = 4$, $AB = 3$, and $DC = 2$.

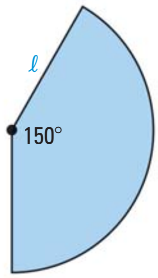


a. Prove $\triangle ABC \sim \triangle DEC$.

b. Find BC, DE, and EC.

c. Find the surface areas of the larger cone and the smaller cone in terms of π. *Compare* the surface areas using a percent.

32. MULTI-STEP PROBLEM The sector shown can be rolled to form the lateral surface of a right cone. The lateral surface area of the cone is 20 square meters.



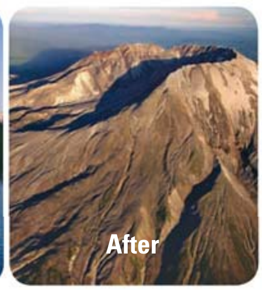
a. Write the formula for the area of a sector.

b. Use the formula in part (a) to find the slant height of the cone. *Explain* your reasoning.

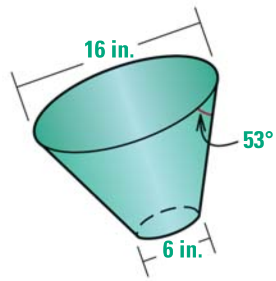
c. Find the radius and height of the cone.

33. VOLCANOES Before 1980, Mount St. Helens was a conic volcano with a height from its base of about 1.08 miles and a base radius of about 3 miles. In 1980, the volcano erupted, reducing its height to about 0.83 mile.

Approximate the lateral area of the volcano after 1980. (*Hint:* The ratio of the radius of the destroyed cone-shaped top to its height is the same as the ratio of the radius of the original volcano to its height.)



34. CHALLENGE An *Elizabethan collar* is used to prevent an animal from irritating a wound. The angle between the opening with a 16 inch diameter and the side of the collar is 53°. Find the surface area of the collar shown.

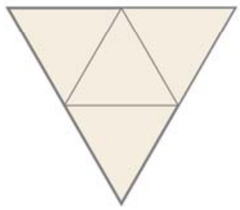


MIXED REVIEW FOR TAKS

TAKS PRACTICE at classzone.com

REVIEW
Lesson 12.3;
TAKS Workbook

35. TAKS PRACTICE Which is closest to the surface area of the net of the right triangular pyramid shown? Use a ruler to measure its dimensions. *TAKS Obj. 8*

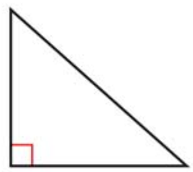


Ⓐ 1 cm^2 Ⓑ 3 cm^2

Ⓒ 4 cm^2 Ⓓ 6 cm^2

REVIEW
Lesson 7.2
TAKS Workbook

36. TAKS PRACTICE Which of the following could be the dimensions of the triangle shown? *TAKS Obj. 7*



Ⓕ 3.5 inches, 4.5 inches, 5.5 inches

Ⓖ 13 feet, 14 feet, 15 feet

Ⓗ 6.2 yards, 8.6 yards, 14.8 yards

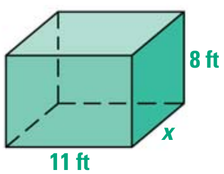
Ⓙ 3.36 meters, 3.77 meters, 5.05 meters

QUIZ *for Lessons 12.1–12.3*

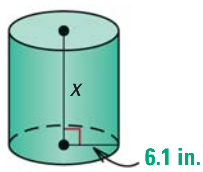
1. A polyhedron has 8 vertices and 12 edges. How many faces does the polyhedron have? *(p. 794)*

Solve for x given the surface area S of the right prism or right cylinder. Round your answer to two decimal places. *(p. 803)*

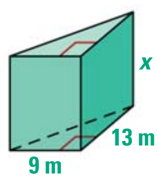
2. $S = 366 \text{ ft}^2$



3. $S = 717 \text{ in.}^2$

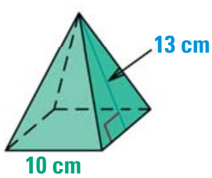


4. $S = 567 \text{ m}^2$

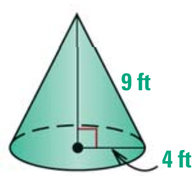


Find the surface area of the regular pyramid or right cone. Round your answer to two decimal places. *(p. 810)*

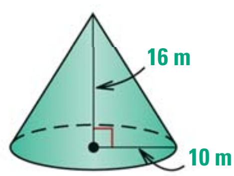
5.



6.



7.



MIXED REVIEW FOR TEKS

Lessons 12.1–12.3

MULTIPLE CHOICE

1. **RIGHT CONE** A right cone is shown. What is the approximate surface area of the cone? *TEKS G.8.D*

Ⓐ 527 in.^{2} Ⓑ 537.3 in.^{2}

Ⓒ 632.4 in.^{2} Ⓓ 1264.7 in.^{2}

2. **PENCILS** Some pencils are made from slats of wood that are machined into right regular hexagonal prisms. Let r represent the side length of a base and let h represent the height of the pencil. Which formula can be used to find the surface area of a new unsharpened pencil? *TEKS G.8.D*

Ⓕ $S = 3\sqrt{3}\, r^2 + 6rh$

Ⓖ $S = 6\sqrt{3}\, r^2 + 6rh$

Ⓗ $S = 6r^2 + 6rh$

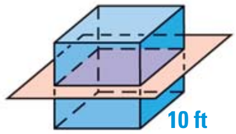
Ⓙ $S = \frac{\sqrt{3}\, r^2}{2} + 6rh$

3. **ALGEBRA** A right cone has a base radius of $3x$ units and a height of $4x$ units. The surface area of the cone is 1944π square units. What is the value of x? *TEKS G.8.D*

Ⓐ 9 Ⓑ $\sqrt{\frac{648}{7}}$

Ⓒ $6\sqrt{3}$ Ⓓ $\sqrt{\frac{648}{5}}$

4. **CROSS SECTION** The figure shows a plane intersecting a cube. The cube has a side length of 10 feet. What is the area of the cross section? *TEKS G.8.D*

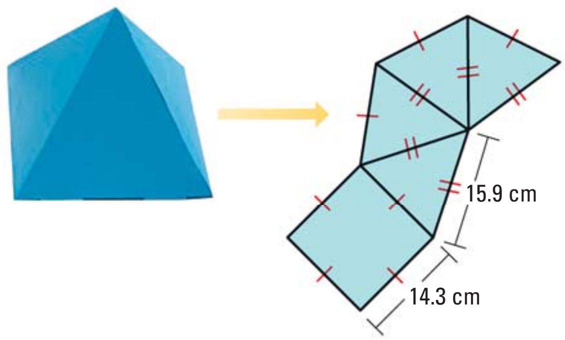


Ⓕ 40 ft^2 Ⓖ 50 ft^2

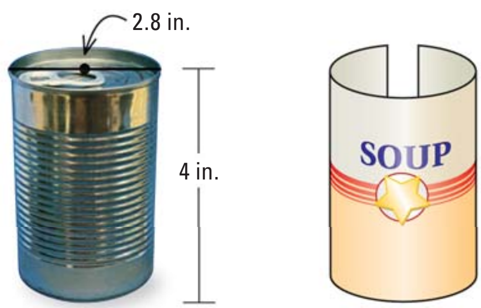
Ⓗ 100 ft^2 Ⓙ 200 ft^2

GRIDDED ANSWER

5. **GIFT BOX** Teresa unfolds a paper gift box shaped like a right square pyramid as shown. What is the area of the unfolded box? Round your answer to the nearest square centimeter. *TEKS G.8.D*



6. **SOUP CAN** The amount of paper needed for a soup can label is approximately equal to the lateral area of the can. Find the lateral area of the can in square inches. Round your answer to two decimal places. *TEKS G.8.D*



12.4 Volume of Prisms and Cylinders

TEKS G.1.A, G.1.B, G.6.A, G.8.D

Before You found surface areas of prisms and cylinders.

Now You will find volumes of prisms and cylinders.

Why So you can determine volume of water in an aquarium, as in Ex. 33.

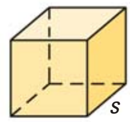
Key Vocabulary
- volume

The **volume** of a solid is the number of cubic units contained in its interior. Volume is measured in cubic units, such as cubic centimeters (cm^3).

POSTULATES *For Your Notebook*

POSTULATE 27 Volume of a Cube Postulate

The volume of a cube is the cube of the length of its side.



$V = s^3$

POSTULATE 28 Volume Congruence Postulate

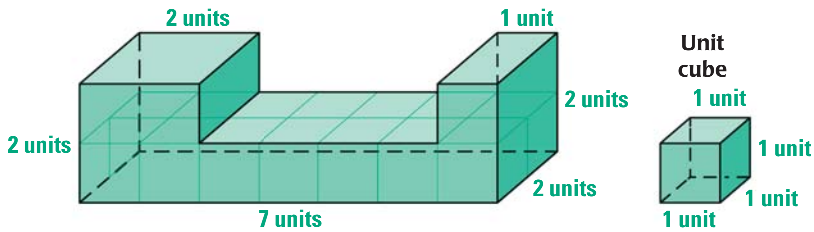
If two polyhedra are congruent, then they have the same volume.

POSTULATE 29 Volume Addition Postulate

The volume of a solid is the sum of the volumes of all its nonoverlapping parts.

EXAMPLE 1 Find the number of unit cubes

3-D PUZZLE **Find the volume of the puzzle piece in cubic units.**



Solution

To find the volume, find the number of unit cubes it contains. Separate the piece into three rectangular boxes as follows:

The *base* is 7 units by 2 units. So, it contains 7 • 2, or 14 unit cubes.

The *upper left box* is 2 units by 2 units. So, it contains 2 • 2, or 4 unit cubes.

The *upper right box* is 1 unit by 2 units. So, it contains 1 • 2, or 2 unit cubes.

▶ By the Volume Addition Postulate, the total volume of the puzzle piece is $14 + 4 + 2 = 20$ cubic units.

VOLUME FORMULAS The volume of any right prism or right cylinder can be found by multiplying the area of its base by its height.

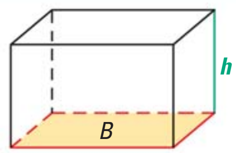
THEOREMS — *For Your Notebook*

THEOREM 12.6 Volume of a Prism

The volume V of a prism is

$V = Bh,$

where B is the area of a base and h is the height.



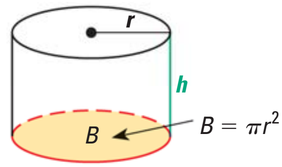
$V = Bh$

THEOREM 12.7 Volume of a Cylinder

The volume V of a cylinder is

$V = Bh = \pi r^2 h,$

where B is the area of a base, h is the height, and r is the radius of a base.

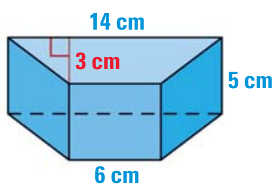


$V = Bh = \pi r^2 h$

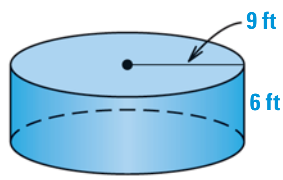
EXAMPLE 2 Find volumes of prisms and cylinders

Find the volume of the solid.

a. Right trapezoidal prism



b. Right cylinder



Solution

REVIEW AREA
For help with finding the area of a trapezoid, see p. 730.

a. The area of a base is $\frac{1}{2}(3)(6 + 14) = 30 \text{ cm}^2$ and $h = 5$ cm.

$V = Bh = 30(5) = 150 \text{ cm}^3$

b. The area of the base is $\pi \cdot 9^2$, or $81\pi \text{ ft}^2$. Use $h = 6$ ft to find the volume.

$V = Bh = 81\pi(6) = 486\pi \approx 1526.81 \text{ ft}^3$

EXAMPLE 3 Use volume of a prism

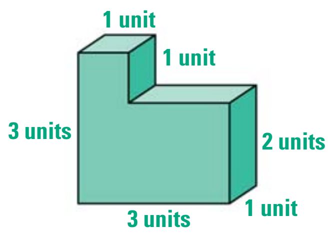
xy ALGEBRA The volume of the cube is 90 cubic inches. Find the value of x.

Solution

A side length of the cube is x inches.

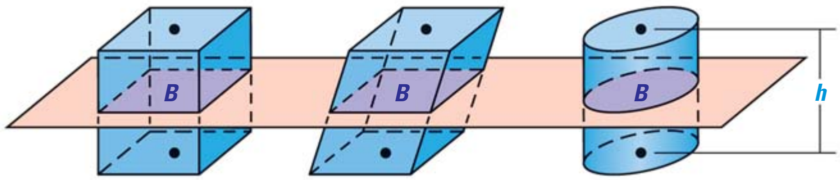
$V = x^3$	Formula for volume of a cube
$90 \text{ in.}^3 = x^3$	Substitute for V.
$4.48 \text{ in.} \approx x$	Find the cube root.

✓ GUIDED PRACTICE for Examples 1, 2, and 3



1. Find the volume of the puzzle piece shown in cubic units.
2. Find the volume of a square prism that has a base edge length of 5 feet and a height of 12 feet.
3. The volume of a right cylinder is 684π cubic inches and the height is 18 inches. Find the radius.

USING CAVALIERI'S PRINCIPLE Consider the solids below. All three have equal heights h and equal cross-sectional areas B. Mathematician Bonaventura Cavalieri (1598–1647) claimed that all three of the solids have the same volume. This principle is stated below.



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THEOREM — For Your Notebook

THEOREM 12.8 Cavalieri's Principle

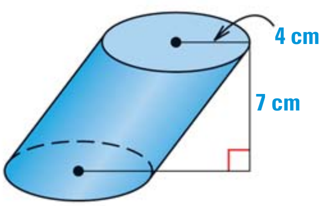
If two solids have the same height and the same cross-sectional area at every level, then they have the same volume.

EXAMPLE 4 Find the volume of an oblique cylinder

APPLY THEOREMS

Cavalieri's Principle tells you that the volume formulas on page 820 work for oblique prisms and cylinders.

Find the volume of the oblique cylinder.



Solution

Cavalieri's Principle allows you to use Theorem 12.7 to find the volume of the oblique cylinder.

$V = \pi r^2 h$	Formula for volume of a cylinder
$= \pi(4^2)(7)$	Substitute known values.
$= 112\pi$	Simplify.
≈ 351.86	Use a calculator.

▶ The volume of the oblique cylinder is about 351.86 cm^3.

EXAMPLE 5 Solve a real-world problem

SCULPTURE The sculpture is made up of 13 beams. In centimeters, suppose the dimensions of each beam are 30 by 30 by 90. Find its volume.

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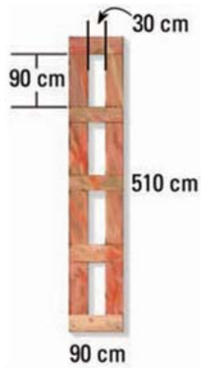
ANOTHER WAY
For alternative methods for solving the problem in Example 5, turn to page 826 for the **Problem Solving Workshop**.

Solution

The area of the base B can be found by subtracting the area of the small rectangles from the area of the large rectangle.



B = Area of large rectangle − 4 • Area of small rectangle

$= 90 \cdot 510 - 4(30 \cdot 90)$

$= 35{,}100 \text{ cm}^2$

Use the formula for the volume of a prism.

