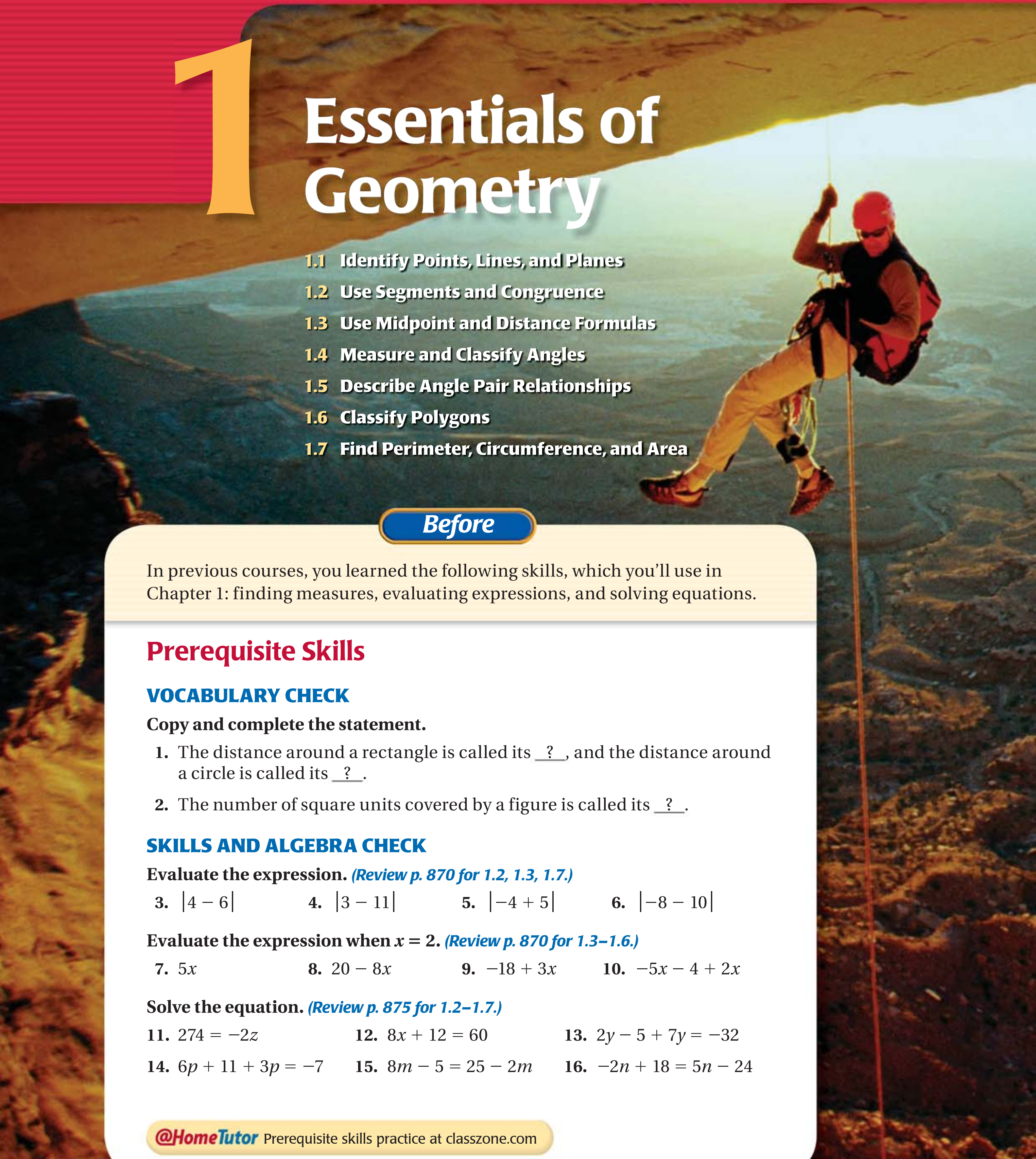


1 Essentials of Geometry

Before

In previous courses, you learned the following skills, which you'll use in Chapter 1: finding measures, evaluating expressions, and solving equations.

Prerequisite Skills

VOCABULARY CHECK

Copy and complete the statement.

1. The distance around a rectangle is called its ___?___, and the distance around a circle is called its ___?___.

2. The number of square units covered by a figure is called its ___?___.

SKILLS AND ALGEBRA CHECK

Evaluate the expression. *(Review p. 870 for 1.2, 1.3, 1.7.)*

3. $|4 - 6|$ **4.** $|3 - 11|$ **5.** $|-4 + 5|$ **6.** $|-8 - 10|$

Evaluate the expression when $x = 2$. *(Review p. 870 for 1.3–1.6.)*

7. $5x$ **8.** $20 - 8x$ **9.** $-18 + 3x$ **10.** $-5x - 4 + 2x$

Solve the equation. *(Review p. 875 for 1.2–1.7.)*

11. $274 = -2z$ **12.** $8x + 12 = 60$ **13.** $2y - 5 + 7y = -32$

14. $6p + 11 + 3p = -7$ **15.** $8m - 5 = 25 - 2m$ **16.** $-2n + 18 = 5n - 24$

@HomeTutor Prerequisite skills practice at classzone.com

Now

In Chapter 1, you will apply the big ideas listed below and reviewed in the Chapter Summary on page 59. You will also use the key vocabulary listed below.

Big Ideas

1. **Describing geometric figures**
2. **Measuring geometric figures**
3. **Understanding equality and congruence**

KEY VOCABULARY

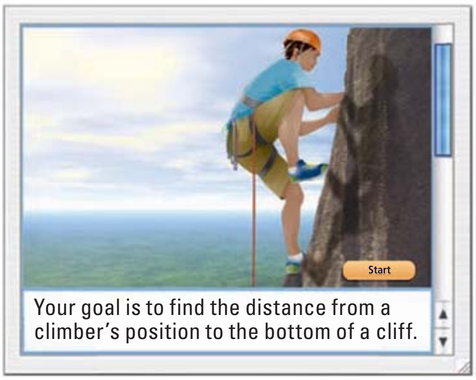
- undefined terms, *p. 2*
  point, line, plane
- defined terms, *p. 3*
- line segment, endpoints, *p. 3*
- ray, opposite rays, *p. 3*
- postulate, axiom, *p. 9*
- congruent segments, *p. 11*
- midpoint, *p. 15*
- segment bisector, *p. 15*
- acute, right, obtuse, straight angles, *p. 25*
- congruent angles, *p. 26*
- angle bisector, *p. 28*
- linear pair, *p. 37*
- vertical angles, *p. 37*
- polygon, *p. 42*
- convex, concave, *p. 42*
- *n*-gon, *p. 43*
- equilateral, equiangular, regular, *p. 43*

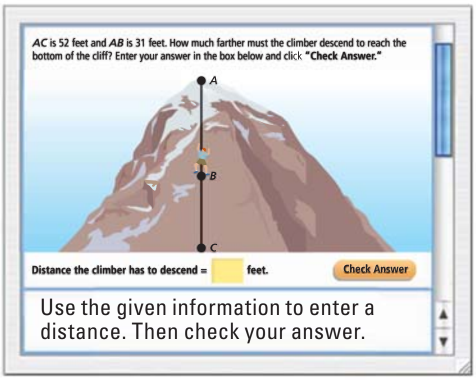
Why?

Geometric figures can be used to represent real-world situations. For example, you can show a climber's position along a stretched rope by a point on a line segment.

Animated Geometry

The animation illustrated below for Exercise 35 on page 14 helps you answer this question: How far must a climber descend to reach the bottom of a cliff?





Animated **Geometry** at classzone.com

Other animations for Chapter 1: pages 3, 21, 25, 43, and 52

1.1 Identify Points, Lines, and Planes

TEKS G.1.A, G.1.B, G.7.A

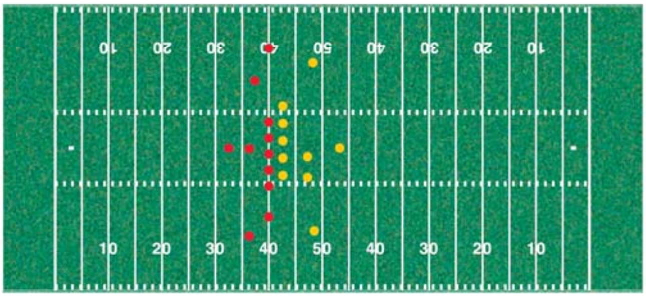
Before You studied basic concepts of geometry.

Now You will name and sketch geometric figures.

Why So you can use geometry terms in the real world, as in Ex. 13.

Key Vocabulary
- **undefined terms**
  point, line, plane
- **collinear points**
- **coplanar points**
- **defined terms**
- **line segment**
- **endpoints**
- **ray**
- **opposite rays**
- **intersection**

In the diagram of a football field, the positions of players are represented by *points*. The yard lines suggest *lines*, and the flat surface of the playing field can be thought of as a *plane*.



In geometry, the words *point*, *line*, and *plane* are **undefined terms**. These words do not have formal definitions, but there is agreement about what they mean.

TAKE NOTES
When you write new concepts and yellow-highlighted vocabulary in your notebook, be sure to copy all associated diagrams.

KEY CONCEPT — *For Your Notebook*

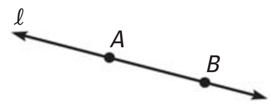
Undefined Terms

Point A **point** has no dimension. It is represented by a dot.

point *A*

Line A **line** has one dimension. It is represented by a line with two arrowheads, but it extends without end.

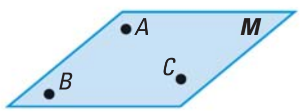
Through any two points, there is exactly one line. You can use any two points on a line to name it.



line ℓ, line *AB* ($\overleftrightarrow{AB}$), or line *BA* ($\overleftrightarrow{BA}$)

Plane A **plane** has two dimensions. It is represented by a shape that looks like a floor or a wall, but it extends without end.

Through any three points not on the same line, there is exactly one plane. You can use three points that are not all on the same line to name a plane.



plane *M* or plane *ABC*

Collinear points are points that lie on the same line. **Coplanar points** are points that lie in the same plane.

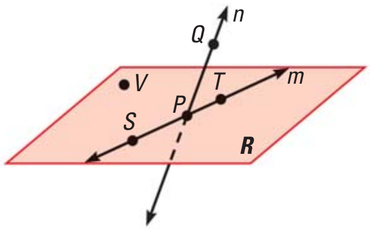
EXAMPLE 1 Name points, lines, and planes

VISUAL REASONING

There is a line through points *S* and *Q* that is not shown in the diagram. Try to imagine what plane *SPQ* would look like if it were shown.

a. Give two other names for $\overleftrightarrow{PQ}$ and for plane *R*.

b. Name three points that are collinear. Name four points that are coplanar.



Solution

a. Other names for $\overleftrightarrow{PQ}$ are $\overleftrightarrow{QP}$ and line *n*. Other names for plane *R* are plane *SVT* and plane *PTV*.

b. Points *S*, *P*, and *T* lie on the same line, so they are collinear. Points *S*, *P*, *T*, and *V* lie in the same plane, so they are coplanar.

Animated Geometry at classzone.com

✓ GUIDED PRACTICE for Example 1

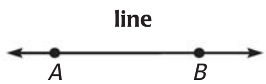
1. Use the diagram in Example 1. Give two other names for $\overleftrightarrow{ST}$. Name a point that is *not* coplanar with points *Q*, *S*, and *T*.

DEFINED TERMS In geometry, terms that can be described using known words such as *point* or *line* are called **defined terms**.

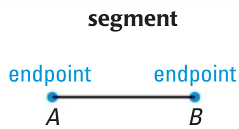
KEY CONCEPT *For Your Notebook*

Defined Terms: Segments and Rays

Line *AB* (written as $\overleftrightarrow{AB}$) and points *A* and *B* are used here to define the terms below.

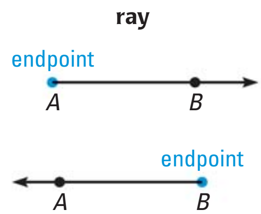


Segment The **line segment** *AB*, or **segment** *AB*, (written as $\overline{AB}$) consists of the **endpoints** *A* and *B* and all points on $\overleftrightarrow{AB}$ that are between *A* and *B*. Note that $\overline{AB}$ can also be named $\overline{BA}$.

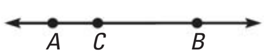


Ray The **ray** *AB* (written as $\overrightarrow{AB}$) consists of the endpoint *A* and all points on $\overleftrightarrow{AB}$ that lie on the same side of *A* as *B*.

Note that $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are different rays.



If point *C* lies on $\overleftrightarrow{AB}$ between *A* and *B*, then $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are **opposite rays**.

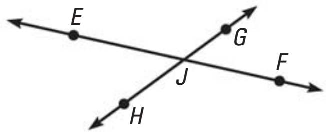


Segments and rays are collinear if they lie on the same line. So, opposite rays are collinear. Lines, segments, and rays are coplanar if they lie in the same plane.

EXAMPLE 2 Name segments, rays, and opposite rays

a. Give another name for $\overline{GH}$.

b. Name all rays with endpoint J. Which of these rays are opposite rays?



AVOID ERRORS

In Example 2, $\overrightarrow{JG}$ and $\overrightarrow{JF}$ have a common endpoint, but are not collinear. So they are *not* opposite rays.

Solution

a. Another name for $\overline{GH}$ is $\overline{HG}$.

b. The rays with endpoint J are $\overrightarrow{JE}$, $\overrightarrow{JG}$, $\overrightarrow{JF}$, and $\overrightarrow{JH}$. The pairs of opposite rays with endpoint J are $\overrightarrow{JE}$ and $\overrightarrow{JF}$, and $\overrightarrow{JG}$ and $\overrightarrow{JH}$.

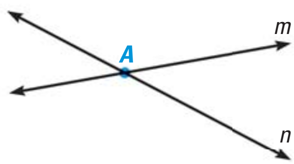
✓ GUIDED PRACTICE for Example 2

Use the diagram in Example 2.

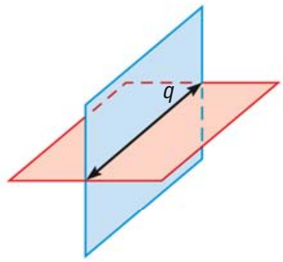
2. Give another name for $\overline{EF}$.

3. Are $\overrightarrow{HJ}$ and $\overrightarrow{JH}$ the same ray? Are $\overrightarrow{HJ}$ and $\overrightarrow{HG}$ the same ray? *Explain.*

INTERSECTIONS Two or more geometric figures *intersect* if they have one or more points in common. The **intersection** of the figures is the set of points the figures have in common. Some examples of intersections are shown below.



The intersection of two different lines is a point.



The intersection of two different planes is a line.

EXAMPLE 3 Sketch intersections of lines and planes

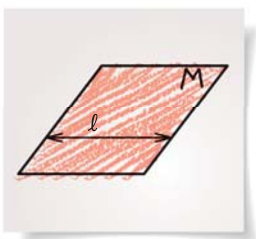
a. Sketch a plane and a line that is in the plane.

b. Sketch a plane and a line that does not intersect the plane.

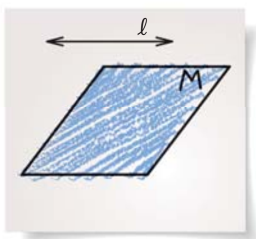
c. Sketch a plane and a line that intersects the plane at a point.

Solution

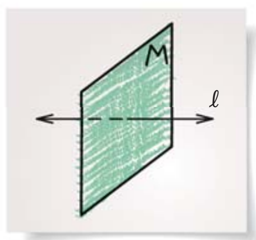
a.



b.



c.



EXAMPLE 4 Sketch intersections of planes

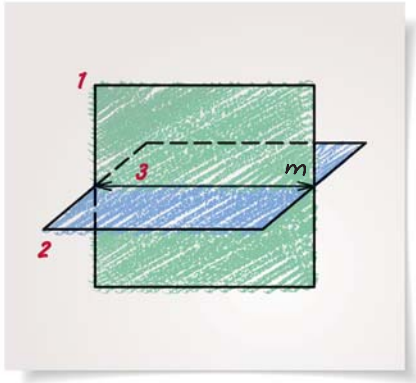
Sketch two planes that intersect in a line.

Solution

STEP 1 **Draw** a vertical plane. Shade the plane.

STEP 2 **Draw** a second plane that is horizontal. Shade this plane a different color. Use dashed lines to show where one plane is hidden.

STEP 3 **Draw** the line of intersection.

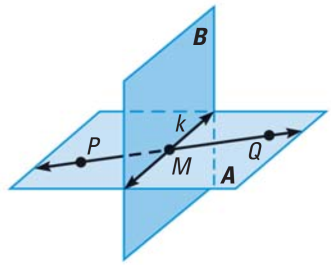


✓ GUIDED PRACTICE for Examples 3 and 4

4. Sketch two different lines that intersect a plane at the same point.

Use the diagram at the right.

5. Name the intersection of $\overleftrightarrow{PQ}$ and line k.
6. Name the intersection of plane A and plane B.
7. Name the intersection of line k and plane A.



1.1 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 15, 19, and 43

★ = **TAKS PRACTICE AND REASONING** Exs. 13, 16, 43, 47, and 48

SKILL PRACTICE

1. **VOCABULARY** Write in words what each of the following symbols means.

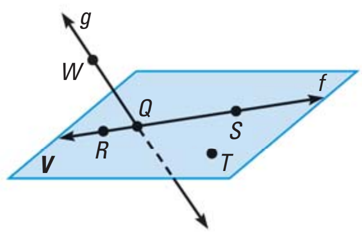
   a. Q **b.** $\overline{MN}$ **c.** $\overrightarrow{ST}$ **d.** $\overleftrightarrow{FG}$

2. **WRITING** *Compare* collinear points and coplanar points. Are collinear points also coplanar? Are coplanar points also collinear? *Explain.*

EXAMPLE 1 on p. 3 for Exs. 3–7

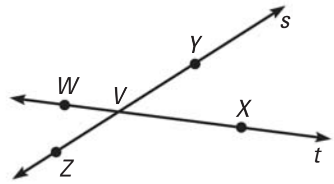
NAMING POINTS, LINES, AND PLANES **In Exercises 3–7, use the diagram.**

3. Give two other names for $\overleftrightarrow{WQ}$.
4. Give another name for plane V.
5. Name three points that are collinear. Then name a fourth point that is *not* collinear with these three points.
6. Name a point that is *not* coplanar with R, S, and T.
7. **WRITING** Is point W coplanar with points Q and R? *Explain.*



EXAMPLE 2
on p. 4
for Exs. 8–13

NAMING SEGMENTS AND RAYS **In Exercises 8–12, use the diagram.**



8. What is another name for $\overline{ZY}$?

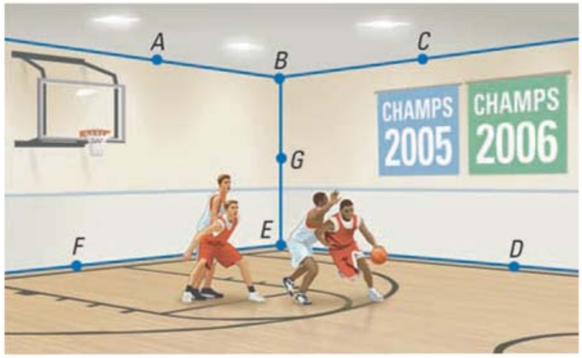
9. Name all rays with endpoint *V*.

10. Name two pairs of opposite rays.

11. Give another name for $\overrightarrow{WV}$.

12. **ERROR ANALYSIS** A student says that $\overrightarrow{VW}$ and $\overrightarrow{VZ}$ are opposite rays because they have the same endpoint. *Describe* the error.

13. **TAKS REASONING** Which statement about the diagram at the right is true?



Ⓐ *A*, *B*, and *C* are collinear.

Ⓑ *C*, *D*, *E*, and *G* are coplanar.

Ⓒ *B* lies on $\overrightarrow{GE}$.

Ⓓ $\overrightarrow{EF}$ and $\overrightarrow{ED}$ are opposite rays.

EXAMPLES 3 and 4
on pp. 4–5
for Exs. 14–23

SKETCHING INTERSECTIONS **Sketch the figure described.**

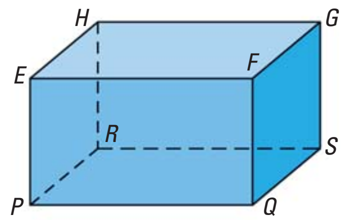
14. Three lines that lie in a plane and intersect at one point

15. One line that lies in a plane, and one line that does not lie in the plane

16. **TAKS REASONING** Line *AB* and line *CD* intersect at point *E*. Which of the following are opposite rays?

Ⓐ $\overrightarrow{EC}$ and $\overrightarrow{ED}$ Ⓑ $\overrightarrow{CE}$ and $\overrightarrow{DE}$ Ⓒ $\overrightarrow{AB}$ and $\overrightarrow{BA}$ Ⓓ $\overrightarrow{AE}$ and $\overrightarrow{BE}$

READING DIAGRAMS **In Exercises 17–22, use the diagram at the right.**



17. Name the intersection of $\overleftrightarrow{PR}$ and $\overleftrightarrow{HR}$.

18. Name the intersection of plane *EFG* and plane *FGS*.

19. Name the intersection of plane *PQS* and plane *HGS*.

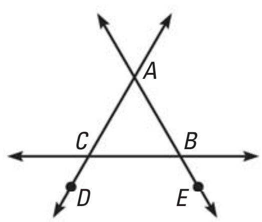
20. Are points *P*, *Q*, and *F* collinear? Are they coplanar?

21. Are points *P* and *G* collinear? Are they coplanar?

22. Name three planes that intersect at point *E*.

23. **SKETCHING PLANES** Sketch plane *J* intersecting plane *K*. Then draw a line ℓ on plane *J* that intersects plane *K* at a single point.

24. **NAMING RAYS** Name 10 different rays in the diagram at the right. Then name 2 pairs of opposite rays.



25. **SKETCHING** Draw three noncollinear points *J*, *K*, and *L*. Sketch $\overline{JK}$ and add a point *M* on $\overline{JK}$. Then sketch $\overrightarrow{ML}$.

26. **SKETCHING** Draw two points *P* and *Q*. Then sketch $\overrightarrow{PQ}$. Add a point *R* on the ray so that *Q* is between *P* and *R*.

REVIEW ALGEBRA
For help with equations of lines, see p. 878.

ALGEBRA In Exercises 27–32, you are given an equation of a line and a point. Use substitution to determine whether the point is on the line.

27. $y = x - 4$; $A(5, 1)$ **28.** $y = x + 1$; $A(1, 0)$ **29.** $y = 3x + 4$; $A(7, 1)$

30. $y = 4x + 2$; $A(1, 6)$ **31.** $y = 3x - 2$; $A(-1, -5)$ **32.** $y = -2x + 8$; $A(-4, 0)$

GRAPHING Graph the inequality on a number line. Tell whether the graph is a *segment*, a *ray* or *rays*, a *point*, or a *line*.

33. $x \le 3$ **34.** $x \ge -4$ **35.** $-7 \le x \le 4$

36. $x \ge 5$ or $x \le -2$ **37.** $x \ge -1$ or $x \le 5$ **38.** $|x| \le 0$

39. CHALLENGE Tell whether each of the following situations involving three planes is possible. If a situation is possible, make a sketch.

a. None of the three planes intersect.

b. The three planes intersect in one line.

c. The three planes intersect in one point.

d. Two planes do not intersect. The third plane intersects the other two.

e. Exactly two planes intersect. The third plane does not intersect the other two.

PROBLEM SOLVING

EXAMPLE 3
on p. 4
for Exs. 40–42

EVERYDAY INTERSECTIONS What kind of geometric intersection does the photograph suggest?

40.

41.

42.

43. TAKS REASONING *Explain* why a four-legged table may rock from side to side even if the floor is level. Would a three-legged table on the same level floor rock from side to side? Why or why not?