$V = Bh$ **Formula for volume of a prism**

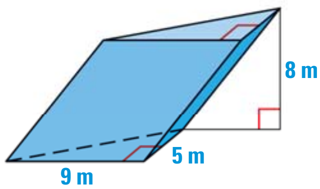
$= 35{,}100(30)$ **Substitute.**

$= 1{,}053{,}000 \text{ cm}^3$ **Simplify.**

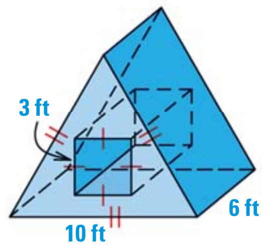
▶ The volume of the sculpture is $1{,}053{,}000 \text{ cm}^3$, or 1.053 m^3.

✓ GUIDED PRACTICE for Examples 4 and 5

4. Find the volume of the oblique prism shown below.



5. Find the volume of the solid shown below.



12.4 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 11, and 29
★ = **TAKS PRACTICE AND REASONING** Exs. 3, 21, 33, 35, and 36

SKILL PRACTICE

1. VOCABULARY In what type of units is the volume of a solid measured?

2. WRITING Two solids have the same surface area. Do they have the same volume? *Explain* your reasoning.

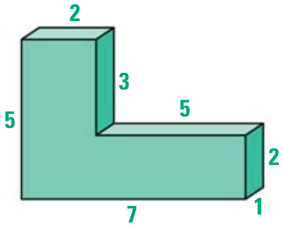
EXAMPLE 1 on p. 819 for Exs. 3–6

3. ★ **TAKS REASONING** How many 3 inch cubes can fit completely in a box that is 15 inches long, 9 inches wide, and 3 inches tall?

Ⓐ 15   Ⓑ 45   Ⓒ 135   Ⓓ 405

USING UNIT CUBES **Find the volume of the solid by determining how many unit cubes are contained in the solid.**

4.



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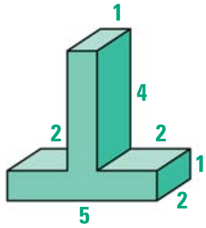

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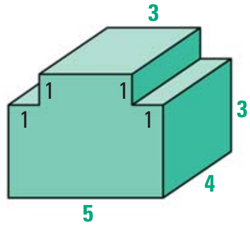
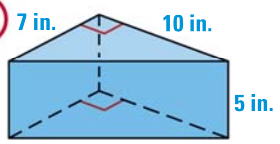


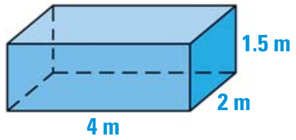
EXAMPLE 2
on p. 820
for Exs. 7–13

FINDING VOLUME **Find the volume of the right prism or right cylinder. Round your answer to two decimal places.**

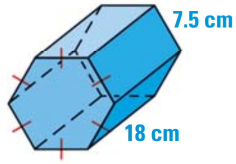
7.



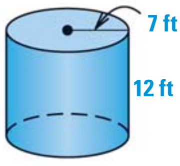
8.



9.



10.



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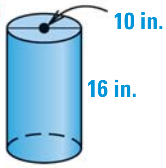

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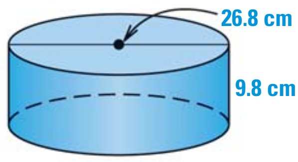


13. **ERROR ANALYSIS** *Describe* and correct the error in finding the volume of a right cylinder with radius 4 feet and height 3 feet.

$V = 2\pi rh$

$= 2\pi(4)(3)$

$= 24\pi\ \text{ft}^3$

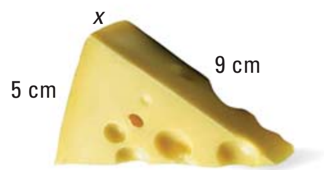
14. **FINDING VOLUME** Sketch a rectangular prism with height 3 feet, width 11 inches, and length 7 feet. Find its volume.

EXAMPLE 3
on p. 820
for Exs. 15–17

ALGEBRA **Find the length x using the given volume V.**

15. $V = 1000\ \text{in.}^3$

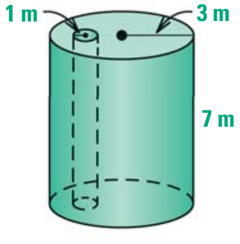
16. $V = 45\ \text{cm}^3$



17. $V = 128\pi\ \text{in.}^3$

COMPOSITE SOLIDS **Find the volume of the solid. The prisms and cylinders are right. Round your answer to two decimal places, if necessary.**

18.



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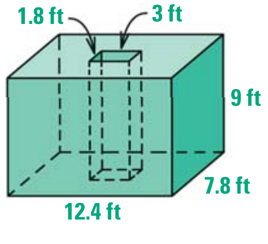

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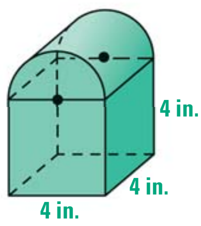


21. **TAKS REASONING** What is the height of a cylinder with radius 4 feet and volume 64π cubic feet?

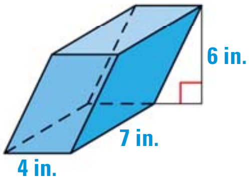
Ⓐ 4 feet Ⓑ 8 feet Ⓒ 16 feet Ⓓ 256 feet

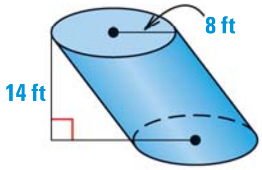
22. **FINDING HEIGHT** The bases of a right prism are right triangles with side lengths of 3 inches, 4 inches, and 5 inches. The volume of the prism is 96 cubic inches. What is the height of the prism?

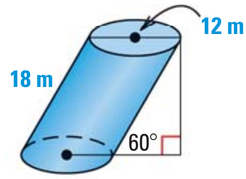
23. **FINDING DIAMETER** A cylinder has height 8 centimeters and volume 1005.5 cubic centimeters. What is the diameter of the cylinder?

EXAMPLE 4
on p. 821
for Exs. 24–26

VOLUME OF AN OBLIQUE SOLID **Use Cavalieri's Principle to find the volume of the oblique prism or cylinder. Round your answer to two decimal places.**

24.


25.


26.


27. **CHALLENGE** The bases of a right prism are rhombuses with diagonals 12 meters and 16 meters long. The height of the prism is 8 meters. Find the lateral area, surface area, and volume of the prism.

PROBLEM SOLVING

EXAMPLE 5
on p. 822
for Exs. 28–30

28. **JEWELRY** The bead at the right is a rectangular prism of length 17 millimeters, width 9 millimeters, and height 5 millimeters. A 3 millimeter wide hole is drilled through the smallest face. Find the volume of the bead.

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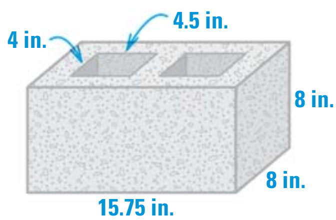
29. **MULTI-STEP PROBLEM** In the concrete block shown, the holes are 8 inches deep.



a. Find the volume of the block using the Volume Addition Postulate.

b. Find the volume of the block using the formula in Theorem 12.6.

c. *Compare* your answers in parts (a) and (b).

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30. **OCEANOGRAPHY** The Blue Hole is a cylindrical trench located on Lighthouse Reef Atoll, an island off the coast of Central America. It is approximately 1000 feet wide and 400 feet deep.

a. Find the volume of the Blue Hole.

b. About how many gallons of water does the Blue Hole contain? ($1\text{ ft}^3 = 7.48$ gallons)

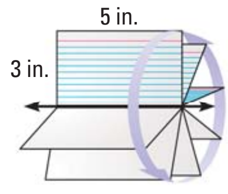
○ = **WORKED-OUT SOLUTIONS** on p. WS1

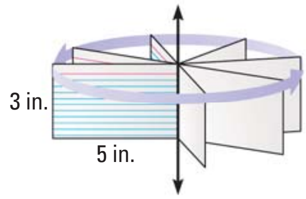
= **TAKS PRACTICE AND REASONING**

31. ARCHITECTURE A cylindrical column in the building shown has circumference 10 feet and height 20 feet. Find its volume. Round your answer to two decimal places.

Animated Geometry at classzone.com

32. ROTATIONS A 3 inch by 5 inch index card is rotated around a horizontal line and a vertical line to produce two different solids, as shown. Which solid has a greater volume? *Explain* your reasoning.





33. TAKS REASONING An aquarium shaped like a rectangular prism has length 30 inches, width 10 inches, and height 20 inches.

a. **Calculate** You fill the aquarium $\frac{3}{4}$ full with water. What is the volume of the water?

b. **Interpret** When you submerge a rock in the aquarium, the water level rises 0.25 inch. Find the volume of the rock.

c. **Interpret** How many rocks of the same size as the rock in part (b) can you place in the aquarium before water spills out?

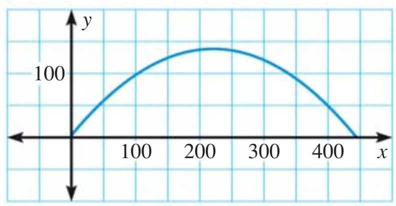
34. CHALLENGE A barn is in the shape of a pentagonal prism with the dimensions shown. The volume of the barn is 9072 cubic feet. Find the dimensions of each half of the roof.

TAKS PRACTICE at classzone.com

MIXED REVIEW FOR TAKS

REVIEW
TAKS Preparation p. 642;
TAKS Workbook

35. TAKS PRACTICE The graph shows the path of a baseball that has been hit in a home run. What is the range of the function? *TAKS Obj. 2*



(A) $0 < x < 445$ (B) $0 \le x \le 445$

(C) $0 < y < 140$ (D) $0 \le y \le 140$

REVIEW
TAKS Preparation p. 714;
TAKS Workbook

36. TAKS PRACTICE If the domain for the function $3x^2 - 5x + 6$ is $\{-5, -3, 0, 6\}$, which is NOT contained in the range? *TAKS Obj. 3*

(F) 48 (G) 84 (H) 106 (J) 144

PROBLEM SOLVING WORKSHOP
LESSON 12.4

TEKS a.6, G.4, G.8.B, G.8.D

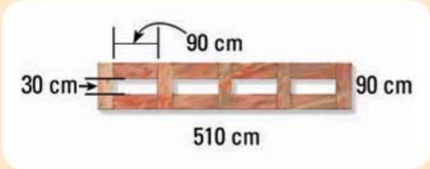
Using ALTERNATIVE METHODS

Another Way to Solve Example 5, page 822

MULTIPLE REPRESENTATIONS In Lesson 12.4, you used volume postulates and theorems to find volumes of prisms and cylinders. Now, you will learn two different ways to solve Example 5 on page 822.

PROBLEM

SCULPTURE The sculpture is made up of 13 beams. In centimeters, suppose the dimensions of each beam are 30 by 30 by 90. Find its volume.



METHOD 1

Finding Volume by Subtracting Empty Spaces One alternative approach is to compute the volume of the prism formed if the holes in the sculpture were filled. Then, to get the correct volume, you must subtract the volume of the four holes.

STEP 1 **Read** the problem. In centimeters, each beam measures 30 by 30 by 90.

The dimensions of the entire sculpture are 30 by 90 by $(4 \cdot 90 + 5 \cdot 30)$, or 30 by 90 by 510.

The dimensions of each hole are equal to the dimensions of one beam.

STEP 2 **Apply** the Volume Addition Postulate. The volume of the sculpture is equal to the volume of the larger prism minus 4 times the volume of a hole.

$$\begin{aligned}\text{Volume } V \text{ of sculpture} &= \text{Volume of larger prism} - \text{Volume of 4 holes}\\ &= 30 \cdot 90 \cdot 510 - 4(30 \cdot 30 \cdot 90)\\ &= 1{,}377{,}000 - 4 \cdot 81{,}000\\ &= 1{,}377{,}000 - 324{,}000\\ &= 1{,}053{,}000\end{aligned}$$

▶ The volume of the sculpture is 1,053,000 cubic centimeters, or 1.053 cubic meters.

STEP 3 **Check** page 822 to verify your new answer, and confirm that it is the same.

METHOD 2 **Finding Volume of Pieces** Another alternative approach is to use the dimensions of each beam.

STEP 1 **Look** at the sculpture. Notice that the sculpture consists of 13 beams, each with the same dimensions. Therefore, the volume of the sculpture will be 13 times the volume of one beam.

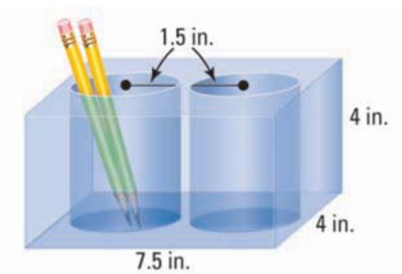
STEP 2 **Write** an expression for the volume of the sculpture and find the volume.

$$\begin{aligned}\text{Volume of sculpture} &= 13(\text{Volume of one beam})\\ &= 13(30 \cdot 30 \cdot 90)\\ &= 13 \cdot 81{,}000\\ &= 1{,}053{,}000\end{aligned}$$

▶ The volume of the sculpture is 1,053,000 cm^3, or 1.053 m^3.

PRACTICE

1. **PENCIL HOLDER** The pencil holder has the dimensions shown.



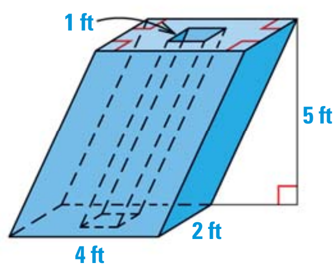
   a. Find its volume using the Volume Addition Postulate.

   b. Use its base area to find its volume.

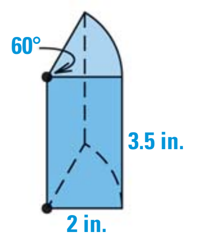
2. **ERROR ANALYSIS** A student solving Exercise 1 claims that the surface area is found by subtracting four times the base area of the cylinders from the surface area of the rectangular prism. *Describe* and correct the student's error.

3. **REASONING** You drill a circular hole of radius r through the base of a cylinder of radius R. Assume the hole is drilled completely through to the other base. You want the volume of the hole to be half the volume of the cylinder. Express r as a function of R.

4. **FINDING VOLUME** Find the volume of the solid shown below. Assume the hole has square cross sections.



5. **FINDING VOLUME** Find the volume of the solid shown to the right.



6. **SURFACE AREA** Refer to the diagram of the sculpture on page 826.

   a. *Describe* a method to find the surface area of the sculpture.

   b. *Explain* why adding the individual surface areas of the beams will give an incorrect result for the total surface area.

Investigating Geometry ACTIVITY *Use before Lesson 12.5*

TEXAS @HomeTutor
classzone.com
Keystrokes

12.5 Investigate the Volume of a Pyramid

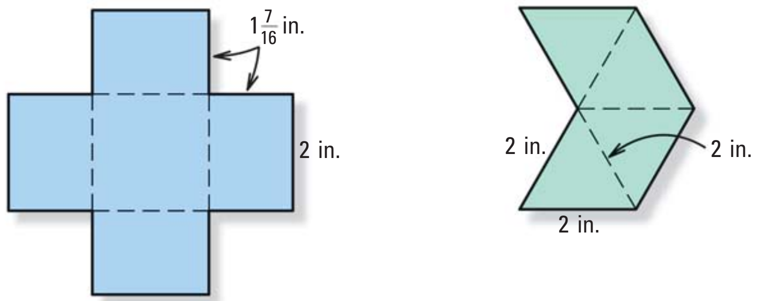
MATERIALS • ruler • poster board • scissors • tape • uncooked rice

TEKS *G.2.A, G.6.B, G.8.D, G.9.D*

QUESTION **How is the volume of a pyramid related to the volume of a prism with the same base and height?**

EXPLORE **Compare the volume of a prism and a pyramid using nets**

STEP 1 ***Draw nets*** Use a ruler to draw the two nets shown below on poster board. (Use $1\frac{7}{16}$ inches to approximate $\sqrt{2}$ inches.)



STEP 2 ***Create an open prism and an open pyramid*** Cut out the nets. Fold along the dotted lines to form an open prism and an open pyramid, as shown below. Tape each solid to hold it in place, making sure that the edges do not overlap.

STEP 3 ***Compare volumes*** Fill the pyramid with uncooked rice and pour it into the prism. Repeat this as many times as needed to fill the prism. How many times did you fill the pyramid? What does this tell you about the volume of the solids?

DRAW CONCLUSIONS **Use your observations to complete these exercises**

1. *Compare* the area of the base of the pyramid to the area of the base of the prism. Placing the pyramid inside the prism will help. What do you notice?
2. *Compare* the heights of the solids. What do you notice?
3. Make a conjecture about the ratio of the volumes of the solids.
4. Use your conjecture to write a formula for the volume of a pyramid that uses the formula for the volume of a prism.

12.5 Volume of Pyramids and Cones

TEKS G.8.C, G.8.D, G.10.B, G.11.C

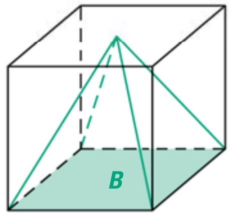
Before You found surface areas of pyramids and cones.

Now You will find volumes of pyramids and cones.

Why? So you can find the edge length of a pyramid, as in Example 2.

Key Vocabulary
- **pyramid,** *p. 810*
- **cone,** *p. 812*
- **volume,** *p. 819*

Recall that the volume of a prism is Bh, where B is the area of a base and h is the height. In the figure at the right, you can see that the volume of a pyramid must be less than the volume of a prism with the same base area and height. As suggested by the Activity on page 828, the volume of a pyramid is one third the volume of a prism.



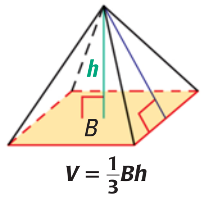
THEOREMS *For Your Notebook*

THEOREM 12.9 Volume of a Pyramid

The volume V of a pyramid is

$$V = \frac{1}{3}Bh,$$

where B is the area of the base and h is the height.

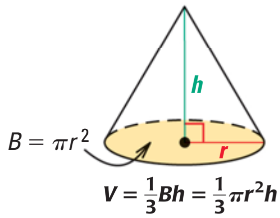


THEOREM 12.10 Volume of a Cone

The volume V of a cone is

$$V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h,$$

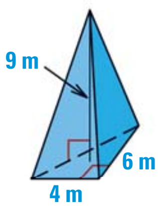
where B is the area of the base, h is the height, and r is the radius of the base.



EXAMPLE 1 Find the volume of a solid

Find the volume of the solid.

a.

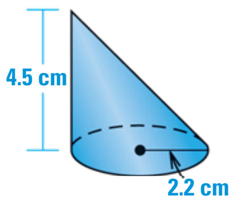


$V = \frac{1}{3}Bh$

$= \frac{1}{3}\left(\frac{1}{2} \cdot 4 \cdot 6\right)(9)$

$= 36 \text{ m}^3$

b.



$V = \frac{1}{3}Bh$

$= \frac{1}{3}(\pi r^2)h$

$= \frac{1}{3}(\pi \cdot 2.2^2)(4.5)$

$= 7.26\pi$

$\approx 22.81 \text{ cm}^3$

APPLY FORMULAS
The formulas given in Theorems 12.9 and 12.10 apply to right and oblique pyramids and cones. This follows from Cavalieri's Principle, stated on page 821.

EXAMPLE 2 Use volume of a pyramid


Khafre's Pyramid, Egypt

ALGEBRA Originally, the pyramid had height 144 meters and volume 2,226,450 cubic meters. Find the side length of the square base.

Solution

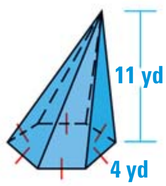
$V = \frac{1}{3}Bh$	**Write formula.**
$2{,}226{,}450 = \frac{1}{3}(x^2)(144)$	**Substitute.**
$6{,}679{,}350 = 144x^2$	**Multiply each side by 3.**
$46{,}384 \approx x^2$	**Divide each side by 144.**
$215 \approx x$	**Find the positive square root.**

▸ Originally, the side length of the base was about 215 meters.

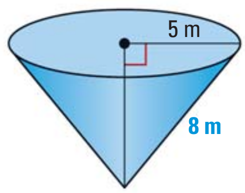
✓ GUIDED PRACTICE for Examples 1 and 2

Find the volume of the solid. Round your answer to two decimal places, if necessary.

1. Hexagonal pyramid



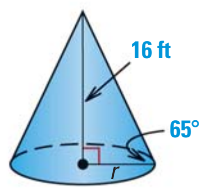
2. Right cone

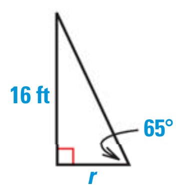


3. The volume of a right cone is 1350π cubic meters and the radius is 18 meters. Find the height of the cone.

EXAMPLE 3 Use trigonometry to find the volume of a cone

Find the volume of the right cone.





Solution

To find the radius r of the base, use trigonometry.

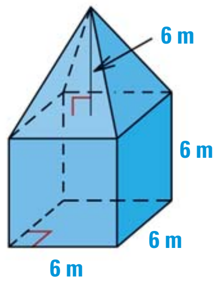
$\tan 65° = \frac{\text{opp.}}{\text{adj.}}$	**Write ratio.**
$\tan 65° = \frac{16}{r}$	**Substitute.**
$r = \frac{16}{\tan 65°} \approx 7.46$	**Solve for *r*.**

Use the formula for the volume of a cone.

$$V = \frac{1}{3}(\pi r^2)h \approx \frac{1}{3}\pi(7.46^2)(16) \approx 932.45 \text{ ft}^3$$

EXAMPLE 4 Find volume of a composite solid

Find the volume of the solid shown.



Solution

Volume of solid	=	Volume of cube	+	Volume of pyramid

$= s^3 + \frac{1}{3}Bh$ **Write formulas.**

$= 6^3 + \frac{1}{3}(6)^2 \cdot 6$ **Substitute.**

$= 216 + 72$ **Simplify.**

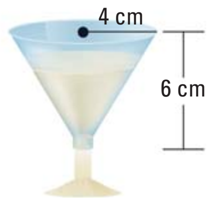
$= 288$ **Add.**

▸ The volume of the solid is 288 cubic meters.

TAKS

EXAMPLE 5 TAKS Reasoning: Multi-Step Problem

SCIENCE You are using the funnel shown to measure the coarseness of a particular type of sand. It takes 2.8 seconds for the sand to empty out of the funnel. Find the flow rate of the sand in milliliters per second. (1 mL = 1 cm^3)



Solution

STEP 1 **Find** the volume of the funnel using the formula for the volume of a cone.

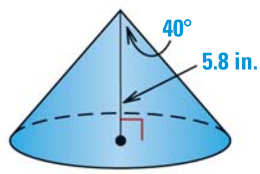
$$V = \frac{1}{3}(\pi r^2)h = \frac{1}{3}\pi(4^2)(6) \approx 101 \text{ cm}^3 = 101 \text{ mL}$$

STEP 2 **Divide** the volume of the funnel by the time it takes the sand to empty out of the funnel.

$$\frac{101 \text{ mL}}{2.8 \text{ s}} \approx 36.07 \text{ mL/s}$$

▸ The flow rate of the sand is about 36.07 milliliters per second.

✓ GUIDED PRACTICE for Examples 3, 4, and 5



4. Find the volume of the cone at the right. Round your answer to two decimal places.

5. A right cylinder with radius 3 centimeters and height 10 centimeters has a right cone on top of it with the same base and height 5 centimeters. Find the volume of the solid. Round your answer to two decimal places.

6. **WHAT IF?** In Example 5, suppose a different type of sand is used that takes 3.2 seconds to empty out of the funnel. Find its flow rate.

12.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 17, and 33

★ = **TAKS PRACTICE AND REASONING** Exs. 11, 18, 35, 43, and 44

◆ = **MULTIPLE REPRESENTATIONS** Ex. 39

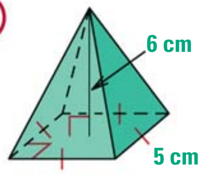
SKILL PRACTICE

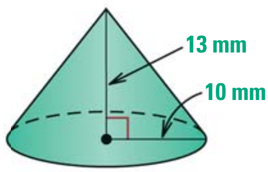
1. **VOCABULARY** *Explain* the difference between a *triangular prism* and a *triangular pyramid*. Draw an example of each.

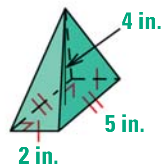
2. **WRITING** *Compare* the volume of a square pyramid to the volume of a square prism with the same base and height as the pyramid.

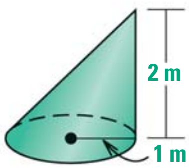
EXAMPLE 1 on p. 829 for Exs. 3–11

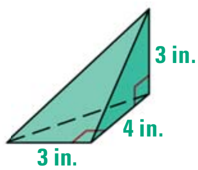
VOLUME OF A SOLID **Find the volume of the solid. Round your answer to two decimal places.**

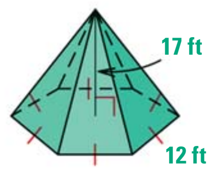
3. 

4. 

5. 

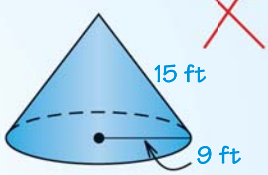
6. 

7. 

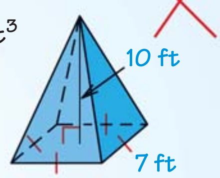
8. 

ERROR ANALYSIS *Describe* **and correct the error in finding the volume of the right cone or pyramid.**

9.
$$V = \frac{1}{3}\pi(9^2)(15)$$
$$= 405\pi$$
$$\approx 1272 \text{ ft}^3$$



10.
$$V = \frac{1}{2}(49)(10)$$
$$= 245 \text{ ft}^3$$



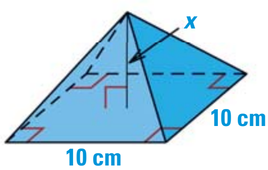
11. ★ **TAKS REASONING** The volume of a pyramid is 45 cubic feet and the height is 9 feet. What is the area of the base?

Ⓐ 3.87 ft^2 Ⓑ 5 ft^2 Ⓒ 10 ft^2 Ⓓ 15 ft^2

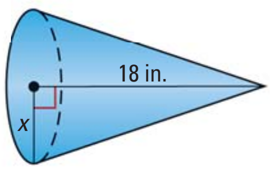
EXAMPLE 2 on p. 830 for Exs. 12–14

xy **ALGEBRA** **Find the value of *x*.**

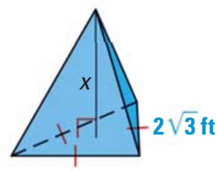
12. Volume = 200 cm^3



13. Volume = 216π in.^{3}

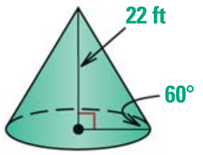


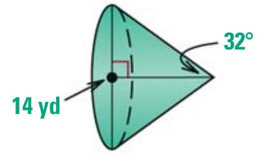
14. Volume = $7\sqrt{3}$ ft^3

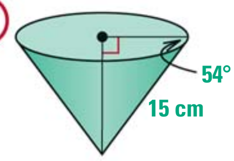


EXAMPLE 3
on p. 830
for Exs. 15–19

VOLUME OF A CONE **Find the volume of the right cone. Round your answer to two decimal places.**

15.


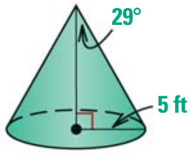
16.


17.


18. **TAKS REASONING** What is the approximate volume of the cone?

Ⓐ 47.23 ft^3 Ⓑ 236.14 ft^3

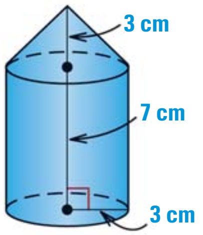
Ⓒ 269.92 ft^3 Ⓓ 354.21 ft^3

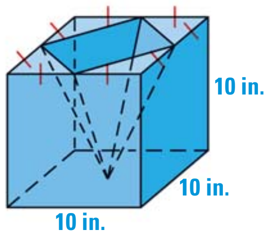


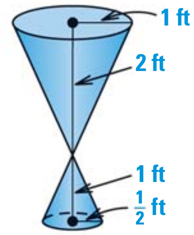
19. **HEIGHT OF A CONE** A cone with a diameter of 8 centimeters has volume 143.6 cubic centimeters. Find the height of the cone. Round your answer to two decimal places.

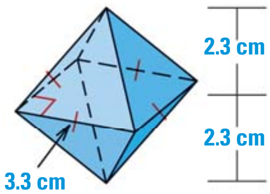
EXAMPLE 4
on p. 831
for Exs. 20–25

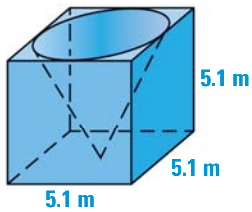
COMPOSITE SOLIDS **Find the volume of the solid. The prisms, pyramids, and cones are right. Round your answer to two decimal places.**

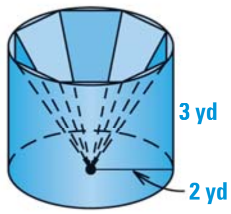
20.


21.


22.


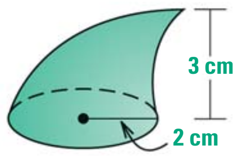
23.


24.


25.


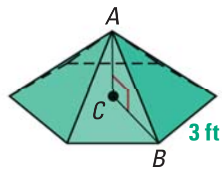
AnimatedGeometry at classzone.com

26. **FINDING VOLUME** The figure at the right is a cone that has been warped but whose cross sections still have the same area as a right cone with equal base area and height. Find the volume of this solid.



27. **FINDING VOLUME** Sketch a regular square pyramid with base edge length 5 meters inscribed in a cone with height 7 meters. Find the volume of the cone. *Explain* your reasoning.

28. **CHALLENGE** Find the volume of the regular hexagonal pyramid. Round your answer to the nearest hundredth of a cubic foot. In the diagram, $m\angle ABC = 35°$.



PROBLEM SOLVING

EXAMPLE 5 on p. 831 for Ex. 30

29. CAKE DECORATION A pastry bag filled with frosting has height 12 inches and radius 4 inches. A cake decorator can make 15 flowers using one bag of frosting.

a. How much frosting is in the pastry bag? Round your answer to the nearest cubic inch.

b. How many cubic inches of frosting are used to make each flower?

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POPCORN A snack stand serves a small order of popcorn in a cone-shaped cup and a large order of popcorn in a cylindrical cup.

30. Find the volume of the small cup.

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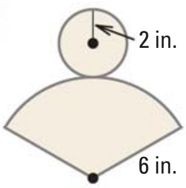
31. How many small cups of popcorn do you have to buy to equal the amount of popcorn in a large container? Do not perform any calculations. *Explain*.

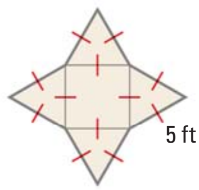
32. Which container gives you more popcorn for your money? *Explain*.

USING NETS In Exercises 33 and 34, use the net to sketch the solid. Then find the volume of the solid. Round your answer to two decimal places.

33.



34.



35. TAKS REASONING A pyramid has height 10 feet and a square base with side length 7 feet.

a. How does the volume of the pyramid change if the base stays the same and the height is doubled?

b. How does the volume of the pyramid change if the height stays the same and the side length of the base is doubled?