TEXAS @HomeTutor for problem solving help at classzone.com

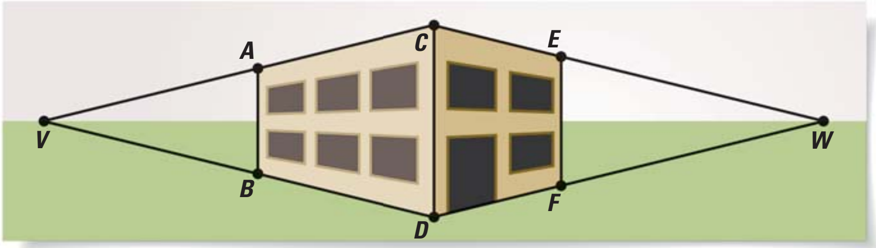
44. SURVEYING A surveying instrument is placed on a tripod. The tripod has three legs whose lengths can be adjusted.

a. When the tripod is sitting on a level surface, are the tips of the legs coplanar?

b. Suppose the tripod is used on a sloping surface. The length of each leg is adjusted so that the base of the surveying instrument is level with the horizon. Are the tips of the legs coplanar? *Explain*.

TEXAS @HomeTutor for problem solving help at classzone.com

45. **MULTI-STEP PROBLEM** In a *perspective drawing*, lines that do not intersect in real life are represented by lines that appear to intersect at a point far away on the horizon. This point is called a *vanishing point*. The diagram shows a drawing of a house with two vanishing points.



a. Trace the black line segments in the drawing. Using lightly dashed lines, join points A and B to the vanishing point W. Join points E and F to the vanishing point V.

b. Label the intersection of $\overleftrightarrow{EV}$ and $\overleftrightarrow{AW}$ as G. Label the intersection of $\overleftrightarrow{FV}$ and $\overleftrightarrow{BW}$ as H.

c. Using heavy dashed lines, draw the hidden edges of the house: $\overline{AG}$, $\overline{EG}$, $\overline{BH}$, $\overline{FH}$, and $\overline{GH}$.

46. **CHALLENGE** Each street in a particular town intersects every existing street exactly one time. Only two streets pass through each intersection.

a. A traffic light is needed at each intersection. How many traffic lights are needed if there are 5 streets in the town? 6 streets?

b. *Describe* a pattern you can use to find the number of additional traffic lights that are needed each time a street is added to the town.

MIXED REVIEW FOR TAKS

TAKS PRACTICE at classzone.com

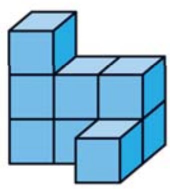
REVIEW
Skills Review Handbook p. 884; TAKS Workbook

47. **TAKS PRACTICE** Which set of coordinates describes a function? *TAKS Obj. 1*

(A) {(2, 2), (4, 0), (6, 0), (2, 0)}
(B) {(0, 3), (1, 0), (1, −1), (3, −3)}
(C) {(4, 0), (−2, 2), (0, 2), (−2, 4)}
(D) {(3, −1), (1, −1), (−3, 0), (−1, 3)}

REVIEW
Skills Review Handbook p. 894; TAKS Workbook

48. **TAKS PRACTICE** How many blocks are visible in the top view of the figure at the right? *TAKS Obj. 7*



(F) 3 blocks
(G) 4 blocks
(H) 5 blocks
(J) 6 blocks

1.2 Use Segments and Congruence

TEKS a.1, a.6, G.1.A, G.7.A

Before You learned about points, lines, and planes.

Now You will use segment postulates to identify congruent **segments.**

Why? So you can calculate flight distances, as in Ex. 33.

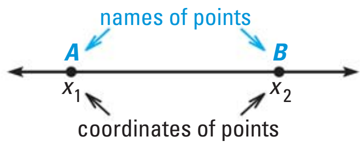
Key Vocabulary
- **postulate, axiom**
- **coordinate**
- **distance**
- **between**
- **congruent segments**

In Geometry, a rule that is accepted without proof is called a **postulate** or **axiom**. A rule that can be proved is called a *theorem*, as you will see later. Postulate 1 shows how to find the distance between two points on a line.

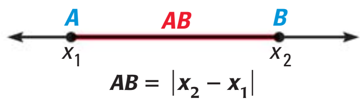
POSTULATE *For Your Notebook*

POSTULATE 1 Ruler Postulate

The points on a line can be matched one to one with the real numbers. The real number that corresponds to a point is the **coordinate** of the point.



The **distance** between points A and B, written as AB, is the absolute value of the difference of the coordinates of A and B.



In the diagrams above, the small numbers in the coordinates x_1 and x_2 are called *subscripts*. The coordinates are read as "x sub one" and "x sub two."

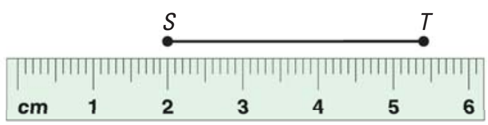
The distance between points A and B, or AB, is also called the *length* of $\overline{AB}$.

EXAMPLE 1 Apply the Ruler Postulate

Measure the length of $\overline{ST}$ to the nearest tenth of a centimeter.

Solution

Align one mark of a metric ruler with S. Then estimate the coordinate of T. For example, if you align S with 2, T appears to align with 5.4.

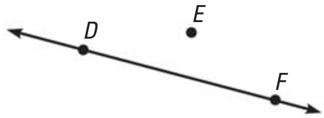


$ST = |5.4 - 2| = 3.4$ **Use Ruler Postulate.**

▶ The length of $\overline{ST}$ is about 3.4 centimeters.

ADDING SEGMENT LENGTHS When three points are collinear, you can say that one point is **between** the other two.

Point *B* is between points *A* and *C*.



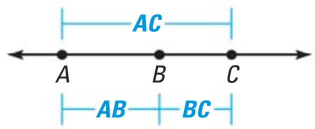
Point *E* is not between points *D* and *F*.

POSTULATE — For Your Notebook

POSTULATE 2 Segment Addition Postulate

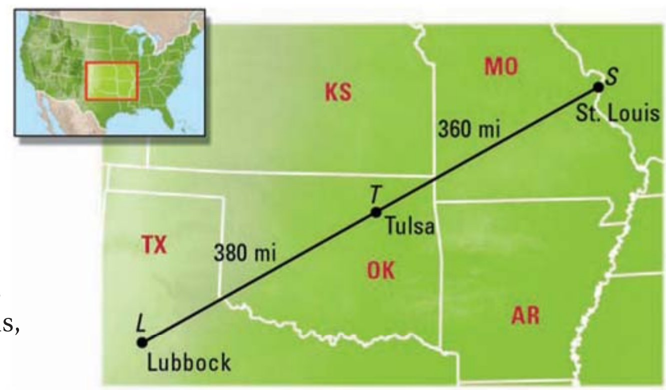
If B is between A and C, then $AB + BC = AC$.

If $AB + BC = AC$, then B is between A and C.



EXAMPLE 2 Apply the Segment Addition Postulate

MAPS The cities shown on the map lie approximately in a straight line. Use the given distances to find the distance from Lubbock, Texas, to St. Louis, Missouri.



Solution

Because Tulsa, Oklahoma, lies between Lubbock and St. Louis, you can apply the Segment Addition Postulate.

$LS = LT + TS = 380 + 360 = 740$

▶ The distance from Lubbock to St. Louis is about 740 miles.

✓ GUIDED PRACTICE for Examples 1 and 2

Use a ruler to measure the length of the segment to the nearest $\frac{1}{8}$ inch.

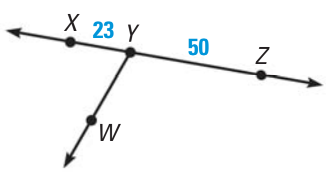
1.

2.

In Exercises 3 and 4, use the diagram shown.

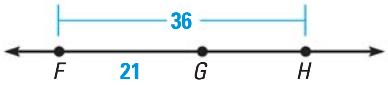
3. Use the Segment Addition Postulate to find XZ.

4. In the diagram, $WY = 30$. Can you use the Segment Addition Postulate to find the distance between points W and Z? *Explain* your reasoning.



EXAMPLE 3 Find a length

Use the diagram to find *GH*.



Solution

Use the Segment Addition Postulate to write an equation. Then solve the equation to find *GH*.

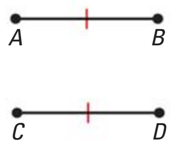
$FH = FG + GH$ **Segment Addition Postulate**

$36 = 21 + GH$ **Substitute 36 for *FH* and 21 for *FG*.**

$15 = GH$ **Subtract 21 from each side.**

CONGRUENT SEGMENTS Line segments that have the same length are called **congruent segments**. In the diagram below, you can say "the length of $\overline{AB}$ is equal to the length of $\overline{CD}$," or you can say "$\overline{AB}$ *is congruent to* $\overline{CD}$." The symbol $\cong$ means "is congruent to."

READ DIAGRAMS
In the diagram, the red tick marks indicate that $\overline{AB} \cong \overline{CD}$.



	Lengths are equal.	Segments are congruent.
	$AB = CD$	$\overline{AB} \cong \overline{CD}$
	"is equal to"	"is congruent to"

EXAMPLE 4 Compare segments for congruence

Plot $J(-3, 4)$, $K(2, 4)$, $L(1, 3)$, and $M(1, -2)$ in a coordinate plane. Then determine whether $\overline{JK}$ and $\overline{LM}$ are congruent.

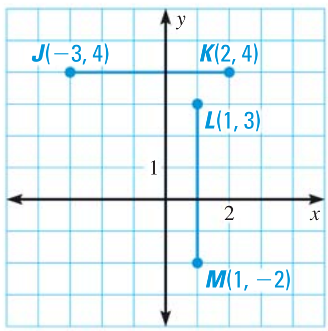
Solution

To find the length of a horizontal segment, find the absolute value of the difference of the *x*-coordinates of the endpoints.

$JK = |2 - (-3)| = 5$ **Use Ruler Postulate.**

To find the length of a vertical segment, find the absolute value of the difference of the *y*-coordinates of the endpoints.

$LM = |-2 - 3| = 5$ **Use Ruler Postulate.**

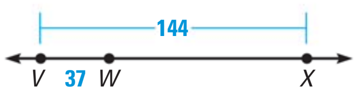


▶ $\overline{JK}$ and $\overline{LM}$ have the same length. So, $\overline{JK} \cong \overline{LM}$.

REVIEW USING A COORDINATE PLANE
For help with using a coordinate plane, see p. 878.

✓ GUIDED PRACTICE for Examples 3 and 4

5. Use the diagram at the right to find *WX*.



6. Plot the points $A(-2, 4)$, $B(3, 4)$, $C(0, 2)$, and $D(0, -2)$ in a coordinate plane. Then determine whether $\overline{AB}$ and $\overline{CD}$ are congruent.

1.2 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 13, 17, and 33
★ = **TAKS PRACTICE AND REASONING** Exs. 20, 27, 34, 37, and 38

SKILL PRACTICE

In Exercises 1 and 2, use the diagram at the right.

1. **VOCABULARY** *Explain* what $\overline{MN}$ means and what MN means.

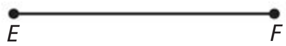
2. **WRITING** *Explain* how you can find PN if you know PQ and QN. How can you find PN if you know MP and MN?

EXAMPLE 1 on p. 9 for Exs. 3–5

MEASUREMENT **Measure the length of the segment to the nearest tenth of a centimeter.**

3. 

4. 

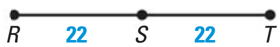
5. 

EXAMPLES 2 and 3 on pp. 10–11 for Exs. 6–12

SEGMENT ADDITION POSTULATE **Find the indicated length.**

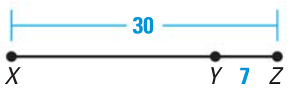
6. Find MP.

7. Find RT.

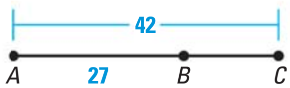


8. Find UW.

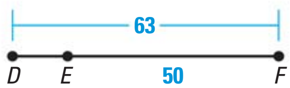
9. Find XY.



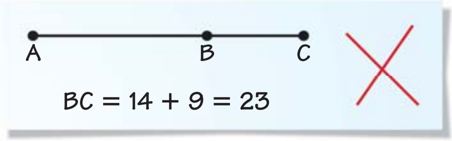
10. Find BC.



11. Find DE.



12. **ERROR ANALYSIS** In the figure at the right, $AC = 14$ and $AB = 9$. *Describe* and correct the error made in finding BC.



EXAMPLE 4 on p. 11 for Exs. 13–19

CONGRUENCE **In Exercises 13–15, plot the given points in a coordinate plane. Then determine whether the line segments named are congruent.**

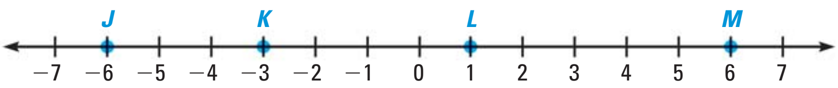
13. $A(0, 1)$, $B(4, 1)$, $C(1, 2)$, $D(1, 6)$; $\overline{AB}$ and $\overline{CD}$

14. $J(-6, -8)$, $K(-6, 2)$, $L(-2, -4)$, $M(-6, -4)$; $\overline{JK}$ and $\overline{LM}$

15. $R(-200, 300)$, $S(200, 300)$, $T(300, -200)$, $U(300, 100)$; $\overline{RS}$ and $\overline{TU}$

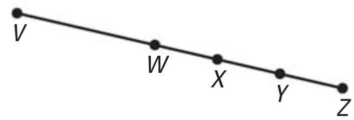
ALGEBRA **Use the number line to find the indicated distance.**

16. JK 17. JL 18. JM 19. KM



20. ★ **TAKS REASONING** Use the diagram. Is it possible to use the Segment Addition Postulate to show that $FB > CB$ or that $AC > DB$? *Explain.*

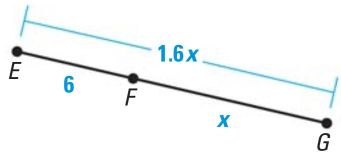
FINDING LENGTHS **In the diagram, points *V, W, X, Y,* and *Z* are collinear, $VZ = 52$, $XZ = 20$, and $WX = XY = YZ$. Find the indicated length.**



21. WX **22.** VW **23.** WY

24. VX **25.** WZ **26.** VY

27. **TAKS REASONING** Use the diagram. What is the length of $\overline{EG}$?



Ⓐ 1 Ⓑ 4.4

Ⓒ 10 Ⓓ 16

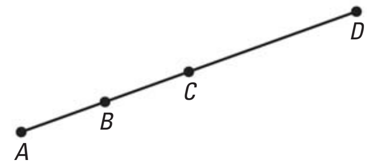
ALGEBRA **Point *S* is between *R* and *T* on $\overline{RT}$. Use the given information to write an equation in terms of *x*. Solve the equation. Then find *RS* and *ST*.**

28. $RS = 2x + 10$
$ST = x - 4$
$RT = 21$

29. $RS = 3x - 16$
$ST = 4x - 8$
$RT = 60$

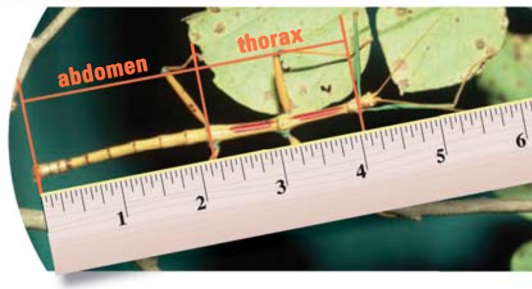
30. $RS = 2x - 8$
$ST = 3x - 10$
$RT = 17$

31. **CHALLENGE** In the diagram, $\overline{AB} \cong \overline{BC}$, $\overline{AC} \cong \overline{CD}$, and $AD = 12$. Find the lengths of all the segments in the diagram. Suppose you choose one of the segments at random. What is the probability that the measure of the segment is greater than 3? *Explain*.



PROBLEM SOLVING

32. **SCIENCE** The photograph shows an insect called a walkingstick. Use the ruler to estimate the length of the abdomen and the length of the thorax to the nearest $\frac{1}{4}$ inch. About how much longer is the walkingstick's abdomen than its thorax?



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EXAMPLE 2
on p. 10
for Ex. 33

33. **MODEL AIRPLANE** In 2003, a remote-controlled model airplane became the first ever to fly nonstop across the Atlantic Ocean. The map shows the airplane's position at three different points during its flight.

a. Find the total distance the model airplane flew.

b. The model airplane's flight lasted nearly 38 hours. Estimate the airplane's average speed in miles per hour.

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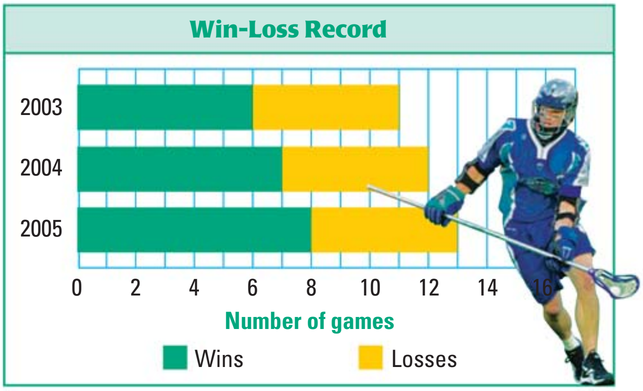
34. **TAKS REASONING** The bar graph shows the win-loss record for a lacrosse team over a period of three years.

a. Use the scale to find the length of the yellow bar for each year. What does the length represent?

b. For each year, find the percent of games lost by the team.

c. *Explain* how you are applying the Segment Addition Postulate when you find information from a stacked bar graph like the one shown.



35. **MULTI-STEP PROBLEM** A climber uses a rope to descend a vertical cliff. Let A represent the point where the rope is secured at the top of the cliff, let B represent the climber's position, and let C represent the point where the rope is secured at the bottom of the cliff.

a. **Model** Draw and label a line segment that represents the situation.

b. **Calculate** If AC is 52 feet and AB is 31 feet, how much farther must the climber descend to reach the bottom of the cliff?

AnimatedGeometry at classzone.com

36. **CHALLENGE** Four cities lie along a straight highway in this order: City A, City B, City C, and City D. The distance from City A to City B is 5 times the distance from City B to City C. The distance from City A to City D is 2 times the distance from City A to City B. Copy and complete the mileage chart.

	City A	City B	City C	City D
City A		?	?	?
City B	?		?	?
City C	?	?		10 mi
City D	?	?	?	

MIXED REVIEW FOR TAKS

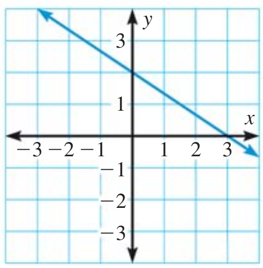
TAKS PRACTICE at classzone.com

REVIEW
Skills Review Handbook p. 878; TAKS Workbook

37. **TAKS PRACTICE** Which function best describes the graph at the right? *TAKS Obj. 3*

Ⓐ $f(x) = -\frac{2}{3}x + 2$ Ⓑ $f(x) = -\frac{3}{2}x - 2$

Ⓒ $f(x) = \frac{2}{3}x + 2$ Ⓓ $f(x) = \frac{3}{2}x - 2$



REVIEW
Skills Review Handbook p. 878; TAKS Workbook

38. **TAKS PRACTICE** Which coordinates represent a point that lies in Quadrant II? *TAKS Obj. 6*

Ⓕ (1, 5) Ⓖ (−1, 5) Ⓗ (1, −5) Ⓙ (−1, −5)

1.3 Use Midpoint and Distance Formulas

TEKS G.5.A, G.7.C, G.8.C

Before You found lengths of segments.

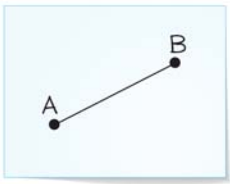
Now You will find lengths of segments in the coordinate plane.

Why? So you can find an unknown length, as in Example 1.

Key Vocabulary
- midpoint
- segment bisector

ACTIVITY FOLD A SEGMENT BISECTOR

STEP 1

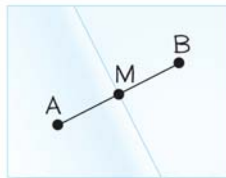


Draw $\overline{AB}$ on a piece of paper.

STEP 2

Fold the paper so that *B* is on top of *A*.

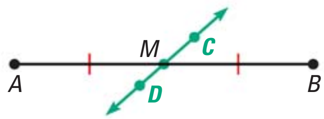
STEP 3



Label point *M*. Compare *AM*, *MB*, and *AB*.

MIDPOINTS AND BISECTORS The **midpoint** of a segment is the point that divides the segment into two congruent segments. A **segment bisector** is a point, ray, line, line segment, or plane that intersects the segment at its midpoint. A midpoint or a segment bisector *bisects* a segment.

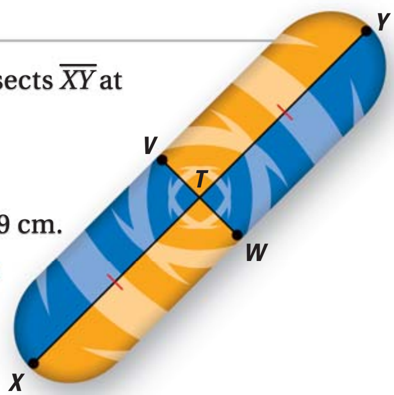
***M* is the midpoint of $\overline{AB}$.**
So, $\overline{AM} \cong \overline{MB}$ and $AM = MB$.



$\overleftrightarrow{CD}$ is a segment bisector of $\overline{AB}$.
So, $\overline{AM} \cong \overline{MB}$ and $AM = MB$.

EXAMPLE 1 Find segment lengths

SKATEBOARD In the skateboard design, $\overline{VW}$ bisects $\overline{XY}$ at point *T*, and $XT = 39.9$ cm. Find *XY*.



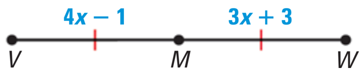
Solution

Point *T* is the midpoint of $\overline{XY}$. So, $XT = TY = 39.9$ cm.

$XY = XT + TY$	Segment Addition Postulate
$= 39.9 + 39.9$	Substitute.
$= 79.8$ cm	Add.

EXAMPLE 2 Use algebra with segment lengths

ALGEBRA **Point *M* is the midpoint of $\overline{VW}$. Find the length of $\overline{VM}$.**



Solution

STEP 1 **Write** and solve an equation. Use the fact that that $VM = MW$.

$VM = MW$	Write equation.
$4x - 1 = 3x + 3$	Substitute.
$x - 1 = 3$	Subtract $3x$ from each side.
$x = 4$	Add 1 to each side.

STEP 2 **Evaluate** the expression for VM when $x = 4$.

$VM = 4x - 1 = 4(4) - 1 = 15$

▶ So, the length of $\overline{VM}$ is 15.

CHECK Because $VM = MW$, the length of $\overline{MW}$ should be 15. If you evaluate the expression for MW, you should find that $MW = 15$.

$MW = 3x + 3 = 3(4) + 3 = 15$ ✓

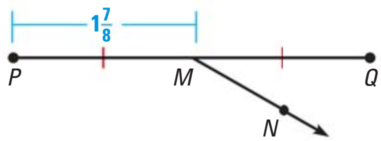
REVIEW ALGEBRA
For help with solving equations, see p. 875.

✓ GUIDED PRACTICE for Examples 1 and 2

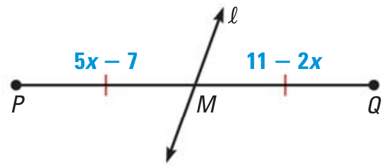
READ DIRECTIONS
Always read direction lines carefully. Notice that this direction line has two parts.

In Exercises 1 and 2, identify the segment bisector of $\overline{PQ}$. Then find PQ.

1.



2.



COORDINATE PLANE You can use the coordinates of the endpoints of a segment to find the coordinates of the midpoint.

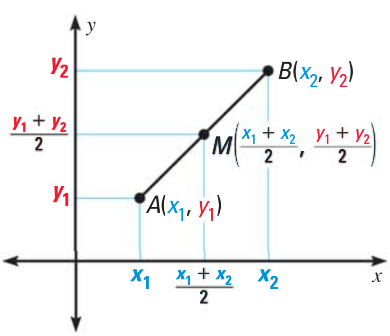
KEY CONCEPT — *For Your Notebook*

The Midpoint Formula

The coordinates of the midpoint of a segment are the averages of the x-coordinates and of the y-coordinates of the endpoints.