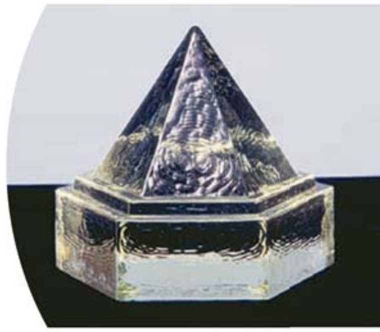
c. *Explain* why your answers to parts (a) and (b) are true for any height and side length.

36. AUTOMATIC FEEDER Assume the automatic pet feeder is a right cylinder on top of a right cone of the same radius. (1 cup = 14.4 in.^{3})

a. Calculate the amount of food in cups that can be placed in the feeder.

b. A cat eats one third of a cup of food, twice per day. How many days will the feeder have food without refilling it?

37. **NAUTICAL PRISMS** The nautical deck prism shown is composed of the following three solids: a regular hexagonal prism with edge length 3.5 inches and height 1.5 inches, a regular hexagonal prism with edge length 3.25 inches and height 0.25 inch, and a regular hexagonal pyramid with edge length 3 inches and height 3 inches. Find the volume of the deck prism.



38. **MULTI-STEP PROBLEM** Calculus can be used to show that the average value of r^2 of a circular cross section of a cone is $\frac{r_b^2}{3}$, where r_b is the radius of the base.

   a. Find the average area of a circular cross section of a cone whose base has radius R.

   b. Show that the volume of the cone can be expressed as follows:

   V_{cone} = (Average area of a circular cross section) • (Height of cone)

39. **MULTIPLE REPRESENTATIONS** Water flows into a reservoir shaped like a right cone at the rate of 1.8 cubic meters per minute. The height and diameter of the reservoir are equal.

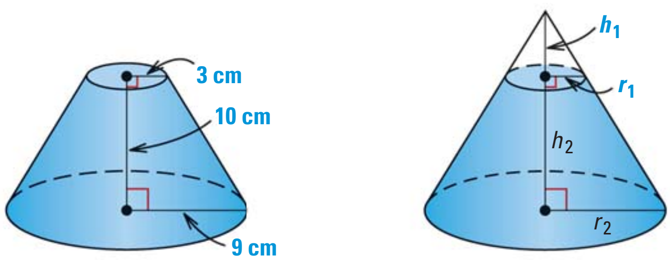
   a. Using Algebra As the water flows into the reservoir, the relationship $h = 2r$ is always true. Using this fact, show that $V = \frac{\pi h^3}{12}$.

   b. Making a Table Make a table that gives the height h of the water after 1, 2, 3, 4, and 5 minutes.

   c. Drawing a Graph Make a graph of height versus time. Is there a linear relationship between the height of the water and time? *Explain.*

FRUSTUM A frustum of a cone is the part of the cone that lies between the base and a plane parallel to the base, as shown. Use the information to complete Exercises 40 and 41.

One method for calculating the volume of a frustum is to add the areas of the two bases to their geometric mean, then multiply the result by $\frac{1}{3}$ the height.



40. Use the measurements in the diagram at the left above to calculate the volume of the frustum.

41. Complete parts (a) and (b) below to write a formula for the volume of a frustum that has bases with radii r_1 and r_2 and a height h_2.

   a. Use similar triangles to find the value of h_1 in terms of h_2, r_1, and r_2.

   b. Write a formula in terms of h_2, r_1, and r_2 for V_{frustum} = (Original volume) − (Removed volume).

   c. Show that your formula in part (b) is equivalent to the formula involving geometric mean described above.

42. **CHALLENGE** A square pyramid is inscribed in a right cylinder so that the base of the pyramid is on a base of the cylinder, and the vertex of the pyramid is on the other base of the cylinder. The cylinder has radius 6 feet and height 12 feet. Find the volume of the pyramid. Round your answer to two decimal places.

MIXED REVIEW FOR TAKS

TAKS PRACTICE at classzone.com

REVIEW
Lesson 7.2;
TAKS Workbook

43. **TAKS PRACTICE** Which of the following are possible dimensions of a right triangle? *TAKS Obj. 7*

(A) 0.3 ft, 0.4 ft, 0.4 ft (B) 5 ft, 12 ft, 13 ft

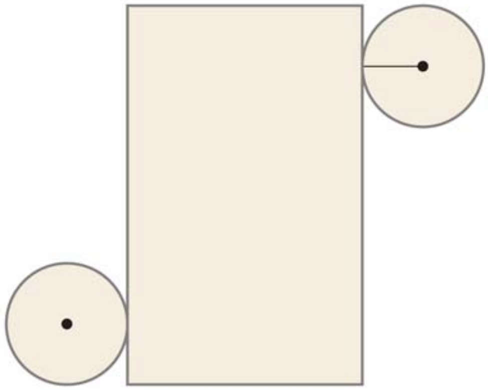
(C) 9 ft, 12 ft, 16 ft (D) 10 ft, 15 ft, 20 ft

REVIEW
Lesson 12.2;
TAKS Workbook

44. **TAKS PRACTICE** The net of a cylinder is shown. Use a ruler to measure the dimensions of the cylinder to the nearest tenth of a centimeter. Which is closest to the surface area of the cylinder? *TAKS Obj. 8*

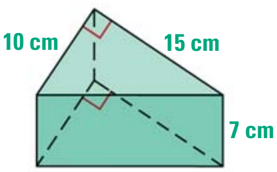
(F) 17 cm^2 (G) 20 cm^2

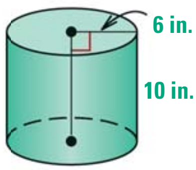
(H) 26 cm^2 (J) 32 cm^2

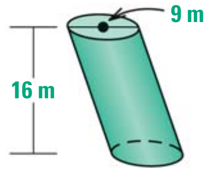


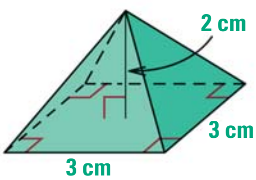
QUIZ *for Lessons 12.4–12.5*

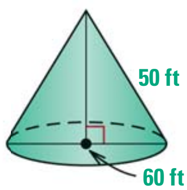
Find the volume of the figure. Round your answer to two decimal places, if necessary. *(pp. 819, 829)*

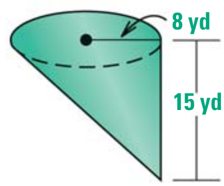
1. 

2. 

3. 

4. 

5. 

6. 

7. Suppose you fill up a cone-shaped cup with water. You then pour the water into a cylindrical cup with the same radius. Both cups have a height of 6 inches. Without doing any calculation, determine how high the water level will be in the cylindrical cup once all of the water is poured into it. *Explain* your reasoning. *(p. 829)*

Spreadsheet **ACTIVITY** *Use after Lesson 12.5*

TEXAS @HomeTutor
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Keystrokes

12.5 Minimize Surface Area

TEKS *a.5, G.3.D, G.4, G.8.D*

MATERIALS • computer

QUESTION **How can you find the minimum surface area of a solid with a given volume?**

A manufacturer needs a cylindrical container with a volume of 72 cubic centimeters. You have been asked to find the dimensions of such a container so that it has a minimum surface area.

EXAMPLE **Use a spreadsheet**

STEP 1 ***Make a table*** Make a table with the four column headings shown in Step 4. The first column is for the given volume V. In cell A2, enter 72. In cell A3, enter the formula "=A2".

STEP 2 ***Enter radius*** The second column is for the radius r. Cell B2 stores the starting value for r. So, enter 2 into cell B2. In cell B3, use the formula "=B2 + 0.05" to increase r in increments of 0.05 centimeter.

STEP 3 ***Enter formula for height*** The third column is for the height. In cell C2, enter the formula "=A2/(PI()*B2^2)". *Note:* Your spreadsheet might use a different expression for π.

STEP 4 ***Enter formula for surface area*** The fourth column is for the surface area. In cell D2, enter the formula "=2*PI()*B2^2+2*PI()*B2*C2".

	A	B	C	D
1	Volume V	Radius r	Height $=V/(\pi r^2)$	Surface area $S=2\pi r^2+2\pi r$
2	72.00	2.00	=A2/(PI()*B2^2)	=2*PI()*B2^2+2*PI()*B2*C2
3	=A2	=B2+0.05		

STEP 5 ***Create more rows*** Use the *Fill Down* feature to create more rows. Rows 3 and 4 of your spreadsheet should resemble the one below.

	A	B	C	D
...				
3	72.00	2.05	5.45	96.65
4	72.00	2.10	5.20	96.28

PRACTICE

1. From the data in your spreadsheet, which dimensions yield a minimum surface area for the given volume? *Explain* how you know.
2. **WHAT IF?** Find the dimensions that give the minimum surface area if the volume of a cylinder is instead 200π cubic centimeters.

12.6 Surface Area and Volume of Spheres

TEKS a.4, a.5, G.6.A, G.8.D

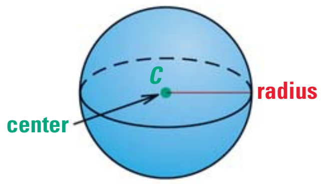
Before You found surface areas and volumes of polyhedra.

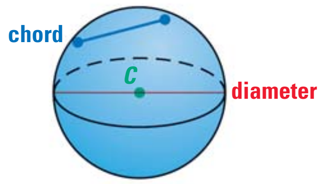
Now You will find surface areas and volumes of spheres.

Why? So you can find the volume of a tennis ball, as in Ex. 33.

Key Vocabulary
- **sphere**
  center, radius, chord, diameter
- **great circle**
- **hemispheres**

A **sphere** is the set of all points in space equidistant from a given point. This point is called the **center** of the sphere. A **radius** of a sphere is a segment from the center to a point on the sphere. A **chord** of a sphere is a segment whose endpoints are on the sphere. A **diameter** of a sphere is a chord that contains the center.





As with circles, the terms radius and diameter also represent distances, and the diameter is twice the radius.

THEOREM *For Your Notebook*

THEOREM 12.11 Surface Area of a Sphere

The surface area S of a sphere is

$S = 4\pi r^2$,

where r is the radius of the sphere.

$S = 4\pi r^2$

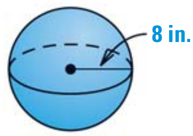
USE FORMULAS
If you understand how a formula is derived, then it will be easier for you to remember the formula.

SURFACE AREA FORMULA To understand how the formula for the surface area of a sphere is derived, think of a baseball. The surface area of a baseball is sewn from two congruent shapes, each of which resembles two joined circles, as shown.

So, the entire covering of the baseball consists of four circles, each with radius r. The area A of a circle with radius r is $A = \pi r^2$. So, the area of the covering can be approximated by $4\pi r^2$. This is the formula for the surface area of a sphere.

EXAMPLE 1 Find the surface area of a sphere

Find the surface area of the sphere.



Solution

$S = 4\pi r^2$ **Formula for surface area of a sphere**

$= 4\pi(8^2)$ **Substitute 8 for *r*.**

$= 256\pi$ **Simplify.**

≈ 804.25 **Use a calculator.**

▶ The surface area of the sphere is about 804.25 square inches.

EXAMPLE 2 TAKS PRACTICE: Multiple Choice

The surface area of a sphere is 182.25π square centimeters. What is the diameter of the sphere?

Ⓐ 3.375 cm Ⓑ 6.75 cm

Ⓒ 13.5 cm Ⓓ 15.5 cm

ANOTHER WAY

Instead of finding the value of *r*, you can check the answer choices by substituting them for *r* in the formula for surface area of a sphere.

Solution

$S = 4\pi r^2$ **Formula for surface area of a sphere**

$182.25\pi = 4\pi r^2$ **Substitute 182.25π for *S*.**

$45.5625 = r^2$ **Divide each side by 4π.**

$6.75 = r$ **Find the positive square root.**

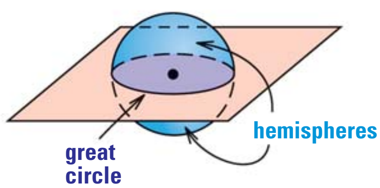
The diameter of the sphere is $2r = 2 \cdot 6.75 = 13.5$ centimeters.

▶ The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

GUIDED PRACTICE for Examples 1 and 2

1. The diameter of a sphere is 40 feet. Find the surface area of the sphere.
2. The surface area of a sphere is 30π square meters. Find the radius of the sphere.

GREAT CIRCLES If a plane intersects a sphere, the intersection is either a single point or a circle. If the plane contains the center of the sphere, then the intersection is a **great circle** of the sphere. The circumference of a great circle is the circumference of the sphere. Every great circle of a sphere separates the sphere into two congruent halves called **hemispheres**.



EXAMPLE 3 Use the circumference of a sphere

EXTREME SPORTS In a sport called *sphereing*, a person rolls down a hill inside an inflatable ball surrounded by another ball. The diameter of the outer ball is 12 feet. Find the surface area of the outer ball.

Solution

The diameter of the outer sphere is 12 feet, so the radius is $\frac{12}{2} = 6$ feet.

Use the formula for the surface area of a sphere.

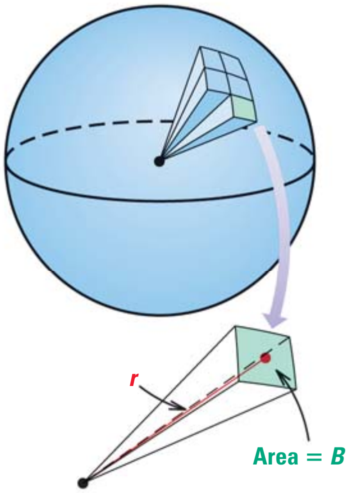
$$S = 4\pi r^2 = 4\pi(6^2) = 144\pi$$

▶ The surface area of the outer ball is 144π, or about 452.39 square feet.

✓ GUIDED PRACTICE for Example 3

3. In Example 3, the circumference of the inner ball is 6π feet. Find the surface area of the inner ball. Round your answer to two decimal places.

VOLUME FORMULA Imagine that the interior of a sphere with radius r is approximated by n pyramids, each with a base area of B and a height of r. The volume of each pyramid is $\frac{1}{3}Br$ and the sum of the base areas is nB. The surface area of the sphere is approximately equal to nB, or $4\pi r^2$. So, you can approximate the volume V of the sphere as follows.



$V \approx n\left(\frac{1}{3}Br\right)$	**Each pyramid has a volume of $\frac{1}{3}Br$.**
$\approx \frac{1}{3}(nB)r$	**Regroup factors.**
$= \frac{1}{3}(4\pi r^2)r$	**Substitute $4\pi r^2$ for nB.**
$= \frac{4}{3}\pi r^3$	**Simplify.**

THEOREM *For Your Notebook*

THEOREM 12.12 Volume of a Sphere

The volume V of a sphere is

$$V = \frac{4}{3}\pi r^3,$$

where r is the radius of the sphere.

$V = \frac{4}{3}\pi r^3$

EXAMPLE 4 Find the volume of a sphere

The soccer ball has a diameter of 9 inches. Find its volume.

Solution

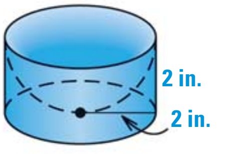
The diameter of the ball is 9 inches, so the radius is $\frac{9}{2} = 4.5$ inches.

$V = \frac{4}{3}\pi r^3$	Formula for volume of a sphere
$= \frac{4}{3}\pi(4.5)^3$	Substitute.
$= 121.5\pi$	Simplify.
≈ 381.70	Use a calculator.

▶ The volume of the soccer ball is 121.5π, or about 381.70 cubic inches.

EXAMPLE 5 Find the volume of a composite solid

Find the volume of the composite solid.



Solution

Volume of solid = Volume of cylinder − Volume of hemisphere

$= \pi r^2 h - \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$	Formulas for volume
$= \pi(2)^2(2) - \frac{2}{3}\pi(2)^3$	Substitute.
$= 8\pi - \frac{2}{3}(8\pi)$	Multiply.
$= \frac{24}{3}\pi - \frac{16}{3}\pi$	Rewrite fractions using least common denominator.
$= \frac{8}{3}\pi$	Simplify.

▶ The volume of the solid is $\frac{8}{3}\pi$, or about 8.38 cubic inches.

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✓ GUIDED PRACTICE for Examples 4 and 5

4. The radius of a sphere is 5 yards. Find the volume of the sphere. Round your answer to two decimal places.

5. A solid consists of a hemisphere of radius 1 meter on top of a cone with the same radius and height 5 meters. Find the volume of the solid. Round your answer to two decimal places.

12.6 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 13, and 31

★ = **TAKS PRACTICE AND REASONING** Exs. 6, 20, 28, 33, 34, 40, 41, and 42

SKILL PRACTICE

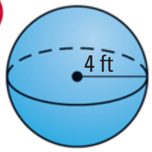
1. **VOCABULARY** What are the formulas for finding the surface area of a sphere and the volume of a sphere?

2. **WRITING** When a plane intersects a sphere, what point in the sphere must the plane contain for the intersection to be a great circle? *Explain.*

EXAMPLE 1 on p. 839 for Exs. 3–5

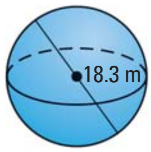
FINDING SURFACE AREA **Find the surface area of the sphere. Round your answer to two decimal places.**

3.



4.

5.



EXAMPLE 2 on p. 839 for Ex. 6

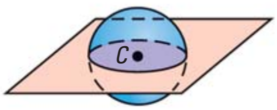
6. ★ **TAKS REASONING** What is the approximate radius of a sphere with surface area 32π square meters?

Ⓐ 2 meters Ⓑ 2.83 meters Ⓒ 4.90 meters Ⓓ 8 meters

EXAMPLE 3 on p. 840 for Exs. 7–11

USING A GREAT CIRCLE **In Exercises 7–9, use the sphere below. The center of the sphere is C and its circumference is 9.6π inches.**

7. Find the radius of the sphere.
8. Find the diameter of the sphere.
9. Find the surface area of one hemisphere.



10. **ERROR ANALYSIS** *Describe* and correct the error in finding the surface area of a hemisphere with radius 5 feet.

$S = 4\pi r^2$
$= 4\pi(5)^2$
$= 100\pi$
$\approx 314.16 \text{ ft}^2$

11. **GREAT CIRCLE** The circumference of a great circle of a sphere is 48.4π centimeters. What is the surface area of the sphere?

EXAMPLE 4 on p. 841 for Exs. 12–15

FINDING VOLUME **Find the volume of the sphere using the given radius r or diameter d. Round your answer to two decimal places.**

12. $r = 6$ in.

13. $r = 40$ mm

14. $d = 5$ cm

15. **ERROR ANALYSIS** *Describe* and correct the error in finding the volume of a sphere with diameter 16 feet.

$$V = \frac{4}{3}\pi r^2$$
$$= \frac{4}{3}\pi (8)^2$$
$$= 85.33\pi \approx 268.08 \text{ ft}^2$$

USING VOLUME In Exercises 16–18, find the radius of a sphere with the given volume *V*. Round your answers to two decimal places.

16. $V = 1436.76 \text{ m}^3$

17. $V = 91.95 \text{ cm}^3$

18. $V = 20{,}814.37 \text{ in.}^3$

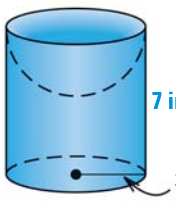
19. **FINDING A DIAMETER** The volume of a sphere is 36π cubic feet. What is the diameter of the sphere?

20. **TAKS REASONING** Let *V* be the volume of a sphere, *S* be the surface area of the sphere, and *r* be the radius of the sphere. Which equation represents the relationship between these three measures?

(A) $V = \frac{rS}{3}$ (B) $V = \frac{r^2S}{3}$ (C) $V = \frac{3}{2}rS$ (D) $V = \frac{3}{2}r^2S$

EXAMPLE 5 on p. 841 for Exs. 21–23

COMPOSITE SOLIDS Find the surface area and the volume of the solid. The cylinders and cones are right. Round your answer to two decimal places.

21.


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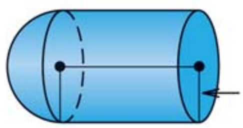

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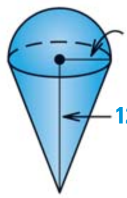


USING A TABLE Copy and complete the table below. Leave your answers in terms of π.

	Radius of sphere	Circumference of great circle	Surface area of sphere	Volume of sphere
24.	10 ft	?	?	?
25.	?	26π in.	?	?
26.	?	?	$2500\pi \text{ cm}^2$	?
27.	?	?	?	$12{,}348\pi \text{ m}^3$

28. **TAKS REASONING** A sphere is inscribed in a cube of volume 64 cubic centimeters. What is the surface area of the sphere?

(A) $4\pi \text{ cm}^2$ (B) $\frac{32}{3}\pi \text{ cm}^2$ (C) $16\pi \text{ cm}^2$ (D) $64\pi \text{ cm}^2$

29. **CHALLENGE** The volume of a right cylinder is the same as the volume of a sphere. The radius of the sphere is 1 inch.

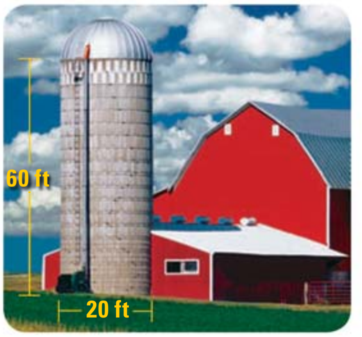
a. Give three possibilities for the dimensions of the cylinder.

b. Show that the surface area of the cylinder is sometimes greater than the surface area of the sphere.

PROBLEM SOLVING

EXAMPLE 5 on p. 841 for Ex. 30

30. GRAIN SILO A grain silo has the dimensions shown. The top of the silo is a hemispherical shape. Find the volume of the grain silo.



TEXAS @HomeTutor for problem solving help at classzone.com

31. GEOGRAPHY The circumference of Earth is about 24,855 miles. Find the surface area of the Western Hemisphere of Earth.

TEXAS @HomeTutor for problem solving help at classzone.com

32. MULTI-STEP PROBLEM A ball has volume 1427.54 cubic centimeters.

a. Find the radius of the ball. Round your answer to two decimal places.

b. Find the surface area of the ball. Round your answer to two decimal places.

33. TAKS REASONING Tennis balls are stored in a cylindrical container with height 8.625 inches and radius 1.43 inches.

a. The circumference of a tennis ball is 8 inches. Find the volume of a tennis ball.

b. There are 3 tennis balls in the container. Find the amount of space within the cylinder not taken up by the tennis balls.

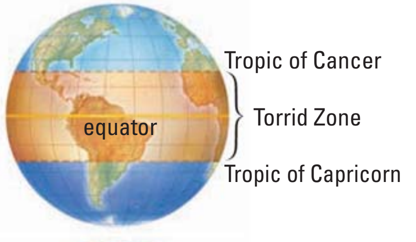
34. TAKS REASONING A partially filled balloon has circumference 27π centimeters. Assume the balloon is a sphere.

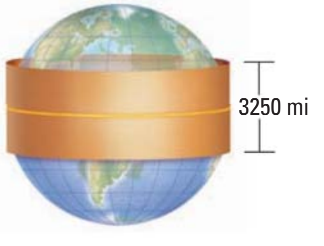
a. Calculate Find the volume of the balloon.

b. Predict Suppose you double the radius by increasing the air in the balloon. *Explain* what you expect to happen to the volume.

c. Justify Find the volume of the balloon with the radius doubled. Was your prediction from part (b) correct? What is the ratio of this volume to the original volume?

35. GEOGRAPHY The Torrid Zone on Earth is the area between the Tropic of Cancer and the Tropic of Capricorn, as shown. The distance between these two tropics is about 3250 miles. You can think of this distance as the height of a cylindrical belt around Earth at the equator, as shown.

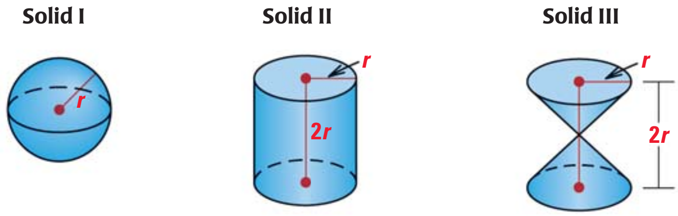




a. Estimate the surface area of the Torrid Zone and the surface area of Earth. (Earth's radius is about 3963 miles at the equator.)

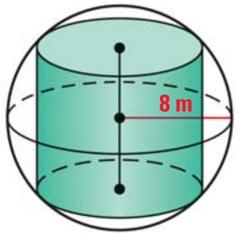
b. A meteorite is equally likely to hit anywhere on Earth. Estimate the probability that a meteorite will land in the Torrid Zone.

36. **REASONING** List the following three solids in order of (a) surface area, and (b) volume, from least to greatest.



37. **ROTATION** A circle with diameter 18 inches is rotated about its diameter. Find the surface area and the volume of the solid formed.

38. **TECHNOLOGY** A cylinder with height $2x$ is inscribed in a sphere with radius 8 meters. The center of the sphere is the midpoint of the altitude that joins the centers of the bases of the cylinder.



a. Show that the volume V of the cylinder is $2\pi x(64 - x^2)$.

b. Use a graphing calculator to graph $V = 2\pi x(64 - x^2)$ for values of x between 0 and 8. Find the value of x that gives the maximum value of V.

c. Use the value for x from part (b) to find the maximum volume of the cylinder.

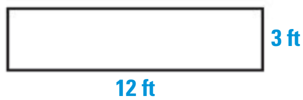
39. **CHALLENGE** A sphere with radius 2 centimeters is inscribed in a right cone with height 6 centimeters. Find the surface area and the volume of the cone.

MIXED REVIEW FOR TAKS

TAKS PRACTICE at classzone.com

REVIEW
Lesson 6.2;
TAKS Workbook

40. **TAKS PRACTICE** Which of the following is a proportional enlargement of the rectangle? *TAKS Obj. 6*



Ⓐ 4 ft by 1.5 ft Ⓑ 15 ft by 6 ft Ⓒ 18 ft by 5.5 ft Ⓓ 30 ft by 7.5 ft

REVIEW
TAKS Preparation
p. 208;
TAKS Workbook

41. **TAKS PRACTICE** Which best describes the effect on the graph of $y = 2x + 3$ if the y-intercept changes to 5? *TAKS Obj. 3*

Ⓕ The slope decreases.

Ⓖ The x-intercept decreases.

Ⓗ The y-intercept decreases.

Ⓙ The new line passes through the origin.

REVIEW
Lesson 9.3;
TAKS Workbook

42. **TAKS PRACTICE** What is the effect on the graph of the equation $y = -5x^2$ when the equation is changed to $y = 5x^2$? *TAKS Obj. 5*

Ⓐ The graph of $y = -5x^2$ is translated 5 units up.

Ⓑ The graph of $y = -5x^2$ is translated 5 units down.

Ⓒ The graph of $y = 5x^2$ is a reflection of $y = -5x^2$ across the x-axis.

Ⓓ The graph of $y = 5x^2$ is a reflection of $y = -5x^2$ across the y-axis.

Investigating Geometry ACTIVITY *Use before Lesson 12.7*

12.7 Investigate Similar Solids

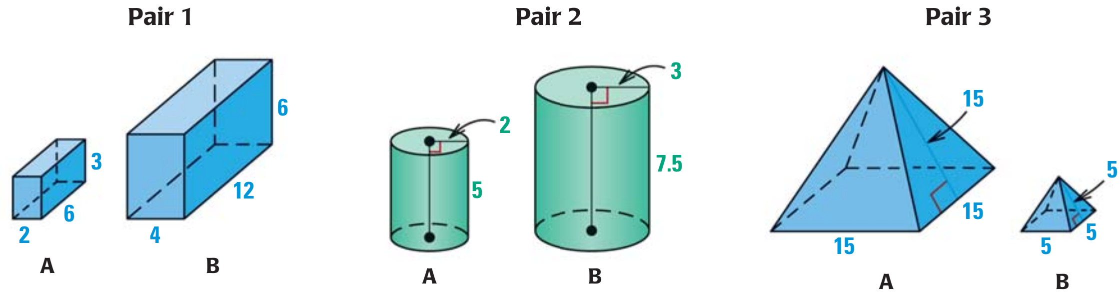
TEKS a.2, G.8.D, G.11.B, G.11.D

MATERIALS • paper • pencil

QUESTION How are the surface areas and volumes of similar solids related?

EXPLORE Compare the surface areas and volumes of similar solids

The solids shown below are *similar*.



STEP 1 ***Make a table*** Copy and complete the table below.

	Scale factor of Solid A to Solid B	Surface area of Solid A, S_A	Surface area of Solid B, S_B	$\frac{S_A}{S_B}$
Pair 1	$\frac{1}{2}$	?	?	?
Pair 2	?	?	63π	?
Pair 3	?	?	?	$\frac{9}{1}$

STEP 2 ***Insert columns*** Insert columns for V_A, V_B, and $\frac{V_A}{V_B}$. Use the dimensions of the solids to find V_A, the volume of Solid A, and V_B, the volume of Solid B. Then find the ratio of these volumes.

STEP 3 ***Compare ratios*** Compare the ratios $\frac{S_A}{S_B}$ and $\frac{V_A}{V_B}$ to the scale factor.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Make a conjecture about how the surface areas and volumes of similar solids are related to the scale factor.
2. Use your conjecture to write a ratio of surface areas and volumes if the dimensions of two similar rectangular prisms are ℓ, w, h, and $k\ell$, kw, kh.

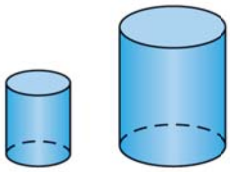
12.7 Explore Similar Solids

TEKS *G.5.B, G.8.D, G.11.B, G.11.D*

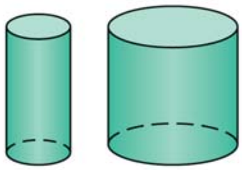
Before	You used properties of similar polygons.
Now	You will use properties of similar solids.
Why	So you can determine a ratio of volumes, as in Ex. 26.

Key Vocabulary
- **similar solids**

Two solids of the same type with equal ratios of corresponding linear measures, such as heights or radii, are called **similar solids**. The common ratio is called the *scale factor* of one solid to the other solid. Any two cubes are similar, as well as any two spheres.



Similar cylinders

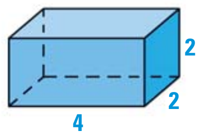


Nonsimilar cylinders

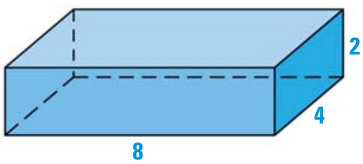
The green cylinders shown above are not similar. Their heights are equal, so they have a 1 : 1 ratio. The radii are different, however, so there is no common ratio.

EXAMPLE 1 Identify similar solids

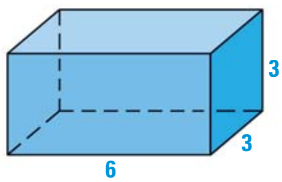
Tell whether the given right rectangular prism is similar to the right rectangular prism shown at the right.



a.



b.