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the midpoint M of $\overline{AB}$ has coordinates

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$



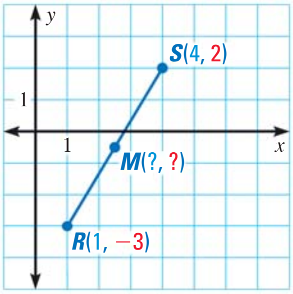
EXAMPLE 3 Use the Midpoint Formula

a. FIND MIDPOINT The endpoints of $\overline{RS}$ are $R(1, -3)$ and $S(4, 2)$. Find the coordinates of the midpoint M.

b. FIND ENDPOINT The midpoint of $\overline{JK}$ is $M(2, 1)$. One endpoint is $J(1, 4)$. Find the coordinates of endpoint K.

Solution

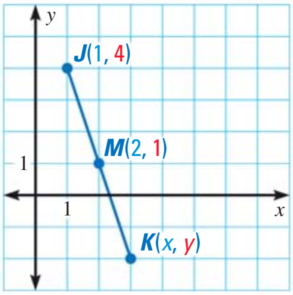
a. FIND MIDPOINT Use the Midpoint Formula.



$$M\left(\frac{1+4}{2}, \frac{-3+2}{2}\right) = M\left(\frac{5}{2}, -\frac{1}{2}\right)$$

▶ The coordinates of the midpoint M are $\left(\frac{5}{2}, -\frac{1}{2}\right)$.

b. FIND ENDPOINT Let (x, y) be the coordinates of endpoint K. Use the Midpoint Formula.



CLEAR FRACTIONS
Multiply each side of the equation by the denominator to clear the fraction.

STEP 1 Find x.	***STEP 2*** Find y.
$\frac{1+x}{2} = 2$	$\frac{4+y}{2} = 1$
$1 + x = 4$	$4 + y = 2$
$x = 3$	$y = -2$

▶ The coordinates of endpoint K are $(3, -2)$.

✓ GUIDED PRACTICE for Example 3

3. The endpoints of $\overline{AB}$ are $A(1, 2)$ and $B(7, 8)$. Find the coordinates of the midpoint M.

4. The midpoint of $\overline{VW}$ is $M(-1, -2)$. One endpoint is $W(4, 4)$. Find the coordinates of endpoint V.

DISTANCE FORMULA The Distance Formula is a formula for computing the distance between two points in a coordinate plane.

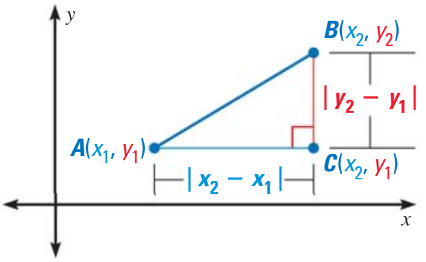
READ DIAGRAMS
The red mark at one corner of the triangle shown indicates a right triangle.

KEY CONCEPT *For Your Notebook*

The Distance Formula

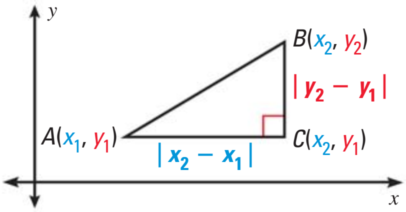
If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the distance between A and B is

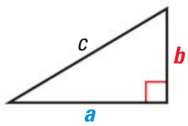
$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$



The Distance Formula is based on the *Pythagorean Theorem,* which you will see again when you work with right triangles in Chapter 7.

Distance Formula	**Pythagorean Theorem**
$(AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$	$c^2 = a^2 + b^2$





EXAMPLE 4 TAKS PRACTICE: Multiple Choice

What is the approximate length of $\overline{RS}$, with endpoints $R(3, 1)$ and $S(-1, -5)$?

Ⓐ 1.6 units Ⓑ 3.2 units Ⓒ 4.8 units Ⓓ 7.2 units

ELIMINATE CHOICES
Drawing a diagram can help you eliminate wrong choices. You can see that choices *A* and *B* are not large enough to be *RS*.

READ SYMBOLS
The symbol ≈ means "is approximately equal to."

Solution

$RS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ **Distance formula**

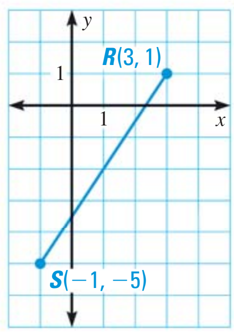
$= \sqrt{[(-1) - 3]^2 + [(-5) - 1]^2}$ **Substitute.**

$= \sqrt{(-4)^2 + (-6)^2}$ **Subtract.**

$= \sqrt{16 + 36}$ **Evaluate powers.**

$= \sqrt{52}$ **Add.**

≈ 7.21 **Use a calculator to approximate the square root.**



▶ The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Example 4

5. In Example 4, does it matter which ordered pair you choose to substitute for (x_1, y_1) and which ordered pair you choose to substitute for (x_2, y_2)? *Explain.*

6. What is the approximate length of $\overline{AB}$, with endpoints $A(-3, 2)$ and $B(1, -4)$?

Ⓐ 6.1 units Ⓑ 7.2 units Ⓒ 8.5 units Ⓓ 10.0 units

1.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 15, 35, and 49

★ = **TAKS PRACTICE AND REASONING** Exs. 34, 41, 42, 53, 55, 56, and 57

SKILL PRACTICE

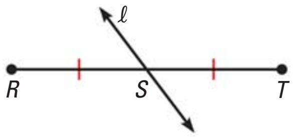
1. VOCABULARY Copy and complete: To find the length of $\overline{AB}$, with endpoints $A(-7, 5)$ and $B(4, -6)$, you can use the __?__.

2. WRITING *Explain* what it means to bisect a segment. Why is it impossible to bisect a line?

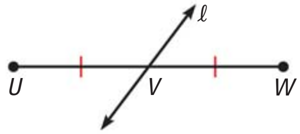
EXAMPLE 1 on p. 15 for Exs. 3–10

FINDING LENGTHS **Line ℓ bisects the segment. Find the indicated length.**

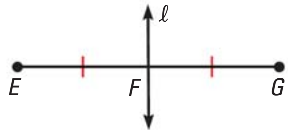
3. Find RT if $RS = 5\frac{1}{8}$ in.



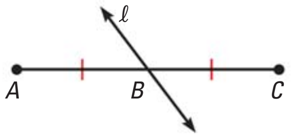
4. Find UW if $VW = \frac{5}{8}$ in.



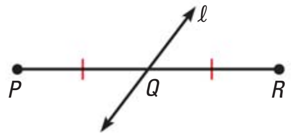
5. Find EG if $EF = 13$ cm.



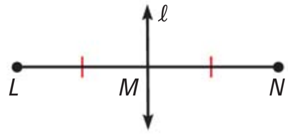
6. Find BC if $AC = 19$ cm.



7. Find QR if $PR = 9\frac{1}{2}$ in.



8. Find LM if $LN = 137$ mm.



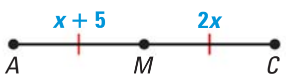
9. SEGMENT BISECTOR Line RS bisects $\overline{PQ}$ at point R. Find RQ if $PQ = 4\frac{3}{4}$ inches.

10. SEGMENT BISECTOR Point T bisects $\overline{UV}$. Find UV if $UT = 2\frac{7}{8}$ inches.

EXAMPLE 2 on p. 16 for Exs. 11–16

xy ALGEBRA **In each diagram, M is the midpoint of the segment. Find the indicated length.**

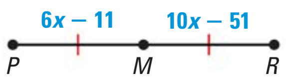
11. Find AM.



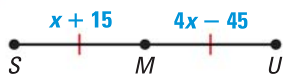
12. Find EM.

13. Find JM.

14. Find PR.



15. Find SU.



16. Find XZ.

EXAMPLE 3 on p. 17 for Exs. 17–30

FINDING MIDPOINTS **Find the coordinates of the midpoint of the segment with the given endpoints.**

17. $C(3, 5)$ and $D(7, 5)$

18. $E(0, 4)$ and $F(4, 3)$

19. $G(-4, 4)$ and $H(6, 4)$

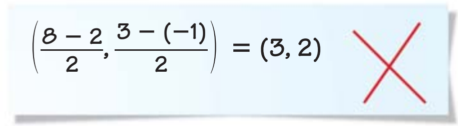
20. $J(-7, -5)$ and $K(-3, 7)$

21. $P(-8, -7)$ and $Q(11, 5)$

22. $S(-3, 3)$ and $T(-8, 6)$

23. WRITING Develop a formula for finding the midpoint of a segment with endpoints $A(0, 0)$ and $B(m, n)$. *Explain* your thinking.

24. **ERROR ANALYSIS** *Describe* the error made in finding the coordinates of the midpoint of a segment with endpoints $S(8, 3)$ and $T(2, -1)$.

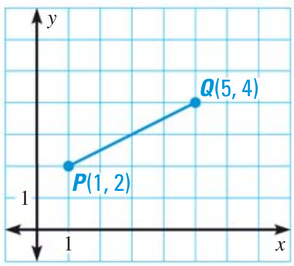


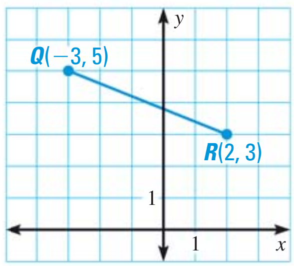
FINDING ENDPOINTS **Use the given endpoint R and midpoint M of $\overline{RS}$ to find the coordinates of the other endpoint S.**

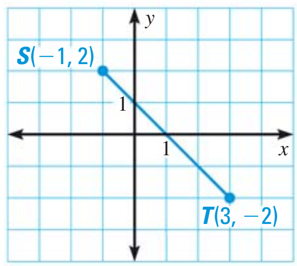
25. $R(3, 0), M(0, 5)$
26. $R(5, 1), M(1, 4)$
27. $R(6, -2), M(5, 3)$
28. $R(-7, 11), M(2, 1)$
29. $R(4, -6), M(-7, 8)$
30. $R(-4, -6), M(3, -4)$

EXAMPLE 4
on p. 18
for Exs. 31–34

DISTANCE FORMULA **Find the length of the segment. Round to the nearest tenth of a unit.**

31.


32.


33.


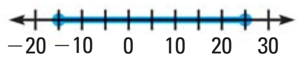
34. **TAKS REASONING** The endpoints of $\overline{MN}$ are $M(-3, -9)$ and $N(4, 8)$. What is the approximate length of $\overline{MN}$?

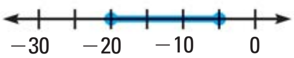
Ⓐ 1.4 units Ⓑ 7.2 units Ⓒ 13 units Ⓓ 18.4 units

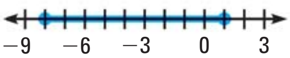
NUMBER LINE **Find the length of the segment. Then find the coordinate of the midpoint of the segment.**

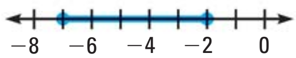
35.


36.


37.


38.


39.


40.


41. **TAKS REASONING** The endpoints of $\overline{LF}$ are $L(-2, 2)$ and $F(3, 1)$. The endpoints of $\overline{JR}$ are $J(1, -1)$ and $R(2, -3)$. What is the approximate difference in the lengths of the two segments?

Ⓐ 2.24 Ⓑ 2.86 Ⓒ 5.10 Ⓓ 7.96

42. **TAKS REASONING** One endpoint of $\overline{PQ}$ is $P(-2, 4)$. The midpoint of $\overline{PQ}$ is $M(1, 0)$. *Explain* how to find PQ.

COMPARING LENGTHS **The endpoints of two segments are given. Find each segment length. Tell whether the segments are congruent.**

43. $\overline{AB}$: $A(0, 2), B(-3, 8)$
$\overline{CD}$: $C(-2, 2), D(0, -4)$
44. $\overline{EF}$: $E(1, 4), F(5, 1)$
$\overline{GH}$: $G(-3, 1), H(1, 6)$
45. $\overline{JK}$: $J(-4, 0), K(4, 8)$
$\overline{LM}$: $L(-4, 2), M(3, -7)$

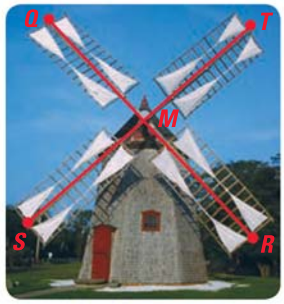
46. xy **ALGEBRA** Points S, T, and P lie on a number line. Their coordinates are 0, 1, and x, respectively. Given $SP = PT$, what is the value of x?

47. **CHALLENGE** M is the midpoint of $\overline{JK}$, $JM = \frac{x}{8}$, and $JK = \frac{3x}{4} - 6$. Find MK.

PROBLEM SOLVING

EXAMPLE 1 on p. 15 for Ex. 48

48. WINDMILL In the photograph of a windmill, $\overline{ST}$ bisects $\overline{QR}$ at point M. The length of $\overline{QM}$ is $18\frac{1}{2}$ feet. Find QR and MR.



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49. DISTANCES A house and a school are 5.7 kilometers apart on the same straight road. The library is on the same road, halfway between the house and the school. Draw a sketch to represent this situation. Mark the locations of the house, school, and library. How far is the library from the house?

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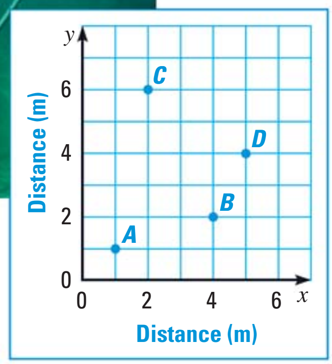
ARCHAEOLOGY The points on the diagram show the positions of objects at an underwater archaeological site. Use the diagram for Exercises 50 and 51.



50. Find the distance between each pair of objects. Round to the nearest tenth of a meter if necessary.

a. A and B **b.** B and C **c.** C and D

d. A and D **e.** B and D **f.** A and C

51. Which two objects are closest to each other? Which two are farthest apart?

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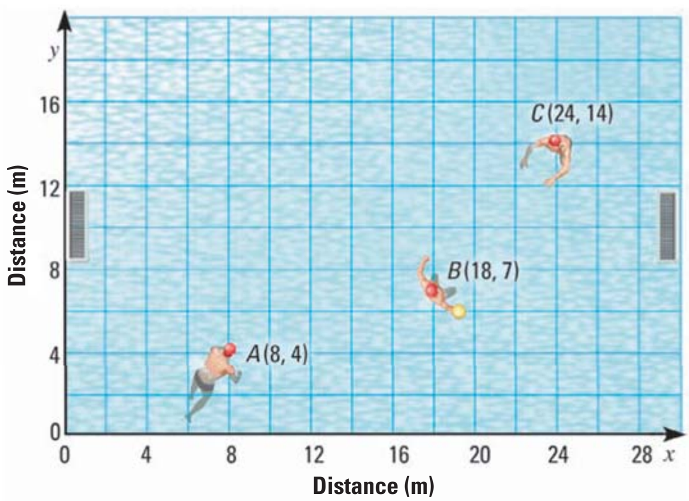
52. WATER POLO The diagram shows the positions of three players during part of a water polo match. Player A throws the ball to Player B, who then throws it to Player C. How far did Player A throw the ball? How far did Player B throw the ball? How far would Player A have thrown the ball if he had thrown it directly to Player C? Round all answers to the nearest tenth of a meter.

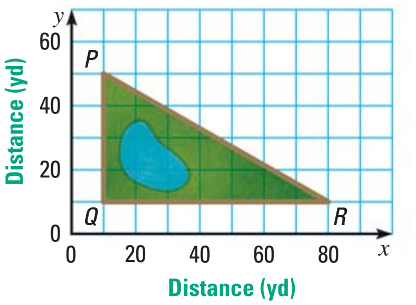


53. **TAKS REASONING** As shown, a path goes around a triangular park.

a. Find the distance around the park to the nearest yard.

b. A new path and a bridge are constructed from point Q to the midpoint M of $\overline{PR}$. Find QM to the nearest yard.

c. A man jogs from P to Q to M to R to Q and back to P at an average speed of 150 yards per minute. About how many minutes does it take? *Explain*.



54. **CHALLENGE** $\overline{AB}$ bisects $\overline{CD}$ at point M, $\overline{CD}$ bisects $\overline{AB}$ at point M, and $AB = 4 \cdot CM$. *Describe* the relationship between AM and CD.

MIXED REVIEW FOR TAKS

TAKS PRACTICE at classzone.com

REVIEW
Skills Review Handbook p. 882; TAKS Workbook

55. **TAKS PRACTICE** What are the solutions to $x^2 + 4x = 5$? *TAKS Obj. 5*

Ⓐ $x = 1$ and $x = 5$ Ⓑ $x = 1$ and $x = -5$

Ⓒ $x = -1$ and $x = 5$ Ⓓ $x = -1$ and $x = -5$

REVIEW
Skills Review Handbook p. 894; TAKS Workbook

56. **TAKS PRACTICE** Juan is putting up a wallpaper border along the top of each wall in his rectangular living room. The border costs $9.25 per roll plus 7.75% sales tax. One roll is 15 feet long. What other information is needed to determine the number of rolls of border he needs to purchase? *TAKS Obj. 10*

Ⓕ The perimeter of the room Ⓖ The total cost of each roll of border

Ⓗ The weight of one roll of border Ⓙ The height of the walls in the room

REVIEW
Skills Review Handbook p. 894; TAKS Workbook

57. **TAKS PRACTICE** Sally can jog at a rate of 6.5 miles per hour. If she continued in a straight path at this rate, what distance would she travel in 24 minutes? *TAKS Obj. 9*

Ⓐ 2.2 miles Ⓑ 2.4 miles Ⓒ 2.6 miles Ⓓ 2.8 miles

QUIZ *for Lessons 1.1–1.3*

1. Sketch two lines that intersect the same plane at two different points. The lines intersect each other at a point not in the plane. *(p. 2)*

In the diagram of collinear points, $AE = 26$, $AD = 15$, and $AB = BC = CD$. Find the indicated length. *(p. 9)*

2. DE 3. AB 4. AC

5. BD 6. CE 7. BE

8. The endpoints of $\overline{RS}$ are $R(-2, -1)$ and $S(2, 3)$. Find the coordinates of the midpoint of $\overline{RS}$. Then find the distance between R and S. *(p. 15)*

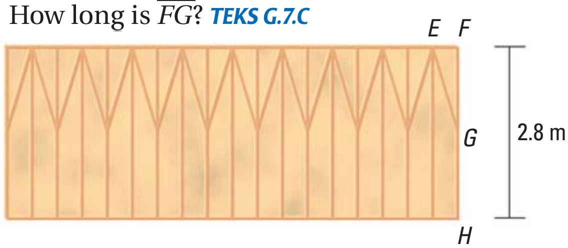
MIXED REVIEW FOR TEKS

TAKS PRACTICE
classzone.com

Lessons 1.1–1.3

1. **MULTI-STEP PROBLEM** All tickets for a

1. **WALL FRAME** The diagram shows the frame for a wall. $\overline{FH}$ represents a vertical board and $\overline{EG}$ represents a brace. The brace bisects $\overline{FH}$. How long is $\overline{FG}$? *TEKS G.7.C*

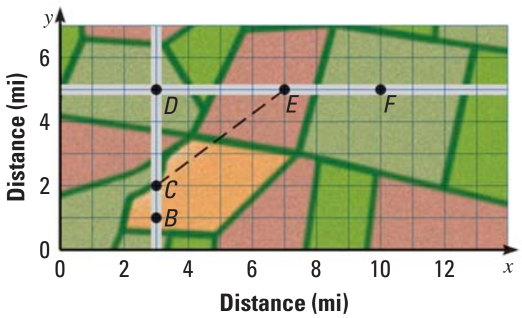


Ⓐ 0.8 meters　Ⓑ 1.4 meters
Ⓒ 4.8 meters　Ⓓ 5.6 meters

2. **COORDINATE PLANE** Point E is the midpoint of $\overline{AB}$ and $\overline{CD}$. The coordinates of A, B, and C are $A(-4, 5)$, $B(6, -5)$, and $C(2, 8)$. What are the coordinates of point D? *TEKS G.7.A*

Ⓕ (0, 2)　Ⓖ (1.5, 4)
Ⓗ (0, −8)　Ⓙ (−10, 2)

3. **NEW ROAD** The diagram shows existing roads and a planned new road, represented by $\overline{CE}$. About how much shorter is a trip from B to F, where possible, using the new road instead of the existing roads? *TEKS G.7.C*

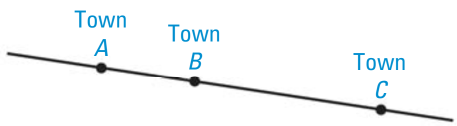


Ⓐ 2 miles　Ⓑ 5 miles
Ⓒ 6 miles　Ⓓ 7 miles

4. **PERIMETER** Rectangle $QRST$ has vertices $Q(3, -3)$, $R(0, -5)$, $S(-4, 1)$, and $T(-1, 3)$. What is the perimeter of rectangle $QRST$? Round to the nearest tenth. *TEKS G.7.C*

Ⓕ 7.2 units　Ⓖ 14.4 units
Ⓗ 21.6 units　Ⓙ 28.8 units

5. **TRAVELING SALESPERSON** Jill is a salesperson who needs to visit Towns A, B, and C. On the map, $AB = 18.7$ km and $BC = 2AB$. Starting at Town A, Jill travels along the road shown to Town B, then Solve to Town C, and returns to Town A. What distance does Jill travel? *TEKS G.7.C*



Ⓐ 56.1 km　Ⓑ 74.8 km
Ⓒ 93.5 km　Ⓓ 112.2 km

6. **CLOCK** In the photo of the clock below, which segment represents the intersection of planes ABC and BFE? *TEKS G.6.C*

Ⓕ $\overline{AB}$　Ⓖ $\overline{BC}$
Ⓗ $\overline{BF}$　Ⓙ $\overline{FG}$

GRIDDED ANSWER

7. **MIDPOINT FORMULA** Point M is the midpoint of $\overline{PQ}$. $PM = (23x + 5)$ inches and $MQ = (25x - 4)$ inches. Find the length (in inches) of $\overline{PQ}$. *TEKS G.7.C*

8. **HIKING TRAIL** Tom is hiking on a trail that lies along a straight railroad track. The total length of the trail is 5.4 kilometers. Starting from the beginning of the trail, he has been walking for 45 minutes at an average speed of 2.4 kilometers per hour. What is the length (in kilometers) to the end of the trail? *TEKS G.7.C*

1.4 Measure and Classify Angles

TEKS a.2, G.5.B

Before You named and measured line segments.

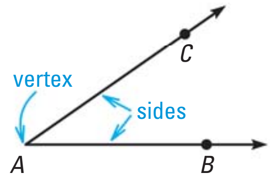
Now You will name, measure, and classify angles.

Why? So you can identify congruent angles, as in Example 4.

Key Vocabulary
- **angle**
  acute, right, obtuse, straight
- **sides, vertex of an angle**
- **measure of an angle**
- **congruent angles**
- **angle bisector**

An **angle** consists of two different rays with the same endpoint. The rays are the **sides** of the angle. The endpoint is the **vertex** of the angle.

The angle with sides $\overrightarrow{AB}$ and $\overrightarrow{AC}$ can be named $\angle BAC$, $\angle CAB$, or $\angle A$. Point A is the vertex of the angle.



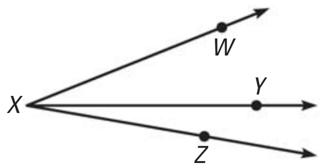
EXAMPLE 1 Name angles

Name the three angles in the diagram.

$\angle WXY$, or $\angle YXW$

$\angle YXZ$, or $\angle ZXY$

$\angle WXZ$, or $\angle ZXW$



You should not name any of these angles $\angle X$ because all three angles have X as their vertex.

MEASURING ANGLES A protractor can be used to approximate the *measure* of an angle. An angle is measured in units called *degrees* (°). For instance, the measure of $\angle WXZ$ in Example 1 above is 32°. You can write this statement in two ways.

Words The measure of $\angle WXZ$ is 32°.

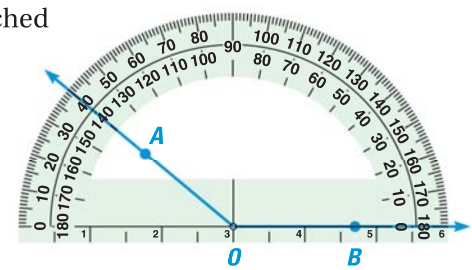
Symbols $m\angle WXZ = 32°$

POSTULATE — For Your Notebook

POSTULATE 3 Protractor Postulate

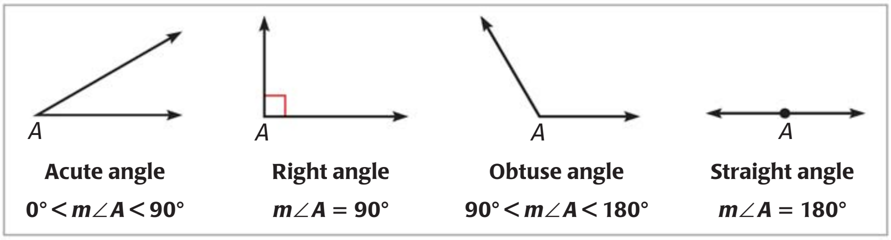
Consider $\overrightarrow{OB}$ and a point A on one side of $\overrightarrow{OB}$. The rays of the form $\overrightarrow{OA}$ can be matched one to one with the real numbers from 0 to 180.

The **measure** of $\angle AOB$ is equal to the absolute value of the difference between the real numbers for $\overrightarrow{OA}$ and $\overrightarrow{OB}$.



CLASSIFYING ANGLES Angles can be classified as **acute**, **right**, **obtuse**, and **straight**, as shown below.

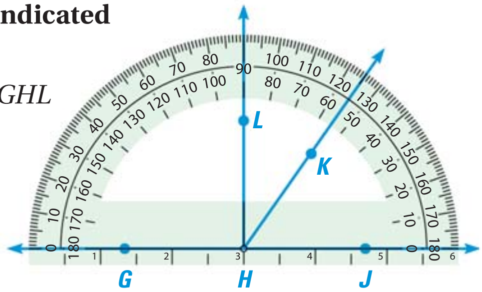
READ DIAGRAMS
A red square inside an angle indicates that the angle is a right angle.



EXAMPLE 2 Measure and classify angles

Use the diagram to find the measure of the indicated angle. Then classify the angle.

a. $\angle KHJ$ **b.** $\angle GHK$ **c.** $\angle GHJ$ **d.** $\angle GHL$



Solution

A protractor has an inner and an outer scale. When you measure an angle, check to see which scale to use.

a. $\overrightarrow{HJ}$ is lined up with the 0° on the inner scale of the protractor. $\overrightarrow{HK}$ passes through 55° on the inner scale. So, $m\angle KHJ = 55°$. It is an acute angle.

b. $\overrightarrow{HG}$ is lined up with the 0° on the outer scale, and $\overrightarrow{HK}$ passes through 125° on the outer scale. So, $m\angle GHK = 125°$. It is an obtuse angle.

c. $m\angle GHJ = 180°$. It is a straight angle.

d. $m\angle GHL = 90°$. It is a right angle.

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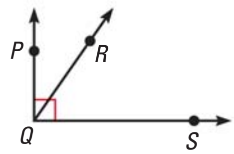
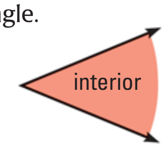
✓ GUIDED PRACTICE for Examples 1 and 2

1. Name all the angles in the diagram at the right. Which angle is a right angle?

2. Draw a pair of opposite rays. What type of angle do the rays form?



READ DIAGRAMS
A point is in the *interior* of an angle if it is between points that lie on each side of the angle.

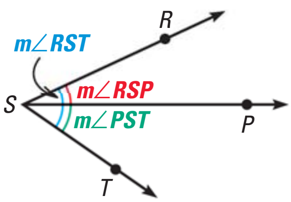


POSTULATE *For Your Notebook*

POSTULATE 4 Angle Addition Postulate

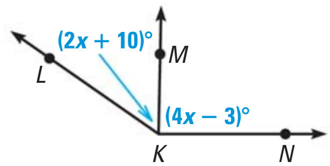
Words If P is in the interior of $\angle RST$, then the measure of $\angle RST$ is equal to the sum of the measures of $\angle RSP$ and $\angle PST$.

Symbols If P is in the interior of $\angle RST$, then
$m\angle RST = m\angle RSP + m\angle PST$.



EXAMPLE 3 Find angle measures

xy ALGEBRA **Given that $m\angle LKN = 145°$, find $m\angle LKM$ and $m\angle MKN$.**



Solution

STEP 1 **Write** and solve an equation to find the value of x.

$m\angle LKN = m\angle LKM + m\angle MKN$	**Angle Addition Postulate**
$145° = (2x + 10)° + (4x - 3)°$	**Substitute angle measures.**
$145 = 6x + 7$	**Combine like terms.**
$138 = 6x$	**Subtract 7 from each side.**
$23 = x$	**Divide each side by 6.**

STEP 2 **Evaluate** the given expressions when $x = 23$.

$$m\angle LKM = (2x + 10)° = (2 \cdot 23 + 10)° = 56°$$

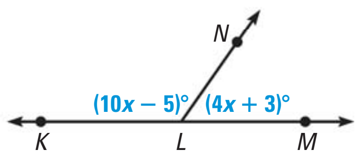
$$m\angle MKN = (4x - 3)° = (4 \cdot 23 - 3)° = 89°$$

▶ So, $m\angle LKM = 56°$ and $m\angle MKN = 89°$.

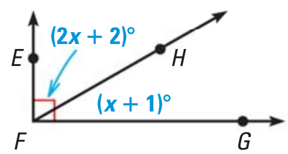
✓ GUIDED PRACTICE for Example 3

Find the indicated angle measures.

3. Given that $\angle KLM$ is a straight angle, find $m\angle KLN$ and $m\angle NLM$.

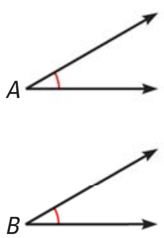


4. Given that $\angle EFG$ is a right angle, find $m\angle EFH$ and $m\angle HFG$.



CONGRUENT ANGLES Two angles are **congruent angles** if they have the same measure. In the diagram below, you can say that "the measure of angle A is equal to the measure of angle B," or you can say "angle A *is congruent to* angle B."

READ DIAGRAMS
Matching arcs are used to show that angles are congruent. If more than one pair of angles are congruent, double arcs are used, as in Example 4 on page 27.



Angle measures are equal.	Angles are congruent.
$m\angle A = m\angle B$	$\angle A \cong \angle B$
↑	↑
"is equal to"	"is congruent to"

EXAMPLE 4 Identify congruent angles

TRAPEZE The photograph shows some of the angles formed by the ropes in a trapeze apparatus. Identify the congruent angles. If $m\angle DEG = 157°$, what is $m\angle GKL$?

Solution

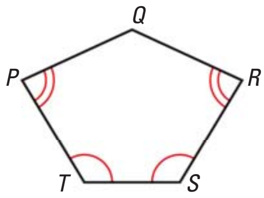
There are two pairs of congruent angles:

$\angle DEF \cong \angle JKL$ and $\angle DEG \cong \angle GKL$.

Because $\angle DEG \cong \angle GKL$, $m\angle DEG = m\angle GKL$. So, $m\angle GKL = 157°$.

✓ GUIDED PRACTICE for Example 4

Use the diagram shown at the right.

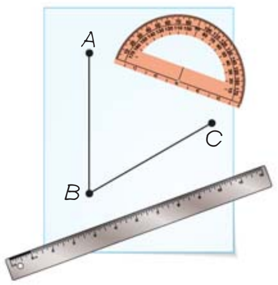


5. Identify all pairs of congruent angles in the diagram.

6. In the diagram, $m\angle PQR = 130°$, $m\angle QRS = 84°$, and $m\angle TSR = 121°$. Find the other angle measures in the diagram.

ACTIVITY FOLD AN ANGLE BISECTOR

STEP 1

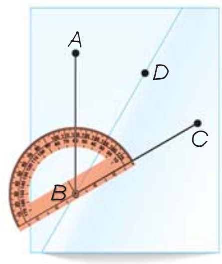


Use a straightedge to draw and label an acute angle, $\angle ABC$.

STEP 2

Fold the paper so that $\overrightarrow{BC}$ is on top of $\overrightarrow{BA}$.

STEP 3

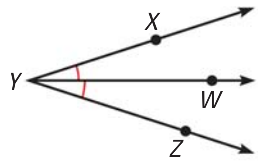


Draw a point D on the fold inside $\angle ABC$. Then measure $\angle ABD$, $\angle DBC$, and $\angle ABC$. What do you observe?

An **angle bisector** is a ray that divides an angle into two angles that are congruent. In the activity on page 27, $\overrightarrow{BD}$ bisects $\angle ABC$. So, $\angle ABD \cong \angle DBC$ and $m\angle ABD = m\angle DBC$.

EXAMPLE 5 Double an angle measure

In the diagram at the right, $\overrightarrow{YW}$ bisects $\angle XYZ$, and $m\angle XYW = 18°$. Find $m\angle XYZ$.



Solution

By the Angle Addition Postulate, $m\angle XYZ = m\angle XYW + m\angle WYZ$. Because $\overrightarrow{YW}$ bisects $\angle XYZ$, you know that $\angle XYW \cong \angle WYZ$.

So, $m\angle XYW = m\angle WYZ$, and you can write

$$m\angle XYZ = m\angle XYW + m\angle WYZ = 18° + 18° = 36°.$$

✓ GUIDED PRACTICE for Example 5

7. Angle *MNP* is a straight angle, and $\overrightarrow{NQ}$ bisects $\angle MNP$. Draw $\angle MNP$ and $\overrightarrow{NQ}$. Use arcs to mark the congruent angles in your diagram, and give the angle measures of these congruent angles.

1.4 EXERCISES

HOMEWORK KEY

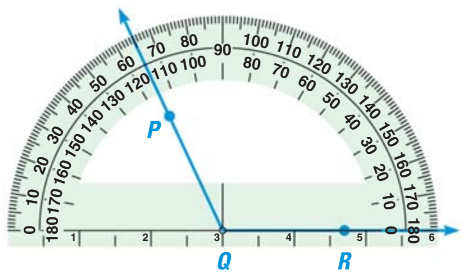
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 15, 23, and 53

★ = **TAKS PRACTICE AND REASONING** Exs. 21, 27, 43, 62, 64, and 65

SKILL PRACTICE

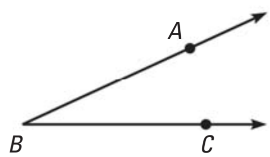
1. **VOCABULARY** Sketch an example of each of the following types of angles: acute, obtuse, right, and straight.

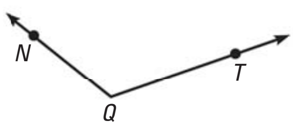
2. **WRITING** *Explain* how to find the measure of $\angle PQR$, shown at the right.

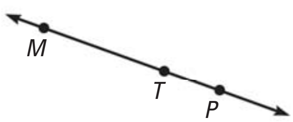


EXAMPLE 1
on p. 24
for Exs. 3–6

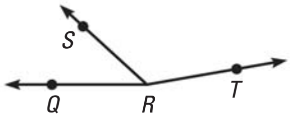
NAMING ANGLES AND ANGLE PARTS **In Exercises 3–5, write three names for the angle shown. Then name the vertex and sides of the angle.**

3.


4.


5.


6. **NAMING ANGLES** Name three different angles in the diagram at the right.

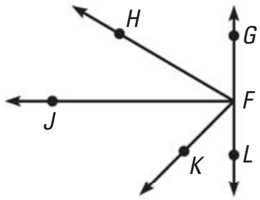


EXAMPLE 2
on p. 25
for Exs. 7–21

CLASSIFYING ANGLES Classify the angle with the given measure as *acute*, *obtuse*, *right*, or *straight*.

7. $m\angle W = 180°$ 8. $m\angle X = 30°$ 9. $m\angle Y = 90°$ 10. $m\angle Z = 95°$

MEASURING ANGLES Trace the diagram and extend the rays. Use a protractor to find the measure of the given angle. Then classify the angle as *acute*, *obtuse*, *right*, or *straight*.



11. $\angle JFL$ 12. $\angle GFH$

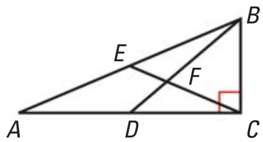
13. $\angle GFK$ 14. $\angle GFL$

NAMING AND CLASSIFYING Give another name for the angle in the diagram below. Tell whether the angle appears to be *acute*, *obtuse*, *right*, or *straight*.

15. $\angle ACB$ 16. $\angle ABC$

17. $\angle BFD$ 18. $\angle AEC$

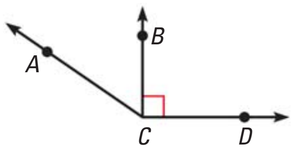
19. $\angle BDC$ 20. $\angle BEC$



21. **TAKS REASONING** Which is a correct name for the obtuse angle in the diagram?

Ⓐ $\angle ACB$ Ⓑ $\angle ACD$

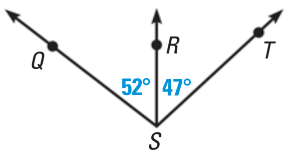
Ⓒ $\angle BCD$ Ⓓ $\angle C$

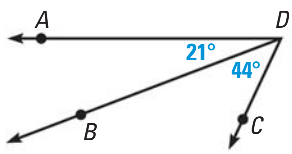


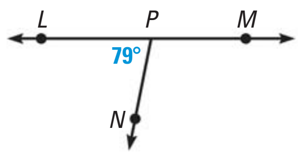
EXAMPLE 3
on p. 26
for Exs. 22–27

ANGLE ADDITION POSTULATE Find the indicated angle measure.

22. $m\angle QST =$ __?__ 23. $m\angle ADC =$ __?__ 24. $m\angle NPM =$ __?__

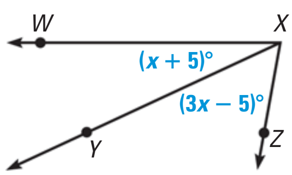


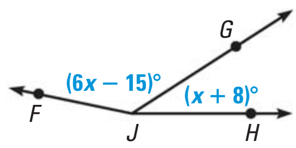




ALGEBRA Use the given information to find the indicated angle measure.

25. Given $m\angle WXZ = 80°$, find $m\angle YXZ$. 26. Given $m\angle FJH = 168°$, find $m\angle FJG$.

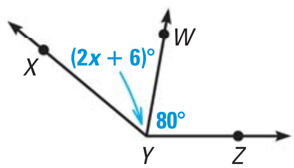




27. **TAKS REASONING** In the diagram, the measure of $\angle XYZ$ is 140°. What is the value of x?

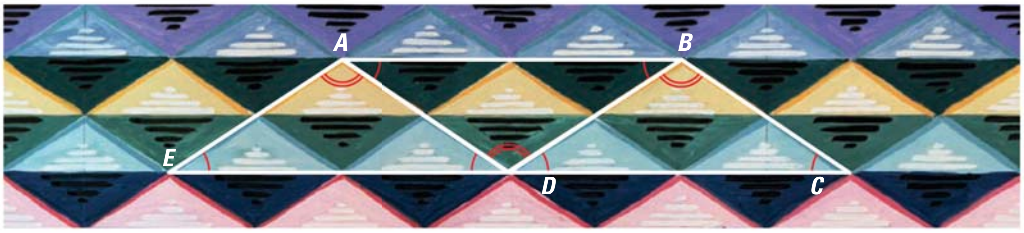
Ⓐ 27 Ⓑ 33

Ⓒ 67 Ⓓ 73



EXAMPLE 4 on p. 27 for Ex. 28

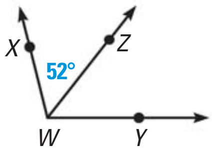
28. **CONGRUENT ANGLES** In the photograph below, $m\angle AED = 34°$ and $m\angle EAD = 112°$. Identify the congruent angles in the diagram. Then find $m\angle BDC$ and $m\angle ADB$.



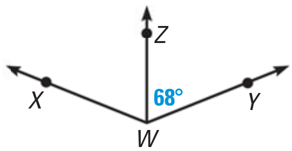
EXAMPLE 5 on p. 28 for Exs. 29–32

ANGLE BISECTORS **Given that $\overrightarrow{WZ}$ bisects $\angle XWY$, find the two angle measures not given in the diagram.**

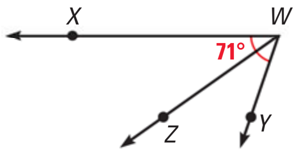
29.



30.



31.



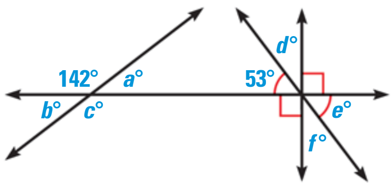
32. **ERROR ANALYSIS** $\overrightarrow{KM}$ bisects $\angle JKL$ and $m\angle JKM = 30°$. *Describe* and correct the error made in stating that $m\angle JKL = 15°$. Draw a sketch to support your answer.

FINDING ANGLE MEASURES **Find the indicated angle measure.**

33. $a°$ **34.** $b°$

35. $c°$ **36.** $d°$

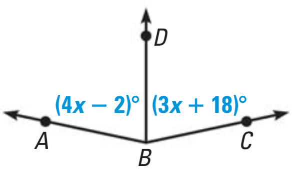
37. $e°$ **38.** $f°$



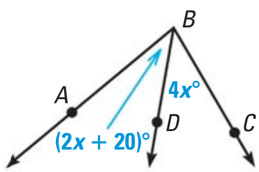
39. **ERROR ANALYSIS** A student states that $\overrightarrow{AD}$ can bisect $\angle AGC$. *Describe* and correct the student's error. Draw a sketch to support your answer.

xy **ALGEBRA** **In each diagram, $\overrightarrow{BD}$ bisects $\angle ABC$. Find $m\angle ABC$.**

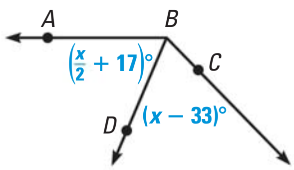
40.



41.



42.



43. **TAKS REASONING** You are measuring $\angle PQR$ with a protractor. When you line up $\overrightarrow{QR}$ with the 20° mark, $\overrightarrow{QP}$ lines up with the 80° mark. Then you move the protractor so that $\overrightarrow{QR}$ lines up with the 15° mark. What mark does $\overrightarrow{QP}$ line up with? *Explain.*

xy **ALGEBRA** **Plot the points in a coordinate plane and draw $\angle ABC$. Classify the angle. Then give the coordinates of a point that lies in the interior of the angle.**

44. $A(3, 3)$, $B(0, 0)$, $C(3, 0)$

45. $A(-5, 4)$, $B(1, 4)$, $C(-2, -2)$

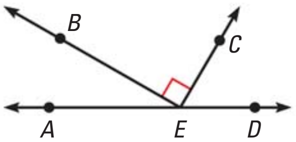
46. $A(-5, 2)$, $B(-2, -2)$, $C(4, -3)$

47. $A(-3, -1)$, $B(2, 1)$, $C(6, -2)$

48. **ALGEBRA** Let $(2x - 12)°$ represent the measure of an acute angle. What are the possible values of x?

49. **CHALLENGE** $\overrightarrow{SQ}$ bisects $\angle RST$, $\overrightarrow{SP}$ bisects $\angle RSQ$, and $\overrightarrow{SV}$ bisects $\angle RSP$. The measure of $\angle VSP$ is 17°. Find $m\angle TSQ$. *Explain.*

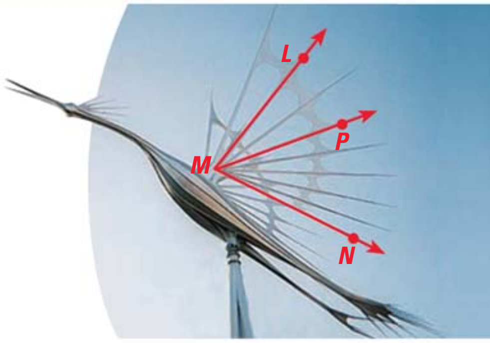
50. **FINDING MEASURES** In the diagram, $m\angle AEB = \frac{1}{2} \cdot m\angle CED$, and $\angle AED$ is a straight angle. Find $m\angle AEB$ and $m\angle CED$.



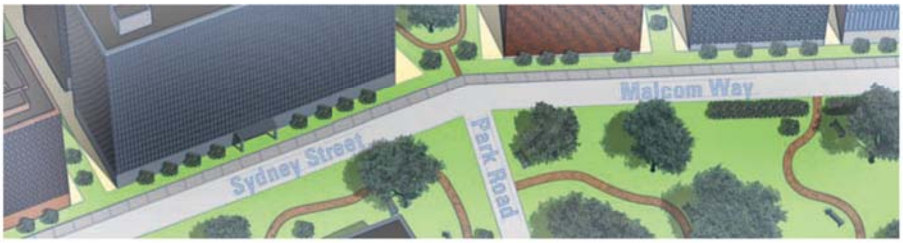
PROBLEM SOLVING

51. **SCULPTURE** In the sculpture shown in the photograph, suppose the measure of $\angle LMN$ is 79° and the measure of $\angle PMN$ is 47°. What is the measure of $\angle LMP$?

TEXAS @HomeTutor for problem solving help at classzone.com



52. **MAP** The map shows the intersection of three roads. Malcom Way intersects Sydney Street at an angle of 162°. Park Road intersects Sydney Street at an angle of 87°. Find the angle at which Malcom Way intersects Park Road.



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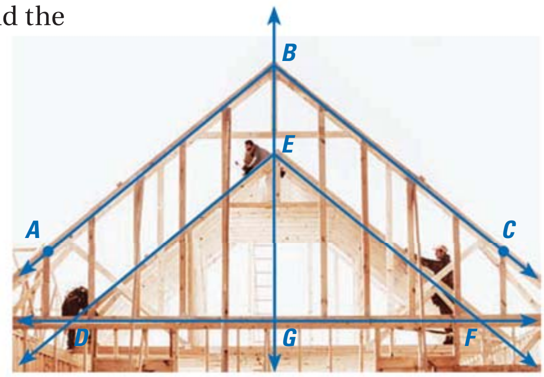
EXAMPLES 4 and 5 on pp. 27–28 for Exs. 53–55

CONSTRUCTION **In Exercises 53–55, use the photograph of a roof truss.**

53. In the roof truss, $\overrightarrow{BG}$ bisects $\angle ABC$ and $\angle DEF$, $m\angle ABC = 112°$, and $\angle ABC \cong \angle DEF$. Find the measure of the following angles.

a. $m\angle DEF$ **b.** $m\angle ABG$

c. $m\angle CBG$ **d.** $m\angle DEG$



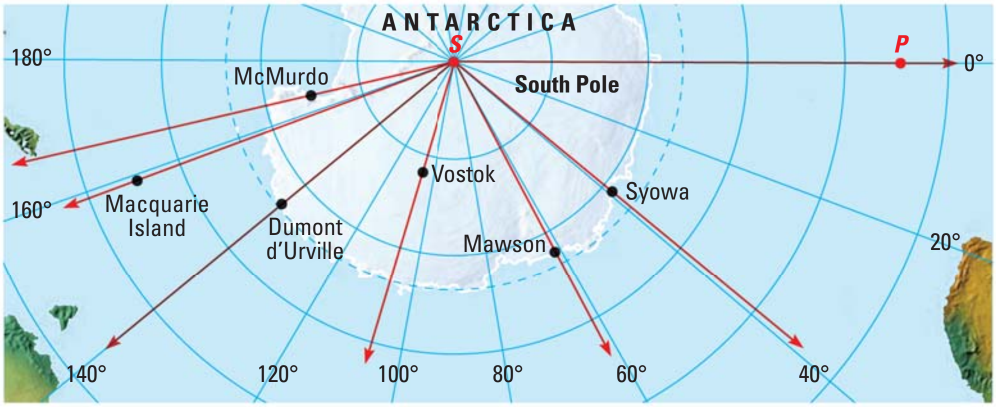
54. In the roof truss, $\overrightarrow{GB}$ bisects $\angle DGF$. Find $m\angle DGE$ and $m\angle FGE$.