Solution

a. Lengths $\frac{4}{8} = \frac{1}{2}$ Widths $\frac{2}{4} = \frac{1}{2}$ Heights $\frac{2}{2} = \frac{1}{1}$

▶ The prisms are not similar because the ratios of corresponding linear measures are not all equal.

b. Lengths $\frac{4}{6} = \frac{2}{3}$ Widths $\frac{2}{3}$ Heights $\frac{2}{3}$

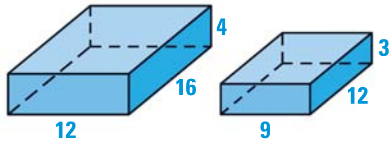
▶ The prisms are similar because the ratios of corresponding linear measures are all equal. The scale factor is 2 : 3.

COMPARE RATIOS
To compare the ratios of corresponding side lengths, write the ratios as fractions in simplest form.

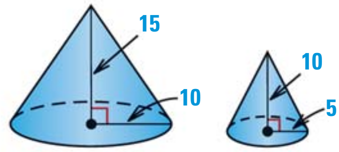
✓ GUIDED PRACTICE for Example 1

Tell whether the pair of right solids is similar. *Explain* your reasoning.

1.



2.



SIMILAR SOLIDS THEOREM The surface areas S and volumes V of the similar solids in Example 1, part (b), are as follows.

Prism	Dimensions	Surface area, $S = 2B + Ph$	Volume, $V = Bh$
Smaller	4 by 2 by 2	$S = 2(8) + 12(2) = 40$	$V = 8(2) = 16$
Larger	6 by 3 by 3	$S = 2(18) + 18(3) = 90$	$V = 18(3) = 54$

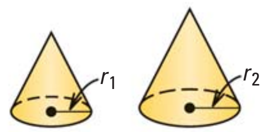
The ratio of side lengths is 2 : 3. Notice that the ratio of surface areas is 40 : 90, or 4 : 9, which can be written as $2^2 : 3^2$, and the ratio of volumes is 16 : 54, or 8 : 27, which can be written as $2^3 : 3^3$. This leads to the following theorem.

READ VOCABULARY
In Theorem 12.13, areas can refer to any pair of corresponding areas in the similar solids, such as lateral areas, base areas, and surface areas.

THEOREM *For Your Notebook*

THEOREM 12.13 Similar Solids Theorem

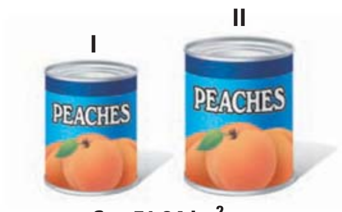
If two similar solids have a scale factor of $a : b$, then corresponding areas have a ratio of $a^2 : b^2$, and corresponding volumes have a ratio of $a^3 : b^3$.



$\frac{r_1}{r_2} = \frac{a}{b}$, $\frac{S_1}{S_2} = \frac{a^2}{b^2}$, $\frac{V_1}{V_2} = \frac{a^3}{b^3}$

EXAMPLE 2 Use the scale factor of similar solids

PACKAGING The cans shown are similar with a scale factor of 87 : 100. Find the surface area and volume of the larger can.



$S = 51.84 \text{ in.}^2$
$V = 28.27 \text{ in.}^3$

Solution

Use Theorem 12.13 to write and solve two proportions.

$$\frac{\text{Surface area of I}}{\text{Surface area of II}} = \frac{a^2}{b^2} \qquad \frac{\text{Volume of I}}{\text{Volume of II}} = \frac{a^3}{b^3}$$

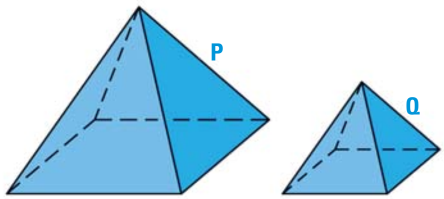
$$\frac{51.84}{\text{Surface area of II}} = \frac{87^2}{100^2} \qquad \frac{28.27}{\text{Volume of II}} = \frac{87^3}{100^3}$$

$$\text{Surface area of II} \approx 68.49 \qquad \text{Volume of II} \approx 42.93$$

▶ The surface area of the larger can is about 68.49 square inches, and the volume of the larger can is about 42.93 cubic inches.

EXAMPLE 3 Find the scale factor

The pyramids are similar. Pyramid P has a volume of 1000 cubic inches and Pyramid Q has a volume of 216 cubic inches. Find the scale factor of Pyramid P to Pyramid Q.



Solution

Use Theorem 12.13 to find the ratio of the two volumes.

$\frac{a^3}{b^3} = \frac{1000}{216}$ **Write ratio of volumes.**

$\frac{a}{b} = \frac{10}{6}$ **Find cube roots.**

$\frac{a}{b} = \frac{5}{3}$ **Simplify.**

▸ The scale factor of Pyramid P to Pyramid Q is 5 : 3.

EXAMPLE 4 Compare similar solids

CONSUMER ECONOMICS A store sells balls of yarn in two different sizes. The diameter of the larger ball is twice the diameter of the smaller ball. If the balls of yarn cost \$7.50 and \$1.50, respectively, which ball of yarn is the better buy?

Solution

STEP 1 **Compute** the ratio of volumes using the diameters.

$$\frac{\text{Volume of large ball}}{\text{Volume of small ball}} = \frac{2^3}{1^3} = \frac{8}{1}, \text{ or } 8:1$$

STEP 2 **Find** the ratio of costs.

$$\frac{\text{Price of large ball}}{\text{Volume of small ball}} = \frac{\$7.50}{\$1.50} = \frac{5}{1}, \text{ or } 5:1$$

STEP 3 **Compare** the ratios in Steps 1 and 2.

If the ratios were the same, neither ball would be a better buy. Comparing the smaller ball to the larger one, the price increase is less than the volume increase. So, you get more yarn for your dollar if you buy the larger ball of yarn.

▸ The larger ball of yarn is the better buy.

✓ GUIDED PRACTICE for Examples 2, 3, and 4

3. Cube C has a surface area of 54 square units and Cube D has a surface area of 150 square units. Find the scale factor of C to D. Find the edge length of C, and use the scale factor to find the volume of D.

4. **WHAT IF?** In Example 4, calculate a new price for the larger ball of yarn so that neither ball would be a better buy than the other.

12.7 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 9, and 27

★ = **TAKS PRACTICE AND REASONING** Exs. 7, 16, 28, 31, 33, 36, 37, and 38

◆ = **MULTIPLE REPRESENTATIONS** Ex. 34

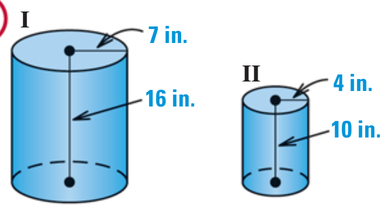
SKILL PRACTICE

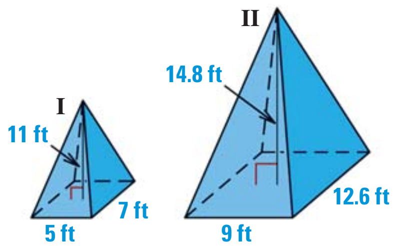
1. **VOCABULARY** What does it mean for two solids to be similar?

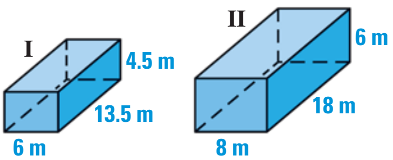
2. **WRITING** How are the volumes of similar solids related?

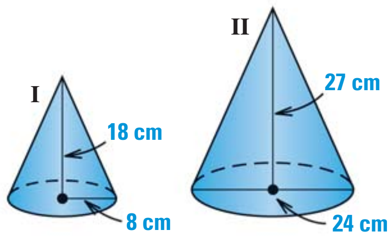
EXAMPLE 1 on p. 847 for Exs. 3–7

IDENTIFYING SIMILAR SOLIDS **Tell whether the pair of right solids is similar. *Explain* your reasoning.**

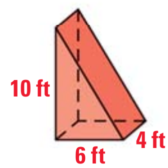
3. 

4. 

5. 

6. 

7. ★ **TAKS REASONING** Which set of dimensions corresponds to a triangular prism that is similar to the prism shown?



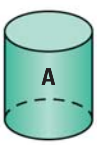
Ⓐ 2 feet by 1 foot by 5 feet Ⓑ 4 feet by 2 feet by 8 feet

Ⓒ 9 feet by 6 feet by 20 feet Ⓓ 15 feet by 10 feet by 25 feet

EXAMPLE 2 on p. 848 for Exs. 8–11

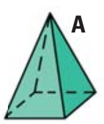
USING SCALE FACTOR **Solid A (shown) is similar to Solid B (not shown) with the given scale factor of A to B. Find the surface area and volume of Solid B.**

8. Scale factor of 1 : 2



$S = 150\pi$ in.^{2}
$V = 250\pi$ in.^{3}

9. Scale factor of 3 : 1



$S = 1500$ m^2
$V = 3434.6$ m^3

10. Scale factor of 5 : 2

$S = 2356.2$ cm^2
$V = 7450.9$ cm^3

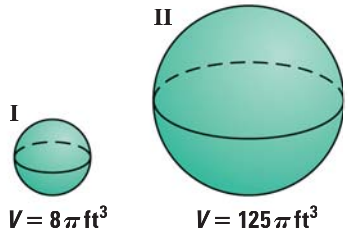
11. **ERROR ANALYSIS** The scale factor of two similar solids is 1 : 4. The volume of the smaller Solid A is 500π. *Describe* and correct the error in writing an equation to find the volume of the larger Solid B.

$$\frac{500\pi}{\text{Volume of B}} = \frac{1^2}{4^2}$$

EXAMPLE 3
on p. 849
for Exs. 12–18

FINDING SCALE FACTOR **In Exercises 12–15, Solid I is similar to Solid II. Find the scale factor of Solid I to Solid II.**

12.



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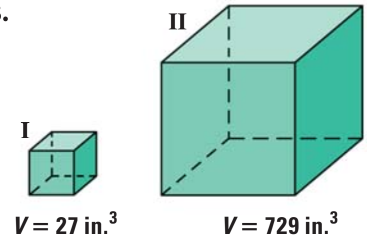

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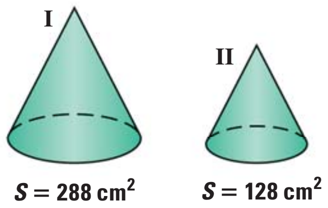
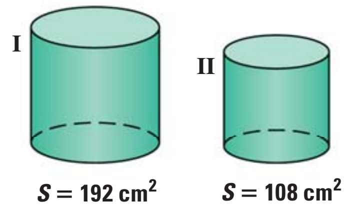


15.



16. **TAKS REASONING** The volumes of two similar cones are 8π and 27π. What is the ratio of the lateral areas of the cones?

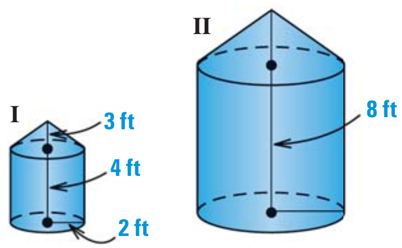
Ⓐ $\frac{8}{27}$ Ⓑ $\frac{1}{3}$ Ⓒ $\frac{4}{9}$ Ⓓ $\frac{2}{3}$

17. **FINDING A RATIO** Two spheres have volumes 2π cubic feet and 16π cubic feet. What is the ratio of the surface area of the smaller sphere to the surface area of the larger sphere?

18. **FINDING SURFACE AREA** Two cylinders have a scale factor of 2 : 3. The smaller cylinder has a surface area of 78π square meters. Find the surface area of the larger cylinder.

COMPOSITE SOLIDS **In Exercises 19–22, Solid I is similar to Solid II. Find the surface area and volume of Solid II.**

19.



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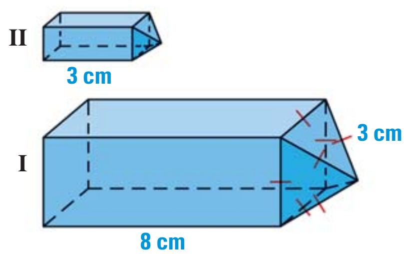

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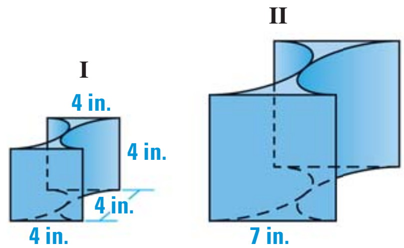

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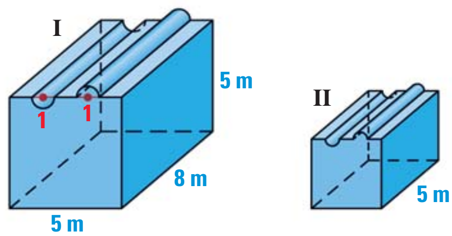


23. **ALGEBRA** Two similar cylinders have surface areas of 54π square feet and 384π square feet. The height of each cylinder is equal to its diameter. Find the radius and height of both cylinders.

24. **CHALLENGE** A plane parallel to the base of a cone divides the cone into two pieces with the dimensions shown. Find each ratio described.

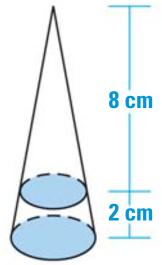
a. The area of the top shaded circle to the area of the bottom shaded circle

b. The slant height of the top part of the cone to the slant height of the whole cone

c. The lateral area of the top part of the cone to the lateral area of the whole cone

d. The volume of the top part of the cone to the volume of the whole cone

e. The volume of the top part of the cone to the volume of the bottom part



PROBLEM SOLVING

EXAMPLE 4 on p. 849 for Exs. 25–27

25. **COFFEE MUGS** The heights of two similar coffee mugs are 3.5 inches and 4 inches. The larger mug holds 12 fluid ounces. What is the capacity of the smaller mug?

TEXAS @HomeTutor for problem solving help at classzone.com

26. **ARCHITECTURE** You have a pair of binoculars that is similar in shape to the structure on page 847. Your binoculars are 6 inches high, and the height of the structure is 45 feet. Find the ratio of the volume of your binoculars to the volume of the structure.

TEXAS @HomeTutor for problem solving help at classzone.com

27. **PARTY PLANNING** Two similar punch bowls have a scale factor of 3 : 4. The amount of lemonade to be added is proportional to the volume. How much lemonade does the smaller bowl require if the larger bowl requires 64 fluid ounces?

28. **TAKS REASONING** Using the scale factor 2 : 5, sketch a pair of solids in the correct proportions. Label the dimensions of the solids.

29. **MULTI-STEP PROBLEM** Two oranges are both spheres with diameters 3.2 inches and 4 inches. The skin on both oranges has an average thickness of $\frac{1}{8}$ inch.

a. Find the volume of each unpeeled orange.

b. *Compare* the ratio of the diameters to the ratio of the volumes.

c. Find the diameter of each orange after being peeled.

d. *Compare* the ratio of surface areas of the peeled oranges to the ratio of the volumes of the peeled oranges.

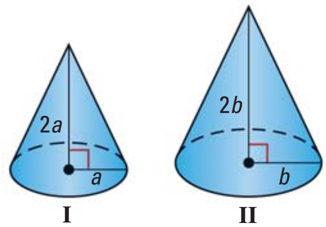
Animated Geometry at classzone.com

30. **ALGEBRA** Use the two similar cones shown.

a. What is the scale factor of Cone I to Cone II? What should the ratio of the volume of Cone I to the volume of Cone II be?

b. Write an expression for the volume of each solid.

c. Write and simplify an expression for the ratio of the volume of Cone I to the volume of Cone II. Does your answer agree with your answer to part (a)? *Explain*.