55. Name an example of each of the following types of angles: *acute*, *obtuse*, *right*, and *straight*.

GEOGRAPHY **For the given location on the map, estimate the measure of $\angle PSL$, where P is on the Prime Meridian (0° longitude), S is the South Pole, and L is the location of the indicated research station.**

56. Macquarie Island **57.** Dumont d'Urville **58.** McMurdo

59. Mawson **60.** Syowa **61.** Vostok

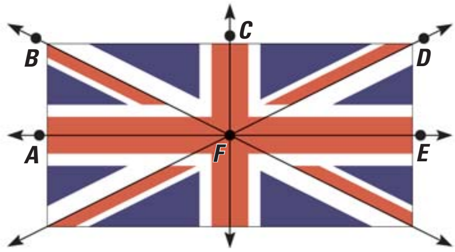


62. **TAKS REASONING** In the flag shown, $\angle AFE$ is a straight angle and $\overrightarrow{FC}$ bisects $\angle AFE$ and $\angle BFD$.

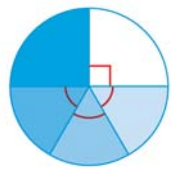
a. Which angles are acute? obtuse? right?

b. Identify the congruent angles.

c. If $m\angle AFB = 26°$, find $m\angle DFE$, $m\angle BFC$, $m\angle CFD$, $m\angle AFC$, $m\angle AFD$, and $m\angle BFD$. *Explain.*



63. **CHALLENGE** Create a set of data that could be represented by the circle graph at the right. *Explain* your reasoning.



TAKS PRACTICE at classzone.com

MIXED REVIEW FOR TAKS

REVIEW
Skills Review Handbook p. 884; TAKS Workbook

64. **TAKS PRACTICE** The equation $y = 2.6x^2 - 3.4x + 1.2$ shows the relationship between x, the number of years since a company began business, and y, the company's profit in millions of dollars. What is the company's profit after they are in business for 8 years? *TAKS Obj. 2*

Ⓐ \$138 million Ⓑ \$140.4 million Ⓒ \$192.4 million Ⓓ \$194.8 million

REVIEW
TAKS Preparation p. 66; TAKS Workbook

65. **TAKS PRACTICE** A cylindrical salt shaker has a height of 7 centimeters and a diameter of 4 centimeters. Mike fills the salt shaker to 0.5 centimeter from the top. Which expression can be used to find the volume of salt in the salt shaker? *TAKS Obj. 8*

Ⓕ $\pi(2^2)(7 - 0.5)$ Ⓖ $\pi(7 - 0.5)^2(2)$ Ⓗ $\pi(4^2)(7 - 0.5)$ Ⓙ $\pi(7 - 0.5)^2(4)$

Investigating Geometry **CONSTRUCTION** *Use after Lesson 1.4*

1.4 Copy and Bisect Segments and Angles

MATERIALS • compass • straightedge

TEKS *a.5, G.2.A*

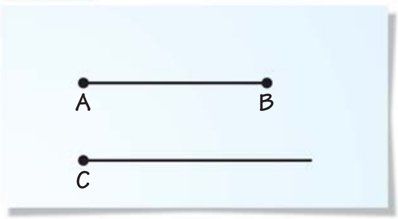
QUESTION **How can you copy and bisect segments and angles?**

A **construction** is a geometric drawing that uses a limited set of tools, usually a *compass* and *straightedge*. You can use a compass and straightedge (a ruler without marks) to construct a segment that is congruent to a given segment, and an angle that is congruent to a given angle.

EXPLORE 1 Copy a segment

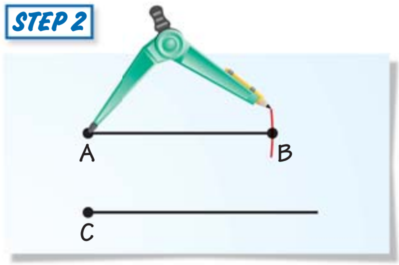
Use the following steps to construct a segment that is congruent to $\overline{AB}$.

STEP 1



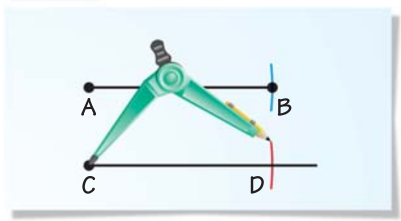
Draw a segment Use a straightedge to draw a segment longer than $\overline{AB}$. Label point C on the new segment.

STEP 2



Measure length Set your compass at the length of $\overline{AB}$.

STEP 3

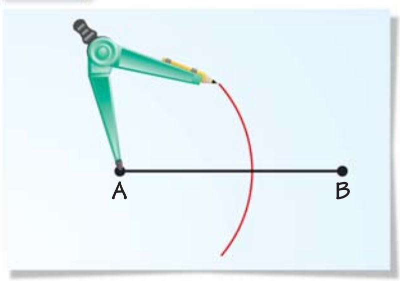


Copy length Place the compass at C. Mark point D on the new segment. $\overline{CD} \cong \overline{AB}$.

EXPLORE 2 Bisect a segment

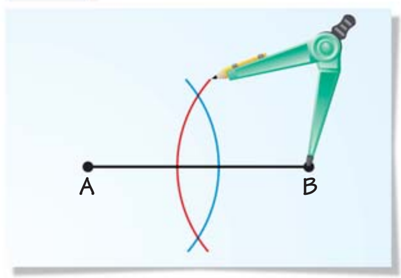
Use the following steps to construct a bisector of $\overline{AB}$ and to find the midpoint M of $\overline{AB}$.

STEP 1



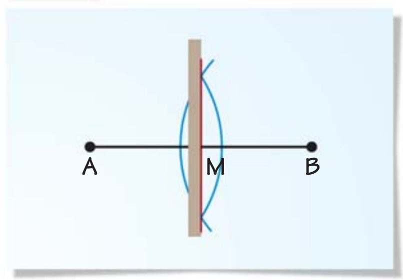
Draw an arc Place the compass at A. Use a compass setting that is greater than half the length of $\overline{AB}$. Draw an arc.

STEP 2



Draw a second arc Keep the same compass setting. Place the compass at B. Draw an arc. It should intersect the other arc at two points.

STEP 3

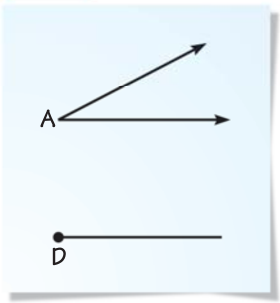


Bisect segment Draw a segment through the two points of intersection. This segment bisects $\overline{AB}$ at M, the midpoint of $\overline{AB}$.

EXPLORE 3 Copy an angle

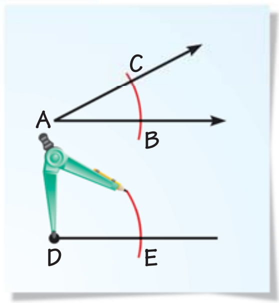
Use the following steps to construct an angle that is congruent to $\angle A$. In this construction, the *radius* of an arc is the distance from the point where the compass point rests (the *center* of the arc) to a point on the arc drawn by the compass.

STEP 1



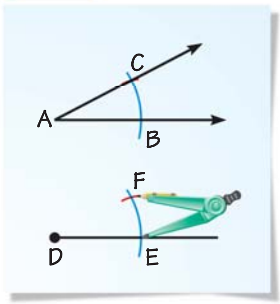
Draw a segment Draw a segment. Label a point D on the segment.

STEP 2



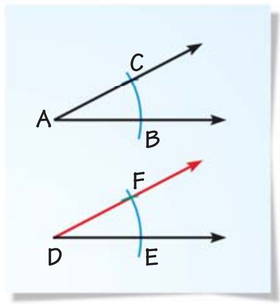
Draw arcs Draw an arc with center A. Using the same radius, draw an arc with center D.

STEP 3



Draw arcs Label B, C, and E. Draw an arc with radius BC and center E. Label the intersection F.

STEP 4

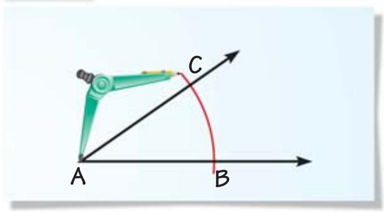


Draw a ray Draw $\overrightarrow{DF}$. $\angle EDF \cong \angle BAC$.

EXPLORE 4 Bisect an angle

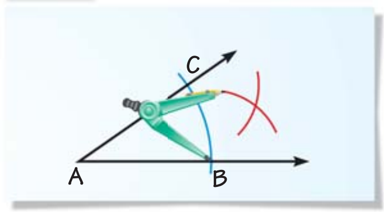
Use the following steps to construct an angle bisector of $\angle A$.

STEP 1



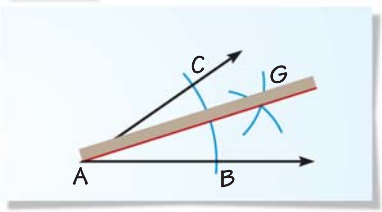
Draw an arc Place the compass at A. Draw an arc that intersects both sides of the angle. Label the intersections C and B.

STEP 2



Draw arcs Place the compass at C. Draw an arc. Then place the compass point at B. Using the same radius, draw another arc.

STEP 3



Draw a ray Label the intersection G. Use a straightedge to draw a ray through A and G. $\overrightarrow{AG}$ bisects $\angle A$.

DRAW CONCLUSIONS Use your observations to complete these exercises

1. *Describe* how you could use a compass and a straightedge to draw a segment that is twice as long as a given segment.

2. Draw an obtuse angle. Copy the angle using a compass and a straightedge. Then bisect the angle using a compass and straightedge.

1.5 Describe Angle Pair Relationships

TEKS a.1, a.4, G.4, G.5.A

Before You used angle postulates to measure and classify angles.

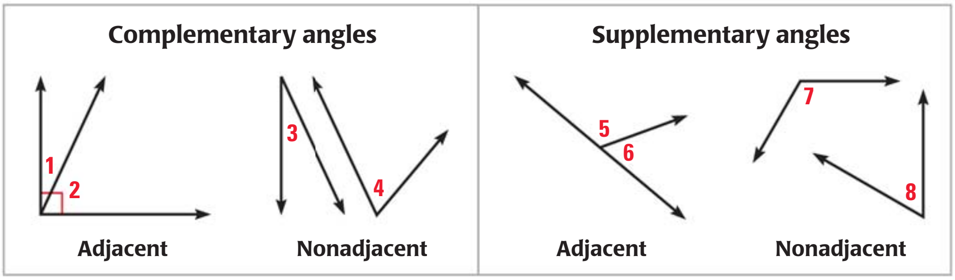
Now You will use special angle relationships to find angle measures.

Why? So you can find measures in a building, as in Ex. 53.

Key Vocabulary
- complementary angles
- supplementary angles
- adjacent angles
- linear pair
- vertical angles

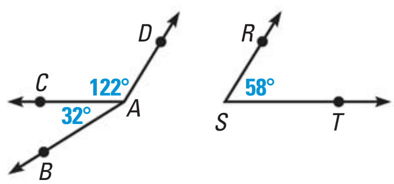
Two angles are **complementary angles** if the sum of their measures is 90°. Each angle is the *complement* of the other. Two angles are **supplementary angles** if the sum of their measures is 180°. Each angle is the *supplement* of the other.

Complementary angles and supplementary angles can be *adjacent angles* or *nonadjacent angles*. **Adjacent angles** are two angles that share a common vertex and side, but have no common interior points.



EXAMPLE 1 Identify complements and supplements

In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.



Solution

Because 32° + 58° = 90°, ∠*BAC* and ∠*RST* are complementary angles.

Because 122° + 58° = 180°, ∠*CAD* and ∠*RST* are supplementary angles.

Because ∠*BAC* and ∠*CAD* share a common vertex and side, they are adjacent.

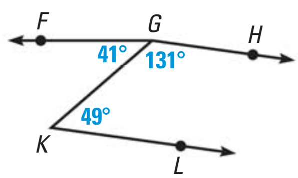
AVOID ERRORS

In Example 1, ∠*DAC* and ∠*DAB* share a common vertex. But they share common interior points, so they are *not* adjacent angles.

✓ GUIDED PRACTICE for Example 1

1. In the figure, name a pair of complementary angles, a pair of supplementary angles, and a pair of adjacent angles.

2. Are ∠*KGH* and ∠*LKG* adjacent angles? Are ∠*FGK* and ∠*FGH* adjacent angles? *Explain.*



EXAMPLE 2 Find measures of a complement and a supplement

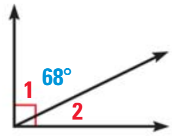
READ DIAGRAMS
Angles are sometimes named with numbers. An angle measure in a diagram has a degree symbol. An angle name does not.

a. Given that $\angle 1$ is a complement of $\angle 2$ and $m\angle 1 = 68°$, find $m\angle 2$.

b. Given that $\angle 3$ is a supplement of $\angle 4$ and $m\angle 4 = 56°$, find $m\angle 3$.

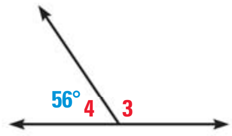
Solution

a. You can draw a diagram with complementary adjacent angles to illustrate the relationship.



$m\angle 2 = 90° - m\angle 1 = 90° - 68° = 22°$

b. You can draw a diagram with supplementary adjacent angles to illustrate the relationship.

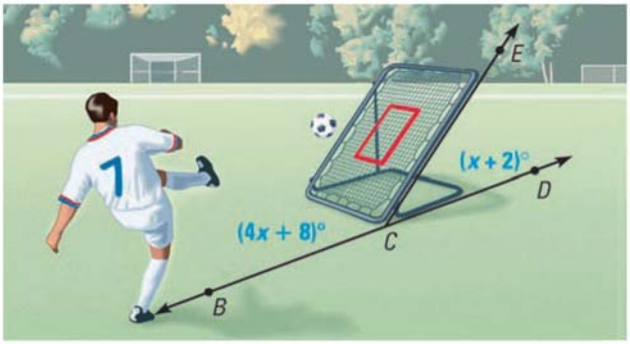


$m\angle 3 = 180° - m\angle 4 = 180° - 56° = 124°$

EXAMPLE 3 Find angle measures

READ DIAGRAMS
In a diagram, you can assume that a line that looks straight *is* straight. In Example 3, *B*, *C*, and *D* lie on $\overleftrightarrow{BD}$. So, $\angle BCD$ is a straight angle.

SPORTS When viewed from the side, the frame of a ball-return net forms a pair of supplementary angles with the ground. Find $m\angle BCE$ and $m\angle ECD$.



Solution

STEP 1 **Use** the fact that the sum of the measures of supplementary angles is 180°.

$m\angle BCE + m\angle ECD = 180°$ **Write equation.**

$(4x + 8)° + (x + 2)° = 180°$ **Substitute.**

$5x + 10 = 180$ **Combine like terms.**

$5x = 170$ **Subtract 10 from each side.**

$x = 34$ **Divide each side by 5.**

STEP 2 **Evaluate** the original expressions when $x = 34$.

$m\angle BCE = (4x + 8)° = (4 \cdot 34 + 8)° = 144°$

$m\angle ECD = (x + 2)° = (34 + 2)° = 36°$

▶ The angle measures are 144° and 36°.

✓ GUIDED PRACTICE for Examples 2 and 3

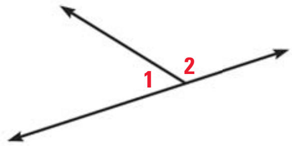
3. Given that $\angle 1$ is a complement of $\angle 2$ and $m\angle 2 = 8°$, find $m\angle 1$.

4. Given that $\angle 3$ is a supplement of $\angle 4$ and $m\angle 3 = 117°$, find $m\angle 4$.

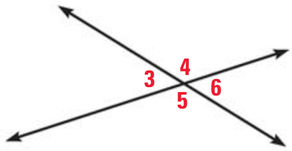
5. $\angle LMN$ and $\angle PQR$ are complementary angles. Find the measures of the angles if $m\angle LMN = (4x - 2)°$ and $m\angle PQR = (9x + 1)°$.

ANGLE PAIRS Two adjacent angles are a **linear pair** if their noncommon sides are opposite rays. The angles in a linear pair are supplementary angles.

Two angles are **vertical angles** if their sides form two pairs of opposite rays.



∠1 and ∠2 are a linear pair.

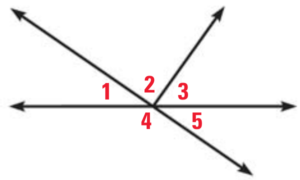


∠3 and ∠6 are vertical angles.
∠4 and ∠5 are vertical angles.

EXAMPLE 4 Identify angle pairs

AVOID ERRORS
In the diagram, one side of ∠1 and one side of ∠3 are opposite rays. But the angles are not a linear pair because they are not adjacent.

Identify all of the linear pairs and all of the vertical angles in the figure at the right.



Solution

To find vertical angles, look for angles formed by intersecting lines.

▶ ∠1 and ∠5 are vertical angles.

To find linear pairs, look for adjacent angles whose noncommon sides are opposite rays.

▶ ∠1 and ∠4 are a linear pair. ∠4 and ∠5 are also a linear pair.

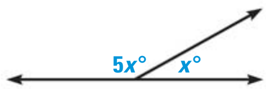
EXAMPLE 5 Find angle measures in a linear pair

ALGEBRA **Two angles form a linear pair. The measure of one angle is 5 times the measure of the other. Find the measure of each angle.**

Solution

DRAW DIAGRAMS
You may find it useful to draw a diagram to represent a word problem like the one in Example 5.

Let $x°$ be the measure of one angle. The measure of the other angle is $5x°$. Then use the fact that the angles of a linear pair are supplementary to write an equation.



$x° + 5x° = 180°$ **Write an equation.**

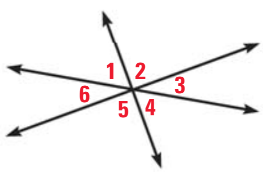
$6x = 180$ **Combine like terms.**

$x = 30$ **Divide each side by 6.**

▶ The measures of the angles are 30° and 5(30°) = 150°.

✓ GUIDED PRACTICE for Examples 4 and 5

6. Do any of the numbered angles in the diagram at the right form a linear pair? Which angles are vertical angles? *Explain.*

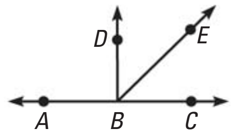


7. The measure of an angle is twice the measure of its complement. Find the measure of each angle.

CONCEPT SUMMARY — *For Your Notebook*

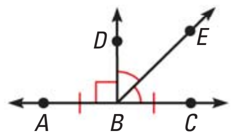
Interpreting a Diagram

There are some things you can conclude from a diagram, and some you cannot. For example, here are some things that you ***can* conclude** from the diagram at the right:



- All points shown are coplanar.
- Points *A*, *B*, and *C* are collinear, and *B* is between *A* and *C*.
- $\overleftrightarrow{AC}$, $\overrightarrow{BD}$, and $\overrightarrow{BE}$ intersect at point *B*.
- $\angle DBE$ and $\angle EBC$ are adjacent angles, and $\angle ABC$ is a straight angle.
- Point *E* lies in the interior of $\angle DBC$.

In the diagram above, you ***cannot* conclude** that $\overline{AB} \cong \overline{BC}$, that $\angle DBE \cong \angle EBC$, or that $\angle ABD$ is a right angle. This information must be indicated, as shown at the right.



1.5 EXERCISES

HOMEWORK KEY

◯ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 9, 21, and 47

= **TAKS PRACTICE AND REASONING** Exs. 16, 30, 53, 57, and 58

= **MULTIPLE REPRESENTATIONS** Ex. 55

SKILL PRACTICE

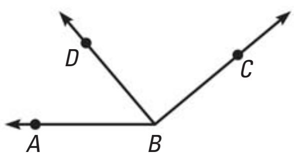
1. VOCABULARY Sketch an example of adjacent angles that are complementary. Are all complementary angles adjacent angles? *Explain*.

2. WRITING Are all linear pairs supplementary angles? Are all supplementary angles linear pairs? *Explain*.

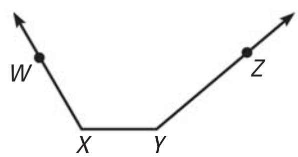
EXAMPLE 1 on p. 35 for Exs. 3–7

IDENTIFYING ANGLES **Tell whether the indicated angles are adjacent.**

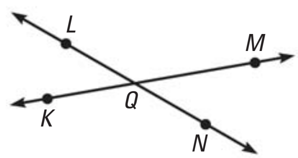
3. $\angle ABD$ and $\angle DBC$



4. $\angle WXY$ and $\angle XYZ$

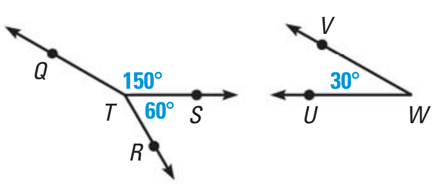


5. $\angle LQM$ and $\angle NQM$

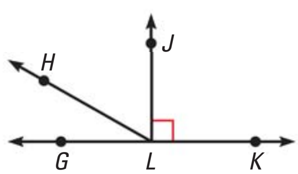


IDENTIFYING ANGLES **Name a pair of complementary angles and a pair of supplementary angles.**

6.



7.



EXAMPLE 2 on p. 36 for Exs. 8–16

COMPLEMENTARY ANGLES **$\angle 1$ and $\angle 2$ are complementary angles. Given the measure of $\angle 1$, find $m\angle 2$.**

8. $m\angle 1 = 43^\circ$ **9.** $m\angle 1 = 21^\circ$ **10.** $m\angle 1 = 89^\circ$ **11.** $m\angle 1 = 5^\circ$

SUPPLEMENTARY ANGLES **$\angle 1$ and $\angle 2$ are supplementary angles. Given the measure of $\angle 1$, find $m\angle 2$.**

12. $m\angle 1 = 60^\circ$ **13.** $m\angle 1 = 155^\circ$ **14.** $m\angle 1 = 130^\circ$ **15.** $m\angle 1 = 27^\circ$

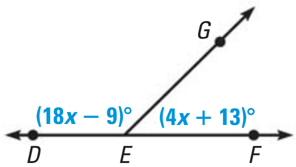
16. **TAKS REASONING** The arm of a crossing gate moves 37° from vertical. How many more degrees does the arm have to move so that it is horizontal?

Ⓐ 37°
Ⓑ 53°
Ⓒ 90°
Ⓓ 143°

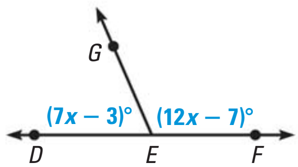
EXAMPLE 3 on p. 36 for Exs. 17–19

ALGEBRA **Find $m\angle DEG$ and $m\angle GEF$.**

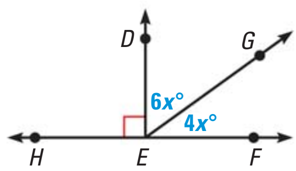
17.



18.



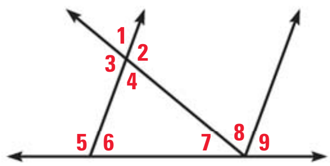
19.