31. **TAKS REASONING** The scale factor of the model car at the right to the actual car is 1 : 18.

a. The model has length 8 inches. What is the length of the actual car?

b. Each tire of the model has a surface area of 12.1 square inches. What is the surface area of each tire of the actual car?

c. The actual car's engine has volume 8748 cubic inches. Find the volume of the model car's engine.

32. **USING VOLUMES** Two similar cylinders have volumes 16π and 432π. The larger cylinder has lateral area 72π. Find the lateral area of the smaller cylinder.

33. **TAKS REASONING** A snow figure is made using three balls of snow with diameters 25 centimeters, 35 centimeters, and 45 centimeters. The smallest weighs about 1.2 kilograms. Find the total weight of the snow used to make the snow figure. *Explain* your reasoning.

34. **MULTIPLE REPRESENTATIONS** A gas is enclosed in a cubical container with side length s in centimeters. Its temperature remains constant while the side length varies. By the *Ideal Gas Law*, the pressure P in atmospheres (atm) of the gas varies inversely with its volume.

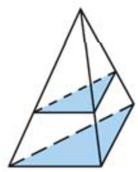
a. **Writing an Equation** Write an equation relating P and s. You will need to introduce a constant of variation k.

b. **Making a Table** Copy and complete the table below for various side lengths. Express the pressure P in terms of the constant k.

Side length s (cm)	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
Pressure P (atm)	?	$8k$	k	?	?

c. **Drawing a Graph** For this particular gas, $k = 1$. Use your table to sketch a graph of P versus s. Place P on the vertical axis and s on the horizontal axis. Does the graph show a linear relationship? *Explain*.

35. **CHALLENGE** A plane parallel to the base of a pyramid separates the pyramid into two pieces with equal volumes. The height of the pyramid is 12 feet. Find the height of the top piece.

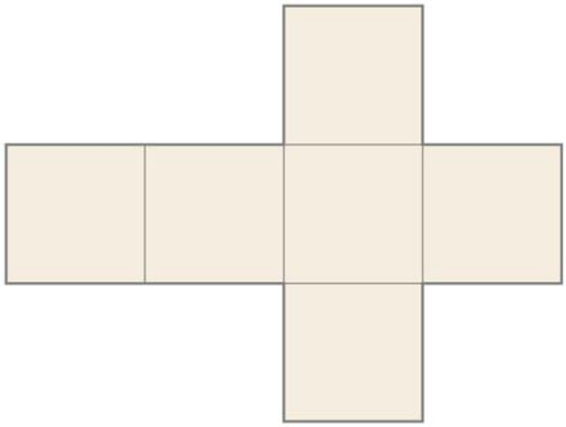


MIXED REVIEW FOR TAKS

TAKS PRACTICE at classzone.com

REVIEW
Lesson 12.4;
TAKS Workbook

36. **TAKS PRACTICE** The net of a cube is shown. Use a ruler to measure the dimensions of the cube to the nearest quarter-inch. What is the approximate volume of the cube? *TAKS Obj. 8*



Ⓐ 0.1 in.^{3}

Ⓑ 0.4 in.^{3}

Ⓒ 3 in.^{3}

Ⓓ 11 in.^{3}

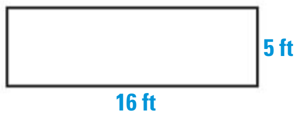
REVIEW
Lesson 3.5;
TAKS Workbook

37. **TAKS PRACTICE** What is the solution to the system of equations $-3x + y = 1$ and $x + y = -3$? *TAKS Obj. 4*

Ⓕ $(-5, 2)$ Ⓖ $(-1, -2)$ Ⓗ $(-0.5, -2.5)$ Ⓙ $(4, 1)$

REVIEW
Lesson 6.3;
TAKS Workbook

38. **TAKS PRACTICE** Which set of dimensions would produce a rectangle similar to the one shown? *TAKS Obj. 8*



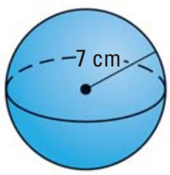
Ⓐ 2.5 ft by 32 ft Ⓑ 10 ft by 64 ft

Ⓒ 15 ft by 48 ft Ⓓ 25 ft by 85 ft

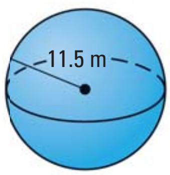
QUIZ *for Lessons 12.6–12.7*

Find the surface area and volume of the sphere. Round your answers to two decimal places. *(p. 838)*

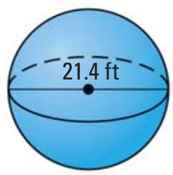
1.



2.

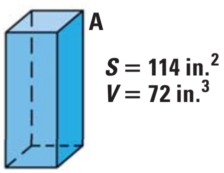


3.

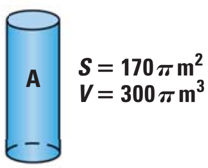


Solid A (shown) is similar to Solid B (not shown) with the given scale factor of A to B. Find the surface area *S* and volume *V* of Solid B. *(p. 847)*

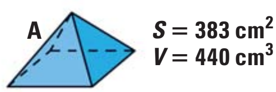
4. Scale factor of 1 : 3



5. Scale factor of 2 : 3



6. Scale factor of 5 : 4



7. Two similar cones have volumes 729π cubic feet and 343π cubic feet. What is the scale factor of the larger cone to the smaller cone? *(p. 847)*

MIXED REVIEW FOR TEKS

TAKS PRACTICE
classzone.com

Lessons 12.4–12.7

MULTIPLE CHOICE

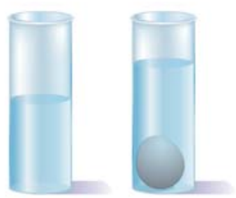
1. **OIL FUNNEL** The funnel shown is used to put oil in a car. Oil flows out of the funnel at a rate of 45 milliliters per second. About how long will it take to empty the funnel when it is full of oil? (1 mL = 1 cm^3) ***TEKS G.8.D***

Ⓐ 0.9 sec Ⓑ 4.4 sec
Ⓒ 6.5 sec Ⓓ 12.2 sec

2. **BEVERAGE CONTAINER** A cup in the shape of a cylinder has an inside diameter of 2.5 inches and an inside height of 7 inches. Jeremy wants to fill the cup with juice to 1 inch below the top. About how many fluid ounces of juice should he pour into the empty cup? (1 $in.^3$ = 0.554 fluid ounces) ***TEKS G.8.D***

Ⓕ 16.3 Ⓖ 19
Ⓗ 29.5 Ⓙ 65.3

3. **ROCKS** To accurately measure the radius of a spherical rock, Jeff places the rock into a cylindrical glass containing water. When he does so, the water level rises 0.14 inch. The radius of the glass is 2 inches. What is the radius of the rock? ***TEKS G.8.D***



Ⓐ 0.40 in. Ⓑ 0.50 in.
Ⓒ 0.65 in. Ⓓ 0.75 in.

4. **BASKETBALLS** An official men's basketball has a circumference of 29.5 inches. An official women's basketball has a circumference of 28.5 inches. What is the difference between the surface area of a men's basketball and a women's basketball? ***TEKS G.11.D***

Ⓕ 12.6 $in.^2$ Ⓖ 18.5 $in.^2$
Ⓗ 42.6 $in.^2$ Ⓙ 73.8 $in.^2$

5. **BOXES** A cardboard container is in the shape of a right rectangular prism with a length of 24 inches, a width of 16 inches, and a height of 20 inches. The container is filled with boxes of cookies. Each box of cookies is in the shape of a right rectangular prism and has a length of 8 inches, a width of 2 inches, and a height of 3 inches. What is the maximum number of boxes of cookies that will fit inside the cardboard container? ***TEKS G.8.D***

Ⓐ 30 Ⓑ 60
Ⓒ 48 Ⓓ 160

GRIDDED ANSWER

6. **SCULPTURE** The lawn of a library in San Antonio features spherical sculptures as shown. Melissa estimates the scale factor of one of the smaller spheres to its larger neighbor to be 4 : 5. She also estimates the diameter of the larger sphere to be 4 feet. According to Melissa's estimates, what is the volume of the smaller sphere in cubic feet? Round your answer to one decimal place. ***TEKS G.11.D***

12 CHAPTER SUMMARY

BIG IDEAS

For Your Notebook

Big Idea 1

TEKS G.5.B

Exploring Solids and Their Properties

Euler's Theorem is useful when finding the number of faces, edges, or vertices on a polyhedron, especially when one of those quantities is difficult to count by hand.

For example, suppose you want to find the number of edges on a regular icosahedron, which has 20 faces. You count 12 vertices on the solid. To calculate the number of edges, use Euler's Theorem:

$F + V = E + 2$ **Write Euler's Theorem.**

$20 + 12 = E + 2$ **Substitute known values.**

$30 = E$ **Solve for *E*.**

Big Idea 2

TEKS G.8.D

Solving Problems Using Surface Area and Volume

Figure	Surface Area	Volume
Right prism	$S = 2B + Ph$	$V = Bh$
Right cylinder	$S = 2B + Ch$	$V = Bh$
Regular pyramid	$S = B + \frac{1}{2}P\ell$	$V = \frac{1}{3}Bh$
Right cone	$S = B + \frac{1}{2}C\ell$	$V = \frac{1}{3}Bh$
Sphere	$S = 4\pi r^2$	$V = \frac{4}{3}\pi r^3$

The volume formulas for prisms, cylinders, pyramids, and cones can be used for oblique solids.

While many of the above formulas can be written in terms of more detailed variables, it is more important to remember the more general formulas for a greater understanding of why they are true.

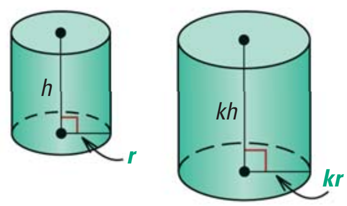
Big Idea 3

TEKS G.11.D

Connecting Similarity to Solids

The similarity concepts learned in Chapter 6 can be extended to 3-dimensional figures as well.

Suppose you have a right cylindrical can whose surface area and volume are known. You are then given a new can whose linear dimensions are k times the dimensions of the original can. If the surface area of the original can is S and the volume of the original can is V, then the surface area and volume of the new can can be expressed as k^2S and k^3V, respectively.



12 CHAPTER REVIEW

TEXAS @HomeTutor
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- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- polyhedron, *p. 794*
  face, edge, vertex, base
- regular polyhedron, *p. 796*
- convex polyhedron, *p. 796*
- Platonic solids, *p. 796*
- tetrahedron, *p. 796*
- cube, *p. 796*
- octahedron, *p. 796*
- dodecahedron, *p. 796*
- icosahedron, *p. 796*
- cross section, *p. 797*
- prism, *p. 803*
  lateral faces, lateral edges
- surface area, *p. 803*
- lateral area, *p. 803*
- net, *p. 803*
- right prism, *p. 804*
- oblique prism, *p. 804*
- cylinder, *p. 805*
- right cylinder, *p. 805*
- pyramid, *p. 810*
- vertex of a pyramid, *p. 810*
- regular pyramid, *p. 810*
- slant height, *p. 810*
- cone, *p. 812*
- vertex of a cone, *p. 812*
- right cone, *p. 812*
- lateral surface, *p. 812*
- volume, *p. 819*
- sphere, *p. 838*
  center, radius, chord, diameter
- great circle, *p. 839*
- hemisphere, *p. 839*
- similar solids, *p. 847*

VOCABULARY EXERCISES

1. Copy and complete: A _?_ is the set of all points in space equidistant from a given point.

2. **WRITING** Sketch a right rectangular prism and an oblique rectangular prism. *Compare* the prisms.

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 12.

12.1 Explore Solids *pp. 794–801*

EXAMPLE

A polyhedron has 16 vertices and 24 edges. How many faces does the polyhedron have?

$F + V = E + 2$ — **Euler's Theorem**

$F + 16 = 24 + 2$ — **Substitute known values.**

$F = 10$ — **Solve for *F*.**

▶ The polyhedron has 10 faces.

EXERCISES

EXAMPLES 2 and 3 on pp. 796–797 for Exs. 3–5

Use Euler's Theorem to find the value of n.

3. Faces: 20
Vertices: n
Edges: 30

4. Faces: n
Vertices: 6
Edges: 12

5. Faces: 14
Vertices: 24
Edges: n

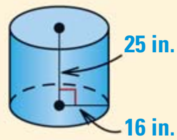
12 CHAPTER REVIEW

12.2 Surface Area of Prisms and Cylinders

pp. 803–809

EXAMPLE

Find the surface area of the right cylinder.



$S = 2\pi r^2 + 2\pi rh$ **Write formula.**

$= 2\pi(16)^2 + 2\pi(16)(25)$ **Substitute for *r* and *h*.**

$= 1312\pi$ **Simplify.**

≈ 4121.77 **Use a calculator.**

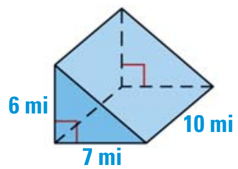
▶ The surface area of the cylinder is about 4121.77 square inches.

EXERCISES

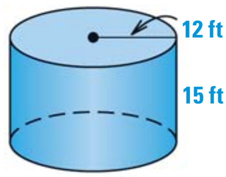
EXAMPLES 2, 3, and 4 on pp. 804–806 for Exs. 6–9

Find the surface area of the right prism or right cylinder. Round your answer to two decimal places, if necessary.

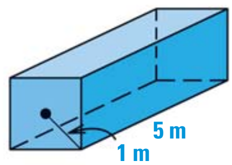
6.



7.



8.



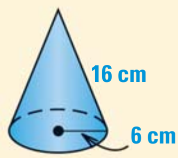
9. A cylinder has a surface area of 44π square meters and a radius of 2 meters. Find the height of the cylinder.

12.3 Surface Area of Pyramids and Cones

pp. 810–817

EXAMPLE

Find the lateral area of the right cone.



Lateral area $= \pi r\ell$ **Write formula.**

$= \pi(6)(16)$ **Substitute for *r* and ℓ.**

$= 96\pi$ **Simplify.**

≈ 301.59 **Use a calculator.**

▶ The lateral area of the cone is about 301.59 square centimeters.

EXERCISES

EXAMPLES 1, 2, and 4 on pp. 810–813 for Exs. 10–12

10. Find the surface area of a right square pyramid with base edge length 2 feet and height 5 feet.

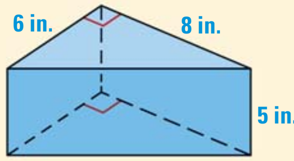
11. The surface area of a cone with height 15 centimeters is 500π square centimeters. Find the radius of the base of the cone. Round your answer to two decimal places.

12. Find the surface area of a right octagonal pyramid with height 2.5 yards, and its base has apothem length 1.5 yards.

12.4 Volume of Prisms and Cylinders *pp. 819–825*

EXAMPLE

Find the volume of the right triangular prism.



The area of the base is $B = \frac{1}{2}(6)(8) = 24$ square inches. Use $h = 5$ to find the volume.

$V = Bh$ **Write formula.**

$= 24(5)$ **Substitute for B and h.**

$= 120$ **Simplify.**

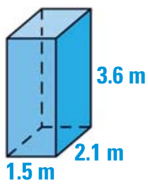
▶ The volume of the prism is 120 cubic inches.

EXERCISES

EXAMPLES 2 and 4 on pp. 820–821 for Exs. 13–15

Find the volume of the right prism or oblique cylinder. Round your answer to two decimal places.

13.



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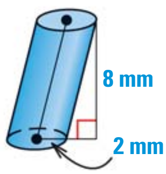

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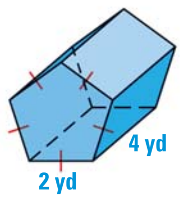
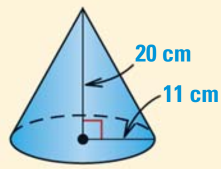


12.5 Volume of Pyramids and Cones *pp. 829–836*

EXAMPLE

Find the volume of the right cone.