EXAMPLE 4 on p. 37 for Exs. 20–27

IDENTIFYING ANGLE PAIRS **Use the diagram below. Tell whether the angles are *vertical angles*, a *linear pair*, or *neither*.**

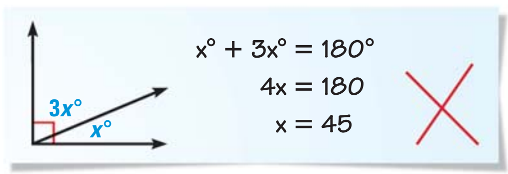
20. $\angle 1$ and $\angle 4$ **21.** $\angle 1$ and $\angle 2$
22. $\angle 3$ and $\angle 5$ **23.** $\angle 2$ and $\angle 3$
24. $\angle 7$, $\angle 8$, and $\angle 9$ **25.** $\angle 5$ and $\angle 6$
26. $\angle 6$ and $\angle 7$ **27.** $\angle 5$ and $\angle 9$



EXAMPLE 5 on p. 37 for Exs. 28–30

28. **ALGEBRA** Two angles form a linear pair. The measure of one angle is 4 times the measure of the other angle. Find the measure of each angle.

29. **ERROR ANALYSIS** *Describe* and correct the error made in finding the value of *x*.

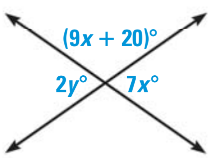


30. **TAKS REASONING** The measure of one angle is 24° greater than the measure of its complement. What are the measures of the angles?

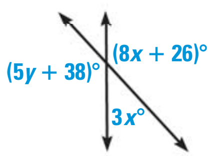
Ⓐ 24° and 66° Ⓑ 24° and 156° Ⓒ 33° and 57° Ⓓ 78° and 102°

ALGEBRA **Find the values of *x* and *y*.**

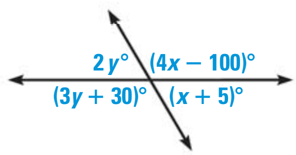
31.



32.



33.



REASONING **Tell whether the statement is *always*, *sometimes*, or *never* true. *Explain* your reasoning.**

34. An obtuse angle has a complement.

35. A straight angle has a complement.

36. An angle has a supplement.

37. The complement of an acute angle is an acute angle.

38. The supplement of an acute angle is an obtuse angle.

FINDING ANGLES **$\angle A$ and $\angle B$ are complementary. Find $m\angle A$ and $m\angle B$.**

39. $m\angle A = (3x + 2)°$
$m\angle B = (x - 4)°$

40. $m\angle A = (15x + 3)°$
$m\angle B = (5x - 13)°$

41. $m\angle A = (11x + 24)°$
$m\angle B = (x + 18)°$

FINDING ANGLES **$\angle A$ and $\angle B$ are supplementary. Find $m\angle A$ and $m\angle B$.**

42. $m\angle A = (8x + 100)°$
$m\angle B = (2x + 50)°$

43. $m\angle A = (2x - 20)°$
$m\angle B = (3x + 5)°$

44. $m\angle A = (6x + 72)°$
$m\angle B = (2x + 28)°$

45. **CHALLENGE** You are given that $\angle GHJ$ is a complement of $\angle RST$ and $\angle RST$ is a supplement of $\angle ABC$. Let $m\angle GHJ$ be $x°$. What is the measure of $\angle ABC$? *Explain* your reasoning.

PROBLEM SOLVING

IDENTIFYING ANGLES **Tell whether the two angles shown are *complementary*, *supplementary*, or *neither*.**

46.


47.


48.


TEXAS @HomeTutor for problem solving help at classzone.com

ARCHITECTURE **The photograph shows the Rock and Roll Hall of Fame in Cleveland, Ohio. Use the photograph to identify an example type of the indicated type of angle pair.**

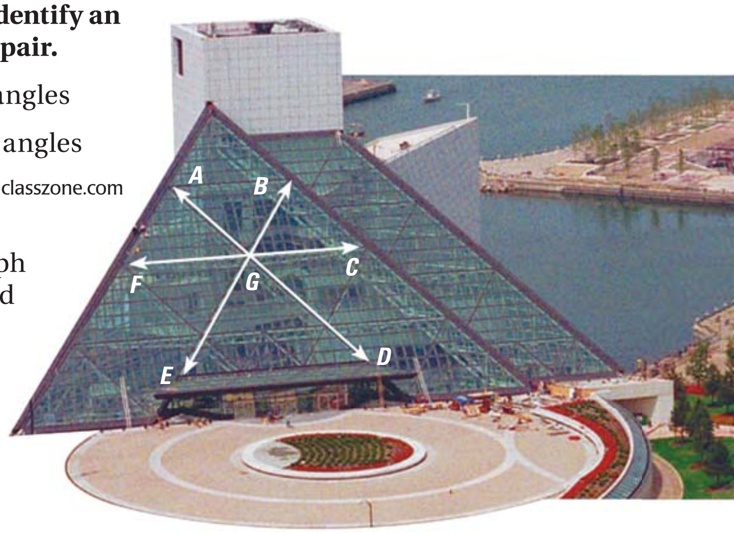
49. Supplementary angles

50. Vertical angles

51. Linear pair

52. Adjacent angles

TEXAS @HomeTutor for problem solving help at classzone.com



53. **TAKS REASONING** Use the photograph shown at the right. Given that $\angle FGB$ and $\angle BGC$ are supplementary angles, and $m\angle FGB = 120°$, *explain* how to find the measure of the complement of $\angle BGC$.

= WORKED-OUT SOLUTIONS on p. WS1

= TAKS PRACTICE AND REASONING

= MULTIPLE REPRESENTATIONS

54. **SHADOWS** The length of a shadow changes as the sun rises. In the diagram below, the length of $\overline{CB}$ is the length of a shadow. The end of the shadow is the vertex of $\angle ABC$, which is formed by the ground and the sun's rays. *Describe* how the shadow and angle change as the sun rises.

55. **MULTIPLE REPRESENTATIONS** Let $x°$ be an angle measure. Let $y_1°$ be the measure of a complement of the angle and let $y_2°$ be the measure of a supplement of the angle.

   a. **Writing an Equation** Write equations for y_1 as a function of x, and for y_2 as a function of x. What is the domain of each function? *Explain*.

   b. **Drawing a Graph** Graph each function and *describe* its range.

56. **CHALLENGE** The sum of the measures of two complementary angles exceeds the difference of their measures by 86°. Find the measure of each angle. *Explain* how you found the angle measures.

TAKS PRACTICE at classzone.com

MIXED REVIEW FOR TAKS

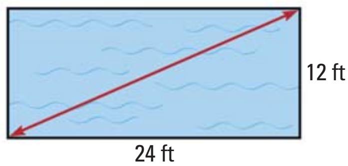
REVIEW
Skills Review Handbook p. 878; TAKS Workbook

57. **TAKS PRACTICE** The point $(-1, y)$ is a solution of the equation $6x + 5y = 19$. What is the value of y? *TAKS Obj. 4*

Ⓐ -3 Ⓑ 1 Ⓒ 4 Ⓓ 5

REVIEW
Lesson 1.3; TAKS Workbook

58. **TAKS PRACTICE** Anna swims diagonal laps in the pool shown. About how many laps must she complete to swim 0.5 mile? *TAKS Obj. 8*

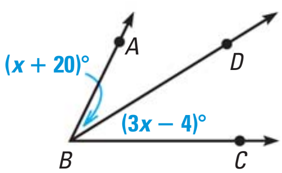


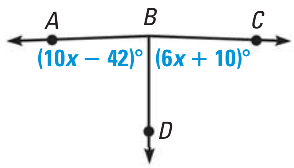
Ⓕ 73 Ⓖ 98

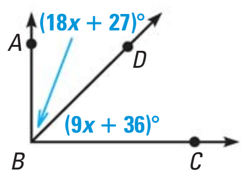
Ⓗ 127 Ⓙ 197

QUIZ *for Lessons 1.4–1.5*

In each diagram, $\overrightarrow{BD}$ bisects $\angle ABC$. Find $m\angle ABD$ and $m\angle DBC$. *(p. 24)*

1. 

2. 

3. 

Find the measure of (a) the complement and (b) the supplement of $\angle 1$. *(p. 35)*

4. $m\angle 1 = 47°$ 5. $m\angle 1 = 19°$ 6. $m\angle 1 = 75°$ 7. $m\angle 1 = 2°$

1.6 Classify Polygons

TEKS a.3, G.2.A

Before	You classified angles.
Now	You will classify polygons.
Why?	So you can find lengths in a floor plan, as in Ex. 32.

Key Vocabulary
- **polygon**
  side, vertex
- **convex**
- **concave**
- ***n*-gon**
- **equilateral**
- **equiangular**
- **regular**

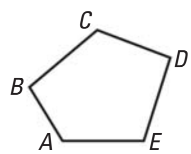
KEY CONCEPT *For Your Notebook*

Identifying Polygons

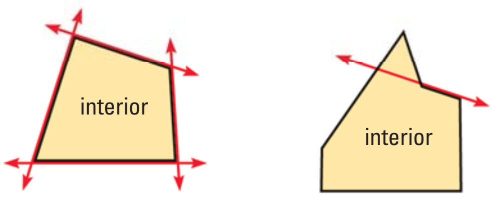
In geometry, a figure that lies in a plane is called a *plane figure*. A **polygon** is a closed plane figure with the following properties.

1. It is formed by three or more line segments called **sides**.
2. Each side intersects exactly two sides, one at each endpoint, so that no two sides with a common endpoint are collinear.

Each endpoint of a side is a **vertex** of the polygon. The plural of vertex is *vertices*. A polygon can be named by listing the vertices in consecutive order. For example, *ABCDE* and *CDEAB* are both correct names for the polygon at the right.



A polygon is **convex** if no line that contains a side of the polygon contains a point in the interior of the polygon. A polygon that is not convex is called *nonconvex* or **concave**.

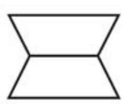


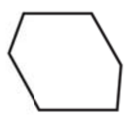
convex polygon   concave polygon

EXAMPLE 1 **Identify polygons**

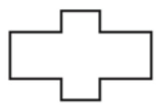
READ VOCABULARY
A *plane figure* is two-dimensional. Later, you will study three-dimensional *space figures* such as prisms and cylinders.

Tell whether the figure is a polygon and whether it is *convex* or *concave*.

a. 

b. 

c. 

d. 

Solution

a. Some segments intersect more than two segments, so it is not a polygon.

b. The figure is a convex polygon.

c. Part of the figure is not a segment, so it is not a polygon.

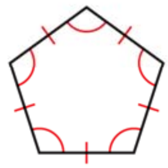
d. The figure is a concave polygon.

CLASSIFYING POLYGONS A polygon is named by the number of its sides.

Number of sides	Type of polygon	Number of sides	Type of polygon
3	Triangle	8	Octagon
4	Quadrilateral	9	Nonagon
5	Pentagon	10	Decagon
6	Hexagon	12	Dodecagon
7	Heptagon	*n*	*n*-gon

The term ***n*-gon**, where *n* is the number of a polygon's sides, can also be used to name a polygon. For example, a polygon with 14 sides is a 14-gon.

In an **equilateral** polygon, all sides are congruent. In an **equiangular** polygon, all angles in the interior of the polygon are congruent. A **regular** polygon is a convex polygon that is both equilateral and equiangular.



regular pentagon

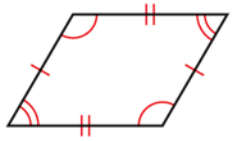
EXAMPLE 2 Classify polygons

READ DIAGRAMS
Double marks are used in part (b) of Example 2 to show that more than one pair of sides are congruent and more than one pair of angles are congruent.

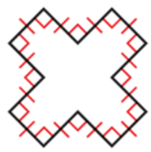
Classify the polygon by the number of sides. Tell whether the polygon is equilateral, equiangular, or regular. Explain your reasoning.

a.

b.



c.



Solution

a. The polygon has 6 sides. It is equilateral and equiangular, so it is a regular hexagon.

b. The polygon has 4 sides, so it is a quadrilateral. It is not equilateral or equiangular, so it is not regular.

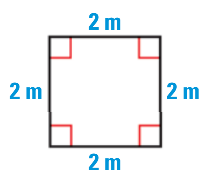
c. The polygon has 12 sides, so it is a dodecagon. The sides are congruent, so it is equilateral. The polygon is not convex, so it is not regular.

Animated Geometry at classzone.com

✓ GUIDED PRACTICE for Examples 1 and 2

1. Sketch an example of a convex heptagon and an example of a concave heptagon.

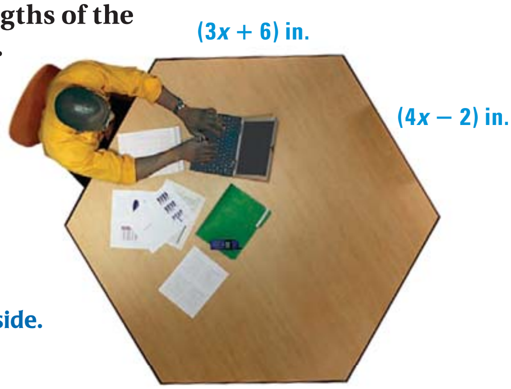
2. Classify the polygon shown at the right by the number of sides. *Explain* how you know that the sides of the polygon are congruent and that the angles of the polygon are congruent.



EXAMPLE 3 Find side lengths

READ VOCABULARY
Hexagonal means "shaped like a hexagon."

ALGEBRA **A table is shaped like a regular hexagon. The expressions shown represent side lengths of the hexagonal table. Find the length of a side.**



Solution

First, write and solve an equation to find the value of x. Use the fact that the sides of a regular hexagon are congruent.

$3x + 6 = 4x - 2$ **Write equation.**

$6 = x - 2$ **Subtract $3x$ from each side.**

$8 = x$ **Add 2 to each side.**

Then find a side length. Evaluate one of the expressions when $x = 8$.

$3x + 6 = 3(8) + 6 = 30$

▶ The length of a side of the table is 30 inches.

✓ GUIDED PRACTICE for Example 3

3. The expressions $8y°$ and $(9y - 15)°$ represent the measures of two of the angles in the table in Example 3. Find the measure of an angle.

1.6 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 13, 19, and 33
★ = **TAKS PRACTICE AND REASONING** Exs. 7, 37, 39, 40, 42, and 43

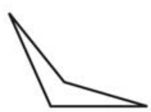
SKILL PRACTICE

1. VOCABULARY *Explain* what is meant by the term n-gon.

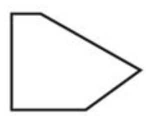
2. WRITING Imagine that you can tie a string tightly around a polygon. If the polygon is convex, will the length of the string be equal to the distance around the polygon? What if the polygon is concave? *Explain.*

EXAMPLE 1
on p. 42
for Exs. 3–7

IDENTIFYING POLYGONS **Tell whether the figure is a polygon. If it is not, *explain* why. If it is a polygon, tell whether it is *convex* or *concave*.**

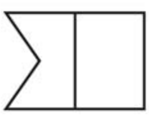
3.


4.

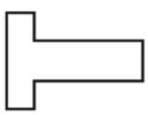

5.


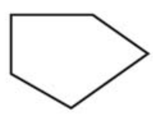
6.


7. ★ **TAKS REASONING** Which of the figures is a concave polygon?

Ⓐ


Ⓑ

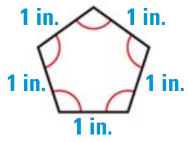

Ⓒ


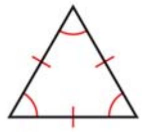
Ⓓ


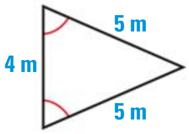
EXAMPLE 2
on p. 43
for Exs. 8–14

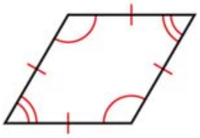
CLASSIFYING **Classify the polygon by the number of sides. Tell whether the polygon is equilateral, equiangular, or regular. *Explain* your reasoning.**

8. 

9. 

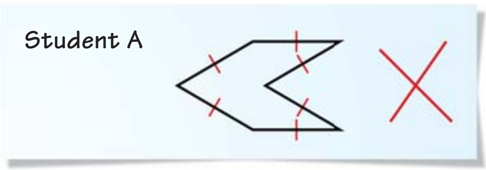
10. 

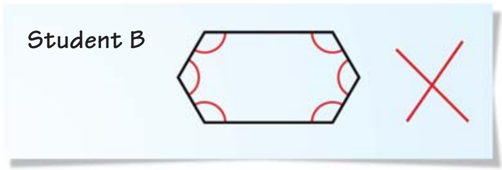
11. 

12. 

13. 

14. **ERROR ANALYSIS** Two students were asked to draw a regular hexagon, as shown below. *Describe* the error made by each student.





EXAMPLE 3
on p. 44
for Exs. 15–17

15. **ALGEBRA** The lengths (in inches) of two sides of a regular pentagon are represented by the expressions $5x - 27$ and $2x - 6$. Find the length of a side of the pentagon.

16. **ALGEBRA** The expressions $(9x + 5)°$ and $(11x - 25)°$ represent the measures of two angles of a regular nonagon. Find the measure of an angle of the nonagon.

17. **ALGEBRA** The expressions $3x - 9$ and $23 - 5x$ represent the lengths (in feet) of two sides of an equilateral triangle. Find the length of a side.

USING PROPERTIES **Tell whether the statement is *always*, *sometimes*, or *never* true.**

18. A triangle is convex.

19. A decagon is regular.

20. A regular polygon is equiangular.

21. A circle is a polygon.

22. A polygon is a plane figure.

23. A concave polygon is regular.

DRAWING **Draw a figure that fits the description.**

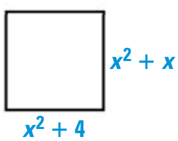
24. A triangle that is not regular

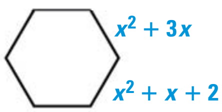
25. A concave quadrilateral

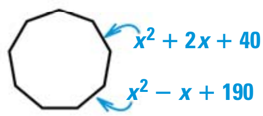
26. A pentagon that is equilateral but not equiangular

27. An octagon that is equiangular but not equilateral

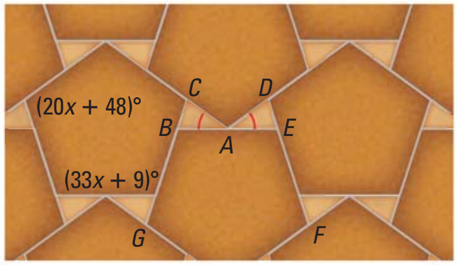
ALGEBRA **Each figure is a regular polygon. Expressions are given for two side lengths. Find the value of x.**

28. 

29. 

30. 

31. **CHALLENGE** Regular pentagonal tiles and triangular tiles are arranged in the pattern shown. The pentagonal tiles are all the same size and shape and the triangular tiles are all the same size and shape. Find the angle measures of the triangular tiles. *Explain* your reasoning.



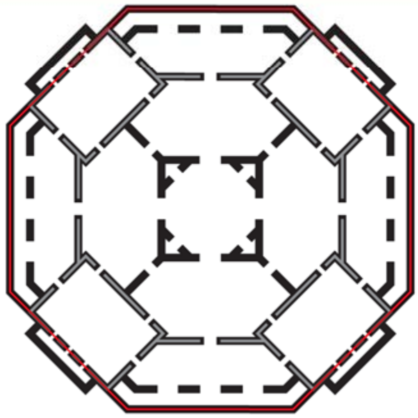
PROBLEM SOLVING

32. **ARCHITECTURE** Longwood House, shown in the photograph on page 42, is located in Natchez, Mississippi. The diagram at the right shows the floor plan of a part of the house.

   a. Tell whether the red polygon in the diagram is *convex* or *concave*.

   b. Classify the red polygon and tell whether it appears to be regular.

   TEXAS @HomeTutor for problem solving help at classzone.com



EXAMPLE 2 on p. 43 for Exs. 33–36

SIGNS Each sign suggests a polygon. Classify the polygon by the number of sides. Tell whether it appears to be *equilateral*, *equiangular*, or *regular*.

33. 

34. 

35. 

36. 

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37. **TAKS REASONING** Two vertices of a regular quadrilateral are $A(0, 4)$ and $B(0, -4)$. Which of the following could be the other two vertices?

   Ⓐ $C(4, 4)$ and $D(4, -4)$

   Ⓑ $C(-4, 4)$ and $D(-4, -4)$

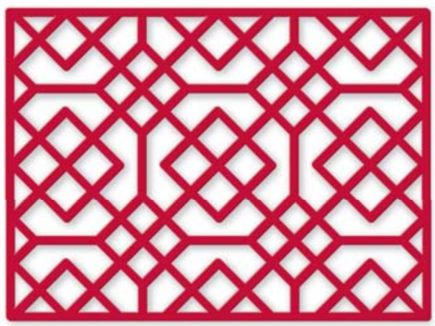
   Ⓒ $C(8, -4)$ and $D(8, 4)$

   Ⓓ $C(0, 8)$ and $D(0, -8)$

38. **MULTI-STEP PROBLEM** The diagram shows the design of a lattice made in China in 1850.

   a. Sketch five different polygons you see in the diagram. Classify each polygon by the number of sides.

   b. Tell whether each polygon you sketched is concave or convex, and whether the polygon appears to be equilateral, equiangular, or regular.

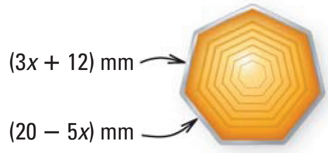


○ = **WORKED-OUT SOLUTIONS** on p. WS1

= **TAKS PRACTICE AND REASONING**

EXAMPLE 3
on p. 44
for Ex. 39

39. **TAKS REASONING** The shape of the button shown is a regular polygon. The button has a border made of silver wire. How many millimeters of silver wire are needed for this border? *Explain*.



40. **TAKS REASONING** A segment that joins two nonconsecutive vertices of a polygon is called a *diagonal*. For example, a quadrilateral has two diagonals, as shown below.

Type of polygon	Diagram	Number of sides	Number of diagonals
Quadrilateral		4	2
Pentagon	?	?	?
Hexagon	?	?	?
Heptagon	?	?	?

a. Copy and complete the table. *Describe* any patterns you see.

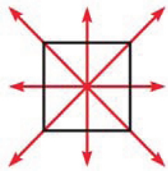
b. How many diagonals does an octagon have? a nonagon? *Explain*.

c. The expression $\frac{n(n-3)}{2}$ can be used to find the number of diagonals in an n-gon. Find the number of diagonals in a 60-gon.

41. **LINE SYMMETRY** A figure has *line symmetry* if it can be folded over exactly onto itself. The fold line is called the *line of symmetry*. A regular quadrilateral has four lines of symmetry, as shown. Find the number of lines of symmetry in each polygon.

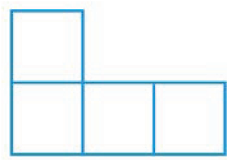
a. A regular triangle **b.** A regular pentagon

c. A regular hexagon **d.** A regular octagon



regular quadrilateral
4 lines of symmetry

42. **CHALLENGE** The diagram shows four identical squares lying edge-to-edge. Sketch all the different ways you can arrange four squares edge-to-edge. Sketch all the different ways you can arrange five identical squares edge-to-edge.



MIXED REVIEW FOR TAKS

TAKS PRACTICE at classzone.com

REVIEW
Skills Review Handbook p. 884;
TAKS Workbook

42. **TAKS PRACTICE** A function $f(x) = 2x^2 + 1$ has {1, 3, 5, 8} as the replacement set for the independent variable x. Which of the following is contained in the corresponding set for the dependent variable? *TAKS Obj. 1*

(A) 0 (B) 5 (C) 15 (D) 19

REVIEW
TAKS Preparation p. 66;
TAKS Workbook

43. **TAKS PRACTICE** The radius of Cylinder A is three times the radius of Cylinder B. The heights of the cylinders are equal. How many times greater is the volume of Cylinder A than the volume of Cylinder B? *TAKS Obj. 8*

(F) 3 (G) 6 (H) 9 (J) 27

Investigating Geometry ACTIVITY *Use before Lesson 1.7*

TEXAS @HomeTutor
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Keystrokes

1.7 Investigate Perimeter and Area

MATERIALS • graph paper • graphing calculator

TEKS *a.5, G.2.A, G.3.D, G.8.A*

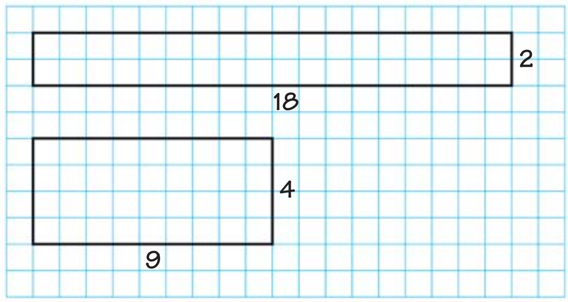
QUESTION **How can you use a graphing calculator to find the smallest possible perimeter for a rectangle with a given area?**

You can use the formulas below to find the perimeter P and the area A of a rectangle with length ℓ and width w.

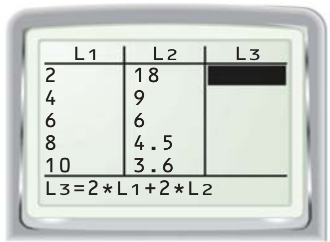
$$P = 2\ell + 2w \qquad A = \ell w$$

EXPLORE Find perimeters of rectangles with fixed areas

STEP 1 ***Draw rectangles*** Draw different rectangles, each with an area of 36 square units. Use lengths of 2, 4, 6, 8, 10, 12, 14, 16, and 18 units.



STEP 2 ***Enter data*** Use the STATISTICS menu on a graphing calculator. Enter the rectangle lengths in List 1. Use the keystrokes below to calculate and enter the rectangle widths and perimeters in Lists 2 and 3.



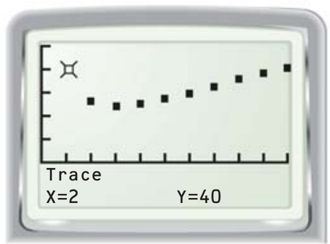
Keystrokes for entering widths in List 2:

36 ÷ 2nd [L1] ENTER

Keystrokes for entering perimeters in List 3:

2 × 2nd [L1] + 2nd 2 × [L2] ENTER

STEP 3 ***Make a scatter plot*** Make a scatter plot using the lengths from List 1 as the x-values and the perimeters from List 3 as the y-values. Choose an appropriate viewing window. Then use the *trace* feature to see the coordinates of each point.



How does the graph show which of your rectangles from Step 1 has the smallest perimeter?

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Repeat the steps above for rectangles with areas of 64 square units.
2. Based on the Explore and your results from Exercise 1, what do you notice about the shape of the rectangle with the smallest perimeter?

1.7 Find Perimeter, Circumference, and Area

Before You classified polygons.

Now You will find dimensions of polygons.

Why? So you can use measures in science, as in Ex. 46.

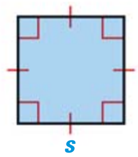
Key Vocabulary
- **perimeter,** *p. 923*
- **circumference,** *p. 923*
- **area,** *p. 923*
- **diameter,** *p. 923*
- **radius,** *p. 923*

Recall that *perimeter* is the distance around a figure, *circumference* is the distance around a circle, and *area* is the amount of surface covered by a figure. Perimeter and circumference are measured in units of length, such as meters (m) and feet (ft). Area is measured in square units, such as square meters (m^2) and square feet (ft^2).

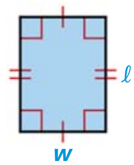
KEY CONCEPT *For Your Notebook*

Formulas for Perimeter *P*, Area *A*, and Circumference *C*

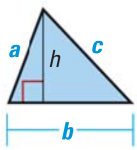
Square
side length s
$P = 4s$
$A = s^2$



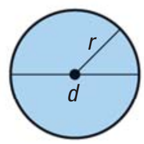
Rectangle
length ℓ and width w
$P = 2\ell + 2w$
$A = \ell w$



Triangle
side lengths a, b, and c, base b, and height h
$P = a + b + c$
$A = \frac{1}{2}bh$



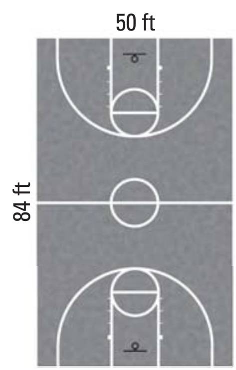
Circle
diameter d and radius r
$C = \pi d = 2\pi r$
$A = \pi r^2$



Pi (π) is the ratio of a circle's circumference to its diameter.

EXAMPLE 1 Find the perimeter and area of a rectangle

BASKETBALL Find the perimeter and area of the rectangular basketball court shown.



Perimeter	**Area**
$P = 2\ell + 2w$	$A = \ell w$
$= 2(84) + 2(50)$	$= 84(50)$
$= 268$	$= 4200$

▶ The perimeter is 268 feet and the area is 4200 square feet.

EXAMPLE 2 Find the circumference and area of a circle

TEAM PATCH **You are ordering circular cloth patches for your soccer team's uniforms. Find the approximate circumference and area of the patch shown.**

Solution

First find the radius. The diameter is 9 centimeters, so the radius is $\frac{1}{2}(9) = 4.5$ centimeters.

Then find the circumference and area.
Use 3.14 to approximate the value of π.

$C = 2\pi r \approx 2(3.14)(4.5) = 28.26$

$A = \pi r^2 \approx 3.14(4.5)^2 = 63.585$

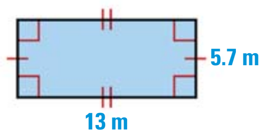
▶ The circumference is about 28.3 cm. The area is about 63.6 cm^2.

APPROXIMATE π
The approximations 3.14 and $\frac{22}{7}$ are commonly used as approximations for the irrational number π. Unless told otherwise, use 3.14 for π.

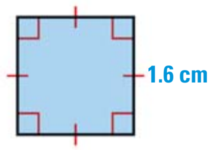
✓ GUIDED PRACTICE for Examples 1 and 2

Find the area and perimeter (or circumference) of the figure. If necessary, round to the nearest tenth.

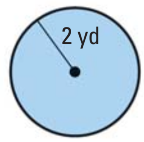
1.



2.



3.

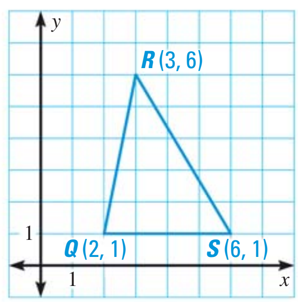


TAKS EXAMPLE 3 TAKS PRACTICE: Multiple Choice

$\triangle QRS$ has vertices at $Q(2, 1)$, $R(3, 6)$, and $S(6, 1)$. What is the approximate perimeter of $\triangle QRS$?

Ⓐ 8 units Ⓑ 8.7 units Ⓒ 14.9 units Ⓓ 29.8 units

Solution



First draw $\triangle QRS$ in a coordinate plane. Then find the side lengths. Because $\overline{QS}$ is horizontal, find QS by using the Ruler Postulate. Use the distance formula to find QR and SR.

$QS = |6 - 2| = 4$ units

$QR = \sqrt{(3-2)^2 + (6-1)^2} = \sqrt{26} \approx 5.1$ units

$RS = \sqrt{(6-3)^2 + (1-6)^2} = \sqrt{34} \approx 5.8$ units

Find the perimeter.

$P = QS + QR + SR \approx 4 + 5.1 + 5.8 = 14.9$ units

▶ The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

AVOID ERRORS
Write down your calculations to make sure you do not make a mistake substituting values in the Distance Formula.

TAKS

EXAMPLE 4 TAKS Reasoning: Multi-Step Problem

SKATING RINK An ice-resurfacing machine is used to smooth the surface of the ice at a skating rink. The machine can resurface about 270 square yards of ice in one minute.

About how many minutes does it take the machine to resurface a rectangular skating rink that is 200 feet long and 90 feet wide?

ANOTHER WAY

For an alternative method for solving the problem in Example 4, turn to page 57 for the **Problem Solving Workshop**.

Solution

The machine can resurface the ice at a rate of 270 square yards per minute. So, the amount of time it takes to resurface the skating rink depends on its area.

STEP 1 **Find** the area of the rectangular skating rink.

Area $= \ell w = 200(90) = 18{,}000 \text{ ft}^2$

The resurfacing rate is in square yards per minute. Rewrite the area of the rink in square yards. There are 3 feet in 1 yard, and $3^2 = 9$ square feet in 1 square yard.

$18{,}000 \cancel{\text{ft}^2} \cdot \dfrac{1 \text{ yd}^2}{9 \cancel{\text{ft}^2}} = 2000 \text{ yd}^2$ **Use unit analysis.**

STEP 2 **Write** a verbal model to represent the situation. Then write and solve an equation based on the verbal model.

Let t represent the total time (in minutes) needed to resurface the skating rink.

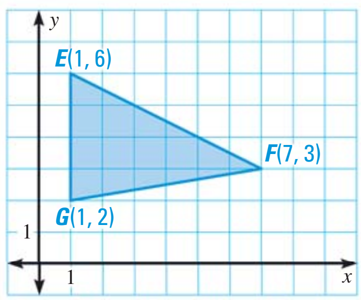
Area of rink (yd^2) = Resurfacing rate (yd^2 per min) × Total time (min)

$2000 = 270 \cdot t$ **Substitute.**

$7.4 \approx t$ **Divide each side by 270.**

▶ It takes the ice-resurfacing machine about 7 minutes to resurface the skating rink.

✓ GUIDED PRACTICE for Examples 3 and 4



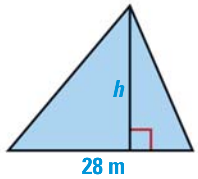
4. Describe how to find the height from F to $\overline{EG}$ in the triangle at the right.

5. Find the perimeter and the area of the triangle shown at the right.

6. **WHAT IF?** In Example 4, suppose the skating rink is twice as long and twice as wide. Will it take an ice-resurfacing machine twice as long to resurface the skating rink? *Explain* your reasoning.

EXAMPLE 5 Find unknown length

The base of a triangle is 28 meters. Its area is 308 square meters. Find the height of the triangle.



Solution

$A = \frac{1}{2}bh$ — Write formula for the area of a triangle.

$308 = \frac{1}{2}(28)h$ — Substitute 308 for *A* and 28 for *b*.

$22 = h$ — Solve for *h*.

▶ The height is 22 meters.

✓ GUIDED PRACTICE for Example 5

7. The area of a triangle is 64 square meters, and its height is 16 meters. Find the length of its base.

1.7 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 7, 21, and 41

★ = **TAKS PRACTICE AND REASONING** Exs. 19, 26, 38, 45, 49, and 50

◆ = **MULTIPLE REPRESENTATIONS** Ex. 45

SKILL PRACTICE

1. VOCABULARY How are the diameter and radius of a circle related?

2. WRITING *Describe* a real-world situation in which you would need to find a perimeter, and a situation in which you would need to find an area. What measurement units would you use in each situation?

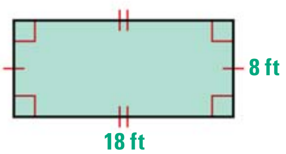
EXAMPLE 1 on p. 49 for Exs. 3–10

3. ERROR ANALYSIS *Describe* and correct the error made in finding the area of a triangle with a height of 9 feet and a base of 52 feet.

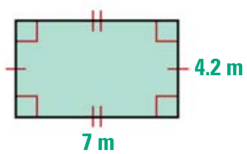
$A = 52(9) = 468\ \text{ft}^2$ ✗

PERIMETER AND AREA **Find the perimeter and area of the shaded figure.**

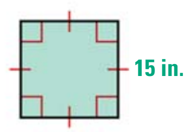
4.



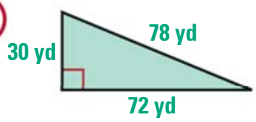
5.



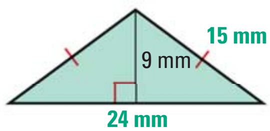
6.



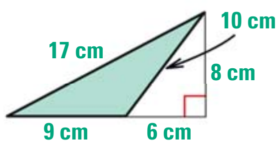
7.



8.



9.



Animated Geometry at classzone.com

10. **DRAWING A DIAGRAM** The base of a triangle is 32 feet. Its height is $16\frac{1}{2}$ feet. Sketch the triangle and find its area.

EXAMPLE 2
on p. 50
for Exs. 11–15

CIRCUMFERENCE AND AREA **Use the given diameter *d* or radius *r* to find the circumference and area of the circle. Round to the nearest tenth.**

11. $d = 27$ cm **12.** $d = 5$ in. **13.** $r = 12.1$ cm **14.** $r = 3.9$ cm

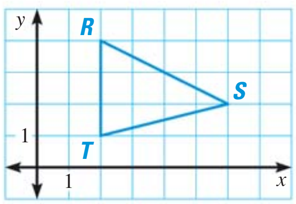



15. **DRAWING A DIAGRAM** The diameter of a circle is 18.9 centimeters. Sketch the circle and find its circumference and area. Round your answers to the nearest tenth.

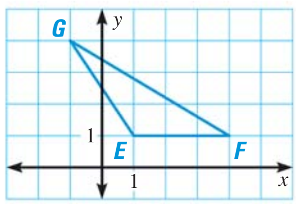
EXAMPLE 3
on p. 50
for Exs. 16–19

DISTANCE FORMULA **Find the perimeter of the figure. Round to the nearest tenth of a unit.**

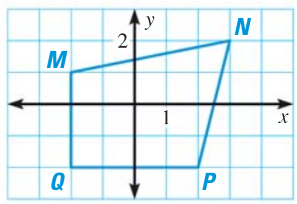
16.



17.



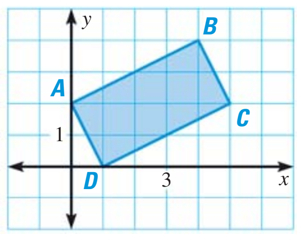
18.



19. **TAKS REASONING** What is the approximate area (in square units) of the rectangle shown at the right?

Ⓐ 6.7 Ⓑ 8.0

Ⓒ 9.0 Ⓓ 10.0



EXAMPLE 4
on p. 51
for Exs. 20–26

CONVERTING UNITS **Copy and complete the statement.**

20. 187 cm^2 = _?_ m^2 **21.** 13 ft^2 = _?_ yd^2 **22.** 18 $in.^2$ = _?_ ft^2

23. 8 km^2 = _?_ m^2 **24.** 12 yd^2 = _?_ ft^2 **25.** 24 ft^2 = _?_ $in.^2$

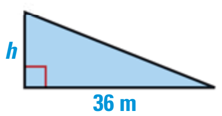
26. **TAKS REASONING** A triangle has an area of 2.25 square feet. What is the area of the triangle in square inches?

Ⓐ 27 $in.^2$ Ⓑ 54 $in.^2$ Ⓒ 144 $in.^2$ Ⓓ 324 $in.^2$

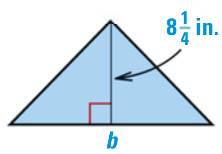
EXAMPLE 5
on p. 52
for Exs. 27–30

UNKNOWN MEASURES **Use the information about the figure to find the indicated measure.**

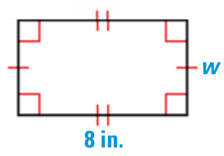
27. Area = 261 m^2
Find the height *h*.



28. Area = 66 $in.^2$
Find the base *b*.



29. Perimeter = 25 in.
Find the width *w*.



30. **UNKNOWN MEASURE** The width of a rectangle is 17 inches. Its perimeter is 102 inches. Find the length of the rectangle.

31. **xy ALGEBRA** The area of a rectangle is 18 square inches. The length of the rectangle is twice its width. Find the length and width of the rectangle.

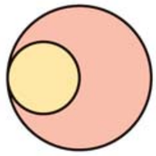
32. **xy ALGEBRA** The area of a triangle is 27 square feet. Its height is three times the length of its base. Find the height and base of the triangle.

33. **xy ALGEBRA** Let x represent the side length of a square. Find a regular polygon with side length x whose perimeter is twice the perimeter of the square. Find a regular polygon with side length x whose perimeter is three times the length of the square. *Explain* your thinking.

FINDING SIDE LENGTHS Find the side length of the square with the given area. Write your answer as a radical in simplest form.

34. $A = 184\text{ cm}^2$ 35. $A = 346\text{ in.}^2$ 36. $A = 1008\text{ mi}^2$ 37. $A = 1050\text{ km}^2$

38. **TAKS REASONING** In the diagram, the diameter of the yellow circle is half the diameter of the red circle. What fraction of the area of the red circle is *not* covered by the yellow circle? *Explain*.



39. **CHALLENGE** The area of a rectangle is 30 cm^2 and its perimeter is 26 cm. Find the length and width of the rectangle.

PROBLEM SOLVING

EXAMPLES 1 and 2 on pp. 49–50 for Exs. 40–41

40. **WATER LILIES** The giant Amazon water lily has a lily pad that is shaped like a circle. Find the circumference and area of a lily pad with a diameter of 60 inches. Round your answers to the nearest tenth.

TEXAS @HomeTutor for problem solving help at classzone.com

41. **LAND** You are planting grass on a rectangular plot of land. You are also building a fence around the edge of the plot. The plot is 45 yards long and 30 yards wide. How much area do you need to cover with grass seed? How many feet of fencing do you need?

TEXAS @HomeTutor for problem solving help at classzone.com

EXAMPLE 4 on p. 51 for Ex. 42

42. **MULTI-STEP PROBLEM** Chris is installing a solar panel. The maximum amount of power the solar panel can generate in a day depends in part on its area. On a sunny day in the city where Chris lives, each square meter of the panel can generate up to 125 watts of power. The flat rectangular panel is 84 centimeters long and 54 centimeters wide.

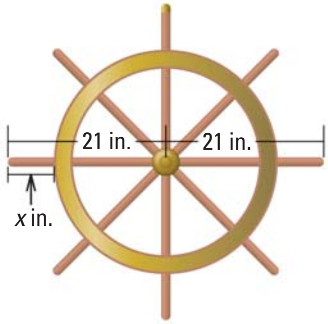
a. Find the area of the solar panel in square meters.

b. What is the maximum amount of power (in watts) that the panel could generate if its area was 1 square meter? 2 square meters? *Explain*.

c. Estimate the maximum amount of power Chris's solar panel can generate. *Explain* your reasoning.

○ = WORKED-OUT SOLUTIONS on p. WS1 = TAKS PRACTICE AND REASONING = MULTIPLE REPRESENTATIONS

43. MULTI-STEP PROBLEM The eight spokes of a ship's wheel are joined at the wheel's center and pass through a large wooden circle, forming handles on the outside of the circle. From the wheel's center to the tip of the handle, each spoke is 21 inches long.



a. The circumference of the outer edge of the large wooden circle is 94 inches. Find the radius of the outer edge of the circle to the nearest inch.

b. Find the length x of a handle on the wheel. *Explain*.

44. MULTIPLE REPRESENTATIONS Let x represent the length of a side of a square. Let y_1 and y_2 represent the perimeter and area of that square.

a. Making a Table Copy and complete the table.

Length, x	1	2	5	10	25
Perimeter, y_1	?	?	?	?	?
Area, y_2	?	?	?	?	?

b. Making a Graph Use the completed table to write two sets of ordered pairs: (x, y_1) and (x, y_2). Graph each set of ordered pairs.

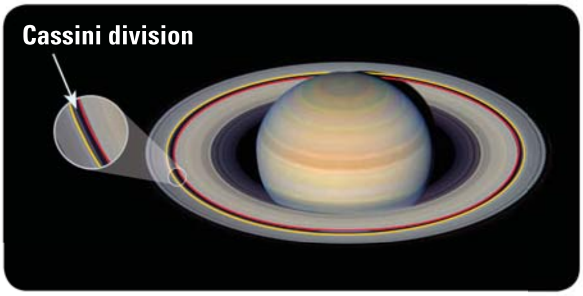
c. Analyzing Data *Describe* any patterns you see in the table from part (a) and in the graphs from part (b).

45. TAKS REASONING The photograph at the right shows the Crown Fountain in Chicago, Illinois. At this fountain, images of faces appear on a large screen. The images are created by light-emitting diodes (LEDs) that are clustered in groups called modules. The LED modules are arranged in a rectangular grid.

a. The rectangular grid is approximately 7 meters wide and 15.2 meters high. Find the area of the grid.

b. Suppose an LED module is a square with a side length of 4 centimeters. How many rows and how many columns of LED modules would be needed to make the Crown Fountain screen? *Explain* your reasoning.

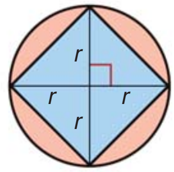
46. ASTRONOMY The diagram shows a gap in Saturn's circular rings. This gap is known as the *Cassini division*. In the diagram, the red circle represents the ring that borders the inside of the Cassini division. The yellow circle represents the ring that borders the outside of the division.



a. The radius of the red ring is 115,800 kilometers. The radius of the yellow ring is 120,600 kilometers. Find the circumference of the red ring and the circumference of the yellow ring. Round your answers to the nearest hundred kilometers.

b. Compare the circumferences of the two rings. About how many kilometers greater is the yellow ring's circumference than the red ring's circumference?

47. **CHALLENGE** In the diagram at the right, how many times as great is the area of the circle as the area of the square? *Explain* your reasoning.



48. **xy ALGEBRA** You have 30 yards of fencing with which to make a rectangular pen. Let x be the length of the pen.

a. Write an expression for the width of the pen in terms of x. Then write a formula for the area y of the pen in terms of x.

b. You want the pen to have the greatest possible area. What length and width should you use? *Explain* your reasoning.

MIXED REVIEW FOR TAKS

TAKS PRACTICE at classzone.com

REVIEW
TAKS Preparation p. 66;
TAKS Workbook

49. **TAKS PRACTICE** Alexis is covering a Styrofoam ball with moss for a science fair project. She knows the radius of the ball and the number of square feet that one bag of moss will cover. Which formula should she use to determine the number of bags of moss needed? *TAKS Obj. 7*

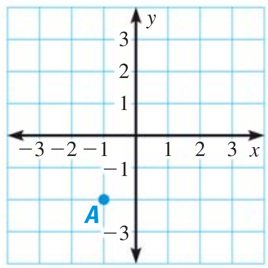
Ⓐ $V = \frac{4}{3}\pi r^3$ Ⓑ $V = \frac{1}{3}\pi r^2 h$ Ⓒ $S = 4\pi r^2$ Ⓓ $S = 4\pi rh$

REVIEW
Skills Review Handbook p. 878;
TAKS Workbook

50. **TAKS PRACTICE** What are the coordinates of A after the translation $(x, y) \rightarrow (x - 1, y + 2)$? *TAKS Obj. 6*

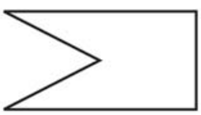
Ⓕ (0, 0) Ⓖ (1, −3)

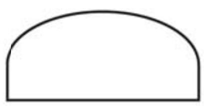
Ⓗ (−2, 0) Ⓙ (−2, −4)

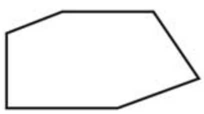


QUIZ *for Lessons 1.6–1.7*

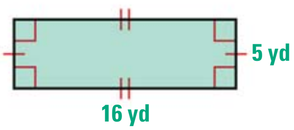
Tell whether the figure is a polygon. If it is not, *explain* why. If it is a polygon, tell whether it is *convex* or *concave*. *(p. 42)*

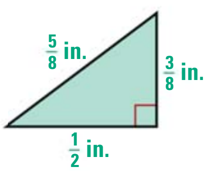
1. 

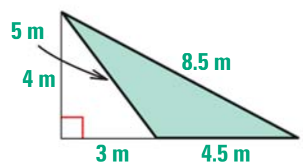
2. 

3. 

Find the perimeter and area of the shaded figure. *(p. 49)*

4. 

5. 

6. 

7. **GARDENING** You are spreading wood chips on a rectangular garden. The garden is $3\frac{1}{2}$ yards long and $2\frac{1}{2}$ yards wide. One bag of wood chips covers 10 square feet. How many bags of wood chips do you need? *(p. 49)*

PROBLEM SOLVING WORKSHOP
LESSON 1.7

Using ALTERNATIVE METHODS

TEKS *a.4, a.5, G.4*

Another Way to Solve Example 4, page 51

MULTIPLE REPRESENTATIONS In Example 4 on page 51, you saw how to use an equation to solve a problem about a skating rink. *Looking for a pattern* can help you write an equation.