The area of the base is $B = \pi r^2 = \pi(11)^2 \approx 380.13 \text{ cm}^2$. Use $h = 20$ to find the volume.

$V = \frac{1}{3}Bh$ **Write formula.**

$\approx \frac{1}{3}(380.13)(20)$ **Substitute for B and h.**

≈ 2534.20 **Simplify.**

▶ The volume of the cone is about 2534.20 cubic centimeters.

EXERCISES

EXAMPLES 1 and 2 on pp. 829–830 for Exs. 16–17

16. A cone with diameter 16 centimeters has height 15 centimeters. Find the volume of the cone. Round your answer to two decimal places.

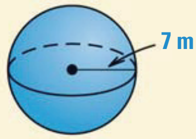
17. The volume of a pyramid is 60 cubic inches and the height is 15 inches. Find the area of the base.

12 CHAPTER REVIEW

12.6 Surface Area and Volume of Spheres *pp. 838–845*

EXAMPLE

Find the surface area of the sphere.



$S = 4\pi r^2$ **Write formula.**

$= 4\pi(7)^2$ **Substitute 7 for *r*.**

$= 196\pi$ **Simplify.**

▶ The surface area of the sphere is 196π, or about 615.75 square meters.

EXERCISES

EXAMPLES 1, 4, and 5 on pp. 839, 841 for Exs. 18–19

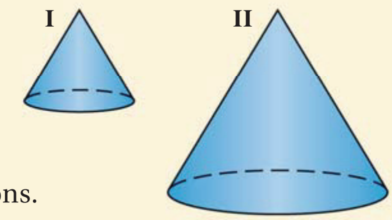
18. ASTRONOMY The shape of Pluto can be approximated as a sphere of diameter 2390 kilometers. Find the surface area and volume of Pluto. Round your answer to two decimal places.

19. A solid is composed of a cube with side length 6 meters and a hemisphere with diameter 6 meters. Find the volume of the solid. Round your answer to two decimal places.

12.7 Explore Similar Solids *pp. 847–854*

EXAMPLE

The cones are similar with a scale factor of 1:2. Find the surface area and volume of Cone II given that the surface area of Cone I is 384π square inches and the volume of Cone I is 768π cubic inches.



Use Theorem 12.13 to write and solve two proportions.

$$\frac{\text{Surface area of I}}{\text{Surface area of II}} = \frac{a^2}{b^2} \qquad \frac{\text{Volume of I}}{\text{Volume of II}} = \frac{a^3}{b^3}$$

$$\frac{384\pi}{\text{Surface area of II}} = \frac{1^2}{2^2} \qquad \frac{768\pi}{\text{Volume of II}} = \frac{1^3}{2^3}$$

$$\text{Surface area of II} = 1536\pi \text{ in.}^2 \qquad \text{Volume of II} = 6144\pi \text{ in.}^3$$

▶ The surface area of Cone II is 1536π, or about 4825.48 square inches, and the volume of Cone II is 6144π, or about 19,301.93 cubic inches.

EXERCISES

EXAMPLE 2 on p. 848 for Exs. 20–22

Solid A is similar to Solid B with the given scale factor of A to B. The surface area and volume of Solid A are given. Find the surface area and volume of Solid B.

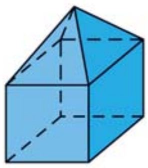
20. Scale factor of 1:4
$S = 62 \text{ cm}^2$
$V = 30 \text{ cm}^3$

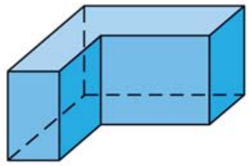
21. Scale factor of 1:3
$S = 112\pi \text{ m}^2$
$V = 160\pi \text{ m}^3$

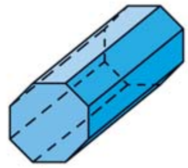
22. Scale factor of 2:5
$S = 144\pi \text{ yd}^2$
$V = 288\pi \text{ yd}^3$

12 CHAPTER TEST

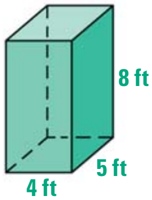
Find the number of faces, vertices, and edges of the polyhedron. Check your answer using Euler's Theorem.

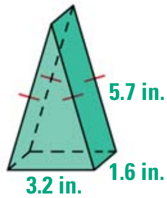
1. 

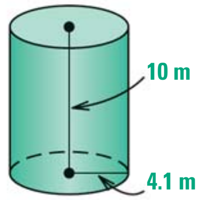
2. 

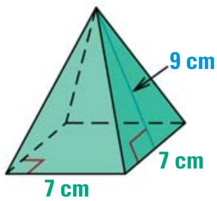
3. 

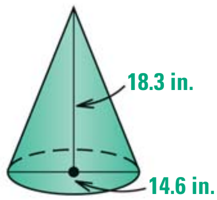
Find the surface area of the solid. The prisms, pyramids, cylinders, and cones are right. Round your answer to two decimal places, if necessary.

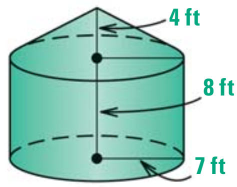
4. 

5. 

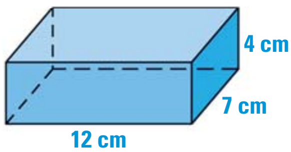
6. 

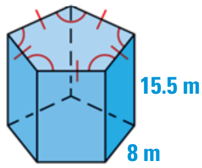
7. 

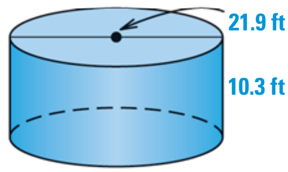
8. 

9. 

Find the volume of the right prism or right cylinder. Round your answer to two decimal places, if necessary.

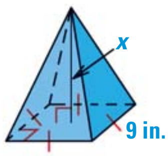
10. 

11. 

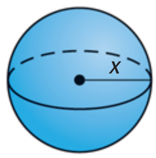
12. 

In Exercises 13–15, solve for *x*.

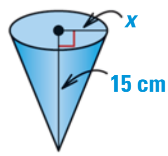
13. Volume $= 324 \text{ in.}^3$



14. Volume $= \frac{32\pi}{3} \text{ ft}^3$



15. Volume $= 180\pi \text{ cm}^3$



16. MARBLES The diameter of the marble shown is 35 millimeters. Find the surface area and volume of the marble.

17. PACKAGING Two similar cylindrical cans have a scale factor of 2 : 3. The smaller can has surface area 308π square inches and volume 735π cubic inches. Find the surface area and volume of the larger can.

12 TAKS PREPARATION

TEXAS TAKS Obj. 10 TEKS 8.14.B, 8.14.C

REVIEWING THE PROBLEM SOLVING PLAN

Some math problems require you to do more than just perform calculations. Approach these types of problems with an organized plan.

A Problem Solving Plan

Step 1 **Understand the problem.** Read the problem carefully. Organize the information you are given and decide what you need to find.

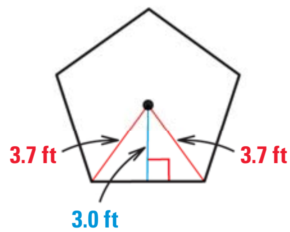
Step 2 **Make a plan to solve the problem.** Choose a problem solving strategy.

Step 3 **Carry out the plan to solve the problem.** Use the problem solving strategy to answer the question.

Step 4 **Evaluate the solution to see if your answer is reasonable.** Reread the problem and see if your answer agrees with the given information.

EXAMPLE

What is the perimeter of the regular pentagon to the nearest foot?



Solution

STEP 1 **Understand the problem.** You want to find the perimeter of the pentagon. The perimeter is the sum of all the side lengths. The pentagon is regular, so all the sides have the same length.

STEP 2 **Make a plan to solve the problem.** Use the Pythagorean Theorem to find the length of the third side of the right triangle. Find the length of one side of the pentagon. Then find its perimeter.

STEP 3 **Carry out the plan.**

$3.0^2 + x^2 = 3.7^2$ **Pythagorean Theorem**

$x^2 = 4.69$ **Simplify.**

$x \approx 2.17$ **Find the square root of each side.**

The third side of the triangle is about 2.17 feet long. The side of the pentagon is about $2 \times 2.17 = 4.34$ feet long. The pentagon is regular, so the perimeter is about $5 \times 4.34 = 21.7$ feet.

STEP 4 **Evaluate the solution.** The question asked for the perimeter to the nearest foot. The perimeter is actually about 21.7 feet, so the answer given is rounded to 22 feet.

PROBLEM SOLVING ON TAKS

Below are examples that test problem solving skills in multiple choice format. Try solving the problems before looking at the solutions. (Cover the solutions with a piece of paper.) Then check your solutions against the ones given.

1. Shannon has 4 more bracelets than Elizabeth. Kara has twice as many bracelets as Shannon. Altogether the girls have 24 bracelets. Which equation can you use to find the number of bracelets each girl has?

A $x + 2x + 4x = 24$

B $x + (x + 4) + 2x = 24$

C $x + (x + 4) + 2(x + 4) = 24$

D $x + 4x + 2(4x) = 24$

Solution

You need to write an equation for this situation. Determine what can be represented by x. Then write expressions relating the given information to x. Finally, set the sum of the expressions equal to the total.

Let x be the number of bracelets Elizabeth has. Then Shannon has $(x + 4)$ bracelets and Kara has $2(x + 4)$ bracelets. The equation that can be used to find how many bracelets each girl has is: $x + (x + 4) + 2(x + 4) = 24$.

So, the correct answer is C.

A B C D

2. Troy wants to use 60 feet of fencing to enclose a vegetable garden. Which shape uses all of the fencing and encloses the largest area?

F A rectangle with a length of 18 feet and a width of 12 feet

G An equilateral triangle with a side length of 17 feet

H A regular hexagon with a side length of 9 feet

J A square with a side length of 15 feet

Solution

Troy has 60 feet of fencing to use. First, you need to determine which of the given shapes has a perimeter of 60 feet. Then determine which shape also encloses the largest area.

The perimeters of the shapes are:

Rectangle: $18 + 12 + 18 + 12 = 60$ feet

Equilateral triangle: $3 \times 17 = 51$ feet

Regular hexagon: $6 \times 9 = 54$ feet

Square: $4 \times 15 = 60$ feet

So, the rectangle and the square use all the fencing.

The area of the rectangle is $18 \times 12 = 216$ ft^2.

The area of the square is $15 \times 15 = 225$ ft^2.

The square uses all of the fencing and encloses the largest area.

So, the correct answer is J.

F G H J

12 TAKS PRACTICE

PRACTICE FOR TAKS OBJECTIVE 10

1. The school dance club plans to attend a ballet in 5 weeks. The total cost is $85 per person. The club has $135 in its account and will divide the money equally among the 9 members who attend the ballet. Sasha is planning to attend the ballet and has already saved $35. How much more money does Sasha need in order to cover her cost to attend the ballet?

A $15

B $35

C $50

D $70

2. The blueprint dimensions for a house are proportional to the actual dimensions of the house. On the blueprint, the foundation of the house is 26 centimeters long by 10 centimeters wide. The actual length of the foundation of the house is 65 feet. What is the actual width of the foundation?

F 25 feet

G 30 feet

H 49 feet

J 169 feet

3. All of the tickets to the annual drama production at Rocco's school were sold. Each adult ticket cost $8 and each student ticket cost $5. The auditorium at the school has 801 seats. Two thirds of the seats were sold at the student rate. The school donated half of the proceeds from the event to charity and used a portion of the remainder to reimburse costs of the production. How much money was donated to charity?

A $403

B $2403

C $4806

D $9612

4. Brian is having lunch at a local diner. The total cost of his lunch is $7.45 including tax. He has $10 and wants to leave a tip equal to 15% of the total bill. Is $10 enough to cover the cost of his lunch and the 15% tip for the server?

F No, he needs $0.10 more.

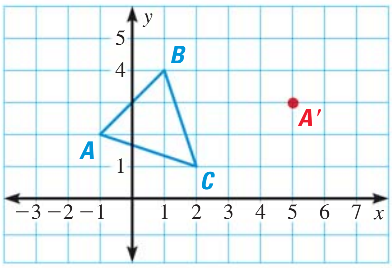
G No, he needs $1.81 more.

H Yes, and he has $1.43 left over.

J Yes, and he has the exact amount.

MIXED TAKS PRACTICE

5. In the diagram below, $\triangle ABC$ is translated so that A is mapped to A'.



Which set of ordered pairs best identifies points B' and C'? *TAKS Obj. 6*

A $B'(7, 5)$, $C'(8, 2)$

B $B'(2, 10)$, $C'(3, 7)$

C $B'(7, 6)$, $C'(8, 3)$

D $B'(4, 5)$, $C'(6, 1)$

6. A cylindrical can has a volume of 352 cubic centimeters and a diameter of 8 centimeters. Which is the best estimate of the height of the can? *TAKS Obj. 8*

F 7 cm

G 14 cm

H 21 cm

J 28 cm

MIXED TAKS PRACTICE

7. The graph of which equation has a y-intercept of 6 and a slope of 3? *TAKS Obj. 3*

A $-12x + 2y = 6$

B $-6x + 2y = 12$

C $6x + 2y = 12$

D $12x + 2y = 6$

8. What is the value of y if $(3, y)$ is a solution of $-x + 2y = 14$? *TAKS Obj. 4*

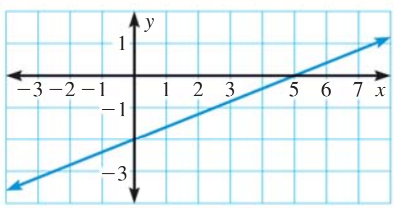
F 5

G 5.5

H 8

J 8.5

9. Which coordinate points represent the x- and y-intercepts of the graph shown? *TAKS Obj. 3*



A $(-2, 0)$ and $(0, 5)$

B $(0, -2)$ and $(5, 0)$

C $(0, 5)$ and $(0, -2)$

D $(5, 0)$ and $(-2, 0)$

10. Which expression is equivalent to $\dfrac{81x^7z^{-4}}{9x^2y^0z^5}$? *TAKS Obj. 5*

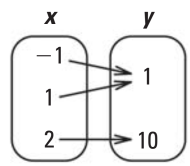
F $\dfrac{9x^5}{z^9}$

G $\dfrac{9x^5}{yz^5}$

H $\dfrac{x^5}{9z^9}$

J $\dfrac{9x^5z^9}{y}$

11. Which function best represents the mapping shown? *TAKS Obj. 1*



A $y = 2x + 3$

B $y = x^2 + 6$

C $y = 3x^2 - 2$

D $y = 4x^2 - 3$

12. What is the effect on the graph of the equation $y = x^2 + 1$ when the equation is changed to $y = x^2 + 4$? *TAKS Obj. 5*

F The graph of $y = x^2 + 1$ is translated 3 units up.

G The graph of $y = x^2 + 1$ is translated 4 units up.

H The graph of $y = x^2 + 1$ is translated 3 units down.

J The graph of $y = x^2 + 1$ is translated 4 units down.

13. Which expression is equivalent to $6(x^2 + 3x) - (x - 5)$? *TAKS Obj. 2*

A $6x^2 + 8x + 5$

B $6x^2 + 8x - 5$

C $6x^2 + 17x + 5$

D $6x^2 + 17x - 5$

14. **GRIDDED ANSWER** Of the 32 students in your class, 75% have brown hair. Of the remaining students, 37.5% have blonde hair. How many students in your class have blonde hair? *TAKS Obj. 9*

Record your answer and fill in the bubbles on your answer document. Be sure to use the correct place value.