PROBLEM

SKATING RINK An ice-resurfacing machine is used to smooth the surface of the ice at a skating rink. The machine can resurface about 270 square yards of ice in one minute. About how many minutes does it take the machine to resurface a rectangular skating rink that is 200 feet long and 90 feet wide?

METHOD

Using a Pattern You can use a table to look for a pattern.

STEP 1 **Find** the area of the rink in square yards. In Example 4 on page 51, you found that the area was 2000 square yards.

STEP 2 **Make** a table that shows the relationship between the time spent resurfacing the ice and the area resurfaced. Look for a pattern.

Time (min)	Area resurfaced (yd^2)
1	$1 \cdot 270 = 270$
2	$2 \cdot 270 = 540$
t	$t \cdot 270 = A$

Use the pattern to write an equation for the area A that has been resurfaced after t minutes.

STEP 3 **Use** the equation to find the time t (in minutes) that it takes the machine to resurface 2000 square yards of ice.

$$270t = A$$
$$270t = 2000$$
$$t \approx 7.4$$

▶ It takes about 7 minutes.

PRACTICE

1. **PLOWING** A square field is $\frac{1}{8}$ mile long on each side. A tractor can plow about 180,000 square feet per hour. To the nearest tenth of an hour, about how long does it take to plow the field? (1 mi = 5280 ft.)

2. **ERROR ANALYSIS** To solve Exercise 1 above, a student writes the equation $660 = 180{,}000t$, where t is the number of hours spent plowing. *Describe* and correct the error in the equation.

3. **PARKING LOT** A rectangular parking lot is 110 yards long and 45 yards wide. It costs about $.60 to pave each square foot of the parking lot with asphalt. About how much will it cost to pave the parking lot?

4. **WALKING** A circular path has a diameter of 120 meters. Your average walking speed is 4 kilometers per hour. About how many minutes will it take you to walk around the path 3 times?

MIXED REVIEW FOR TEKS

TAKS PRACTICE
classzone.com

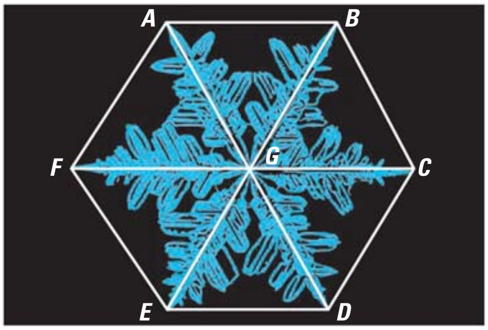
Lessons 1.4–1.7

MULTIPLE CHOICE

1. **ROOFING** Jane is covering the roof of a shed with shingles. The roof is a rectangle that is 4 yards long and 3 yards wide. Asphalt shingles cost $.75 per square foot and wood shingles cost $1.15 per square foot. How much more would Jane pay to use wood shingles instead of asphalt shingles? ***TEKS G.8.A***

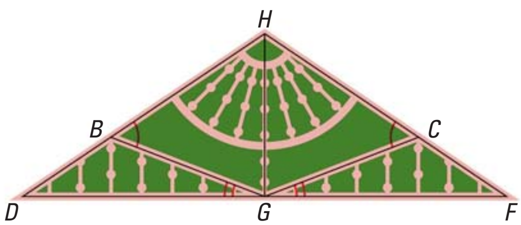
   Ⓐ $4.80   Ⓑ $14.40
   Ⓒ $43.20   Ⓓ $50.09

2. **SNOWFLAKE** The snowflake in the photo below can be circumscribed by a hexagon. Which of the following figures found in the hexagon is a concave polygon? ***TEKS G.9.B***



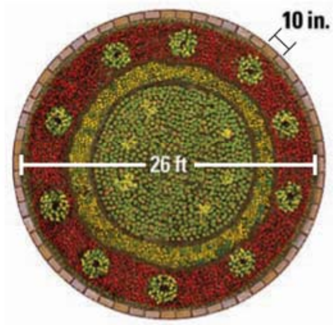
   Ⓕ Triangle *ABG*
   Ⓖ Quadrilateral *ABEF*
   Ⓗ Hexagon *ABCDEG*
   Ⓙ Hexagon *ABCDEF*

3. **DOOR FRAME** The diagram shows a carving on a door frame. $\angle HGD$ and $\angle HGF$ are right angles, $m\angle DGB = 21°$, $m\angle HBG = 55°$, $\angle DGB \cong \angle CGF$, and $\angle HBG \cong \angle HCG$. What is $m\angle HGC$? ***TEKS G.4***



   Ⓐ 21°   Ⓑ 69°
   Ⓒ 111°   Ⓓ 159°

4. **GARDEN** Jim wants to lay bricks end-to-end around the border of the garden as shown below. Each brick is 10 inches long. Which expression can be used to find the number of bricks Jim needs? ***TEKS G.8.B***



   Ⓕ $26\pi \div \frac{10}{12}$ Ⓖ $52\pi \div \frac{12}{10}$
   Ⓗ $13^2\pi \cdot \frac{12}{10}$ Ⓙ $13\pi \cdot \frac{10}{12}$

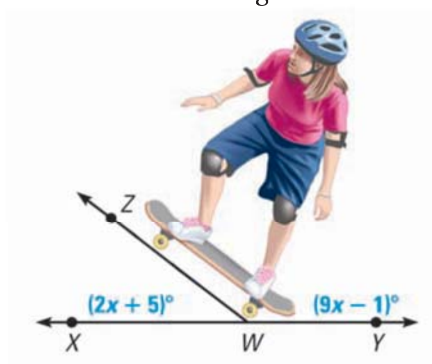
5. **AREA** The points $A(-4,0)$, $B(0,2)$, $C(4,0)$, and $D(0,-2)$ are plotted on a coordinate grid to form the vertices of a quadrilateral. What is the area of quadrilateral *ABCD*? ***TEKS G.8.A***

   Ⓕ 8 square units   Ⓖ 16 square units
   Ⓗ 20 square units   Ⓙ 32 square units

GRIDDED ANSWER

6. **ANGLES** $\angle 1$ and $\angle 2$ are supplementary angles, and $\angle 1$ and $\angle 3$ are complementary angles. If $m\angle 1$ is 28° less than $m\angle 2$, what is $m\angle 3$ in degrees? ***TEKS G.4***

7. **SKATEBOARDING** As shown in the diagram, a skateboarder tilts one end of a skateboard. What is $m\angle ZWX$ in degrees? ***TEKS G.5.A***



1 CHAPTER SUMMARY

BIG IDEAS — For Your Notebook

Big Idea 1 TEKS G.7.A

Describing Geometric Figures

You learned to identify and classify geometric figures.

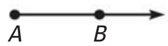
Point A

Line AB ($\overleftrightarrow{AB}$)

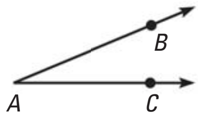
Plane *M*

Segment AB ($\overline{AB}$)

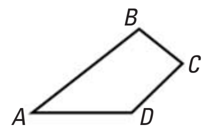
Ray AB ($\overrightarrow{AB}$)



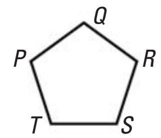
Angle *A* (∠*A*, ∠*BAC*, or ∠*CAB*)



Polygon



Quadrilateral *ABCD*



Pentagon *PQRST*

Big Idea 2 TEKS G.5.B, G.7.C, G.8.A

Measuring Geometric Figures

SEGMENTS You measured segments in the coordinate plane.

Distance Formula

Distance between $A(x_1, y_1)$ and $B(x_2, y_2)$:

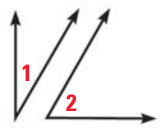
$$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Midpoint Formula

Coordinates of midpoint M of $\overline{AB}$, with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$:

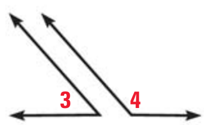
$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

ANGLES You classified angles and found their measures.



Complementary angles

$m\angle 1 + m\angle 2 = 90°$



Supplementary angles

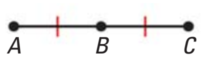
$m\angle 3 + m\angle 4 = 180°$

FORMULAS Perimeter and area formulas are reviewed on page 49.

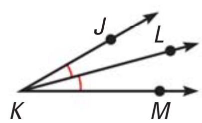
Big Idea 3 TEKS G.9

Understanding Equality and Congruence

Congruent segments have equal lengths. Congruent angles have equal measures.



$\overline{AB} \cong \overline{BC}$ and $AB = BC$



$\angle JKL \cong \angle LKM$ and $m\angle JKL = m\angle LKM$

1 CHAPTER REVIEW

TEXAS @HomeTutor
classzone.com
- Multi-Language Glossary
- Vocabulary practice

REVIEW KEY VOCABULARY

For a list of postulates and theorems, see pp. 926–931.

- undefined terms, *p. 2*
  point, line, plane
- collinear, coplanar points, *p. 2*
- defined terms, *p. 3*
- line segment, endpoints, *p. 3*
- ray, opposite rays, *p. 3*
- intersection, *p. 4*
- postulate, axiom, *p. 9*
- coordinate, *p. 9*
- distance, *p. 9*
- between, *p. 10*
- congruent segments, *p. 11*
- midpoint, *p. 15*
- segment bisector, *p. 15*
- angle, *p. 24*
  sides, vertex, measure
- acute, right, obtuse, straight, *p. 25*
- congruent angles, *p. 26*
- angle bisector, *p. 28*
- construction, *p. 33*
- complementary angles, *p. 35*
- supplementary angles, *p. 35*
- adjacent angles, *p. 35*
- linear pair, *p. 37*
- vertical angles, *p. 37*
- polygon, *p. 42*
  side, vertex
- convex, concave, *p. 42*
- *n*-gon, *p. 43*
- equilateral, equiangular, regular, *p. 43*

VOCABULARY EXERCISES

1. Copy and complete: Points A and B are the __?__ of $\overline{AB}$.

2. Draw an example of a *linear pair.*

3. If Q is between points P and R on $\overleftrightarrow{PR}$, and $PQ = QR$, then Q is the __?__ of $\overline{PR}$.

REVIEW EXAMPLES AND EXERCISES

Use the review examples and exercises below to check your understanding of the concepts you have learned in each lesson of Chapter 1.

1.1 Identify Points, Lines, and Planes *pp. 2–8*

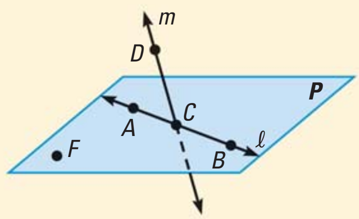
EXAMPLE

Use the diagram shown at the right.

Another name for $\overleftrightarrow{CD}$ is line m.

Points A, B, and C are collinear.

Points A, B, C, and F are coplanar.



EXERCISES

EXAMPLES 1, 2, and 3 on pp. 3–4 for Exs. 4–8

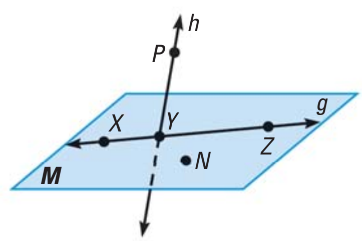
4. Give another name for line g.

5. Name three points that are *not* collinear.

6. Name four points that are coplanar.

7. Name a pair of opposite rays.

8. Name the intersection of line h and plane M.

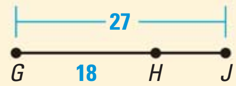


1.2 Use Segments and Congruence

pp. 9–14

EXAMPLE

Find the length of $\overline{HJ}$.



$GJ = GH + HJ$ — Segment Addition Postulate

$27 = 18 + HJ$ — Substitute 27 for GJ and 18 for GH.

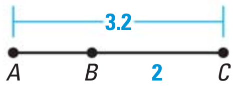
$9 = HJ$ — Subtract 18 from each side.

EXERCISES

EXAMPLES 2, 3, and 4 on pp. 10–11 for Exs. 9–12

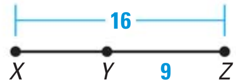
Find the indicated length.

9. Find AB.



10. Find NP.

11. Find XY.



12. The endpoints of $\overline{DE}$ are $D(-4, 11)$ and $E(-4, -13)$. The endpoints of $\overline{GH}$ are $G(-14, 5)$ and $H(-9, 5)$. Are $\overline{DE}$ and $\overline{GH}$ congruent? *Explain.*

1.3 Use Midpoint and Distance Formulas

pp. 15–22

EXAMPLE

$\overline{EF}$ has endpoints $E(1, 4)$ and $F(3, 2)$. Find (a) the length of $\overline{EF}$ rounded to the nearest tenth of a unit, and (b) the coordinates of the midpoint M of $\overline{EF}$.

a. Use the Distance Formula.

$$EF = \sqrt{(3-1)^2 + (2-4)^2} = \sqrt{2^2 + (-2)^2} = \sqrt{8} \approx 2.8 \text{ units}$$

b. Use the Midpoint Formula.

$$M\left(\frac{1+3}{2}, \frac{4+2}{2}\right) = M(2, 3)$$

EXERCISES

EXAMPLES 2, 3, and 4 on pp. 16–18 for Exs. 13–19

13. Point M is the midpoint of $\overline{JK}$. Find JK when $JM = 6x - 7$ and $MK = 2x + 3$.

In Exercises 14–17, the endpoints of a segment are given. Find the length of the segment rounded to the nearest tenth. Then find the coordinates of the midpoint of the segment.

14. $A(2, 5)$ and $B(4, 3)$

15. $F(1, 7)$ and $G(6, 0)$

16. $H(-3, 9)$ and $J(5, 4)$

17. $K(10, 6)$ and $L(0, -7)$

18. Point $C(3, 8)$ is the midpoint of $\overline{AB}$. One endpoint is $A(-1, 5)$. Find the coordinates of endpoint B.

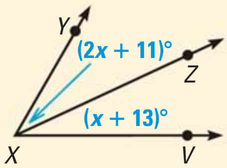
19. The endpoints of $\overline{EF}$ are $E(2, 3)$ and $F(8, 11)$. The midpoint of $\overline{EF}$ is M. Find the length of $\overline{EM}$.

1 CHAPTER REVIEW

1.4 Measure and Classify Angles
pp. 24–32

EXAMPLE

Given that $m\angle YXV$ is 60°, find $m\angle YXZ$ and $m\angle ZXV$.



STEP 1 **Find** the value of x.

$m\angle YXV = m\angle YXZ + m\angle ZXV$ — **Angle Addition Postulate**

$60° = (2x + 11)° + (x + 13)°$ — **Substitute angle measures.**

$x = 12$ — **Solve for x.**

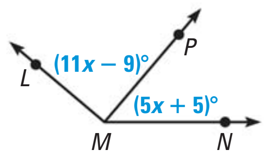
STEP 2 **Evaluate** the given expressions when $x = 12$.

$m\angle YXZ = (2x + 11)° = (2 \cdot 12 + 11)° = 35°$

$m\angle ZXV = (x + 13)° = (12 + 13)° = 25°$

EXERCISES

EXAMPLES 3 and 5 on pp. 26, 28 for Exs. 20–21



20. In the diagram shown at the right, $m\angle LMN = 140°$. Find $m\angle PMN$.

21. $\overrightarrow{VZ}$ bisects $\angle UVW$, and $m\angle UVZ = 81°$. Find $m\angle UVW$. Then classify $\angle UVW$ by its angle measure.

1.5 Describe Angle Pair Relationships
pp. 35–41

EXAMPLE

a. **$\angle 1$ and $\angle 2$ are complementary angles. Given that $m\angle 1 = 37°$, find $m\angle 2$.**

$m\angle 2 = 90° - m\angle 1 = 90° - 37° = 53°$

b. **$\angle 3$ and $\angle 4$ are supplementary angles. Given that $m\angle 3 = 106°$, find $m\angle 4$.**

$m\angle 4 = 180° - m\angle 3 = 180° - 106° = 74°$

EXERCISES

EXAMPLES 2 and 3 on p. 36 for Exs. 22–31

$\angle 1$ and $\angle 2$ are complementary angles. Given the measure of $\angle 1$, find $m\angle 2$.

22. $m\angle 1 = 12°$ **23.** $m\angle 1 = 83°$ **24.** $m\angle 1 = 46°$ **25.** $m\angle 1 = 2°$

$\angle 3$ and $\angle 4$ are supplementary angles. Given the measure of $\angle 3$, find $m\angle 4$.

26. $m\angle 3 = 116°$ **27.** $m\angle 3 = 56°$ **28.** $m\angle 3 = 89°$ **29.** $m\angle 3 = 12°$

30. $\angle 1$ and $\angle 2$ are complementary angles. Find the measures of the angles when $m\angle 1 = (x - 10)°$ and $m\angle 2 = (2x + 40)°$.

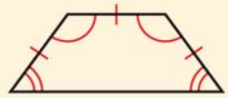
31. $\angle 1$ and $\angle 2$ are supplementary angles. Find the measures of the angles when $m\angle 1 = (3x + 50)°$ and $m\angle 2 = (4x + 32)°$. Then classify $\angle 1$ by its angle measure.

1.6 Classify Polygons

pp. 42–47

EXAMPLE

Classify the polygon by the number of sides. Tell whether it is equilateral, equiangular, or regular. *Explain*.



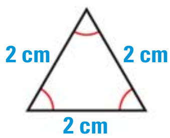
The polygon has four sides, so it is a quadrilateral. It is not equiangular or equilateral, so it is not regular.

EXERCISES

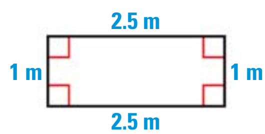
EXAMPLES 2 and 3 on pp. 43–44 for Exs. 32–35

Classify the polygon by the number of sides. Tell whether it is equilateral, equiangular, or regular. *Explain*.

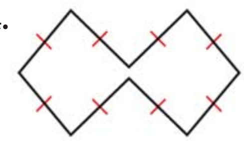
32.



33.



34.



35. Pentagon *ABCDE* is a regular polygon. The length of $\overline{BC}$ is represented by the expression $5x - 4$. The length of $\overline{DE}$ is represented by the expression $2x + 11$. Find the length of $\overline{AB}$.

1.7 Find Perimeter, Circumference, and Area

pp. 49–56

EXAMPLE

The diameter of a circle is 10 feet. Find the circumference and area of the circle. Round to the nearest tenth.

The radius is half of the length of the diameter, so $r = \frac{1}{2}(10) = 5$ ft.

Circumference	**Area**
$C = 2\pi r \approx 2(3.14)(5) = 31.4$ ft	$A = \pi r^2 \approx 3.14(5^2) = 78.5\ \text{ft}^2$

EXERCISES

EXAMPLES 1, 2, and 3 on pp. 49–50 for Exs. 36–40

In Exercises 36–38, find the perimeter (or circumference) and area of the figure described. If necessary, round to the nearest tenth.

36. Circle with diameter 15.6 meters

37. Rectangle with length $4\frac{1}{2}$ inches and width $2\frac{1}{2}$ inches

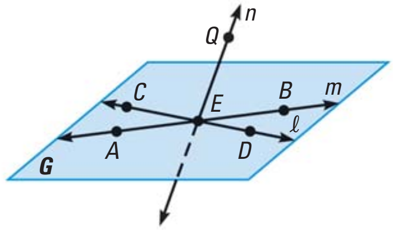
38. Triangle with vertices $U(1, 2)$, $V(-8, 2)$, and $W(-4, 6)$

39. The height of a triangle is 18.6 meters. Its area is 46.5 square meters. Find the length of the triangle's base.

40. The area of a circle is 320 square meters. Find the radius of the circle. Then find the circumference. Round your answers to the nearest tenth.

1 CHAPTER TEST

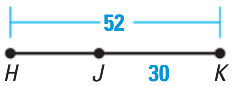
Use the diagram to decide whether the statement is *true* or *false*.



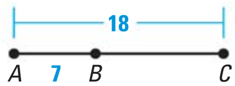
1. Point *A* lies on line *m*.
2. Point *D* lies on line *n*.
3. Points *B*, *C*, *E*, and *Q* are coplanar.
4. Points *C*, *E*, and *B* are collinear.
5. Another name for plane *G* is plane *QEC*.

Find the indicated length.

6. Find *HJ*.



7. Find *BC*.



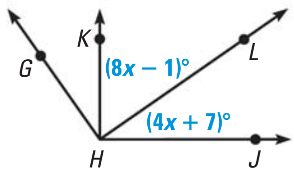
8. Find *XZ*.

In Exercises 9–11, find the distance between the two points.

9. *T*(3, 4) and *W*(2, 7)
10. *C*(5, 10) and *D*(6, −1)
11. *M*(−8, 0) and *N*(−1, 3)

12. The midpoint of $\overline{AB}$ is *M*(9, 7). One endpoint is *A*(3, 9). Find the coordinates of endpoint *B*.
13. Line *t* bisects $\overline{CD}$ at point *M*, $CM = 3x$, and $MD = 27$. Find *CD*.

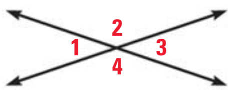
In Exercises 14 and 15, use the diagram.



14. Trace the diagram and extend the rays. Use a protractor to measure $\angle GHJ$. Classify it as *acute*, *obtuse*, *right*, or *straight*.
15. Given $m\angle KHJ = 90°$, find $m\angle LHJ$.

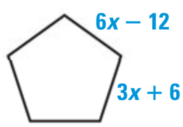
16. The measure of $\angle QRT$ is 154°, and $\overrightarrow{RS}$ bisects $\angle QRT$. What are the measures of $\angle QRS$ and $\angle SRT$?

In Exercises 17 and 18, use the diagram at the right.



17. Name four linear pairs.
18. Name two pairs of vertical angles.

19. The measure of an angle is 64°. What is the measure of its complement? What is the measure of its supplement?
20. A convex polygon has half as many sides as a concave 10-gon. Draw the concave polygon and the convex polygon. Classify the convex polygon by the number of sides it has.
21. Find the perimeter of the regular pentagon shown at the right.



22. **CARPET** You can afford to spend \$300 to carpet a room that is 5.5 yards long and 4.5 yards wide. The cost to purchase and install the carpet you like is \$1.50 per square foot. Can you afford to buy this carpet? *Explain.*

1 ALGEBRA REVIEW

Animated Algebra
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SOLVE LINEAR EQUATIONS AND WORD PROBLEMS

EXAMPLE 1 Solve linear equations

Solve the equation $-3(x + 5) + 4x = 25$.

$-3(x + 5) + 4x = 25$ — **Write original equation.**

$-3x - 15 + 4x = 25$ — **Use the Distributive Property.**

$x - 15 = 25$ — **Group and combine like terms.**

$x = 40$ — **Add 15 to each side.**

EXAMPLE 2 Solve a real-world problem

MEMBERSHIP COSTS A health club charges an initiation fee of $50. Members then pay $45 per month. You have $400 to spend on a health club membership. For how many months can you afford to be a member?

Let n represent the number of months you can pay for a membership.

$400 = Initiation fee + (Monthly Rate × Number of Months)

$400 = 50 + 45n$ — **Substitute.**

$350 = 45n$ — **Subtract 50 from each side.**

$7.8 = n$ — **Divide each side by 45.**

▸ You can afford to be a member at the health club for 7 months.

EXERCISES

EXAMPLE 1 for Exs. 1–9

Solve the equation.

1. $9y + 1 - y = 49$
2. $5z + 7 + z = -8$
3. $-4(2 - t) = -16$
4. $7a - 2(a - 1) = 17$
5. $\frac{4x}{3} + 2(3 - x) = 5$
6. $\frac{2x - 5}{7} = 4$
7. $9c - 11 = -c + 29$
8. $2(0.3r + 1) = 23 - 0.1r$
9. $5(k + 2) = 3(k - 4)$

EXAMPLE 2 for Exs. 10–12

10. **GIFT CERTIFICATE** You have a $50 gift certificate at a store. You want to buy a book that costs $8.99 and boxes of stationery for your friends. Each box costs $4.59. How many boxes can you buy with your gift certificate?

11. **CATERING** It costs $350 to rent a room for a party. You also want to hire a caterer. The caterer charges $8.75 per person. How many people can come to the party if you have $500 to spend on the room and the caterer?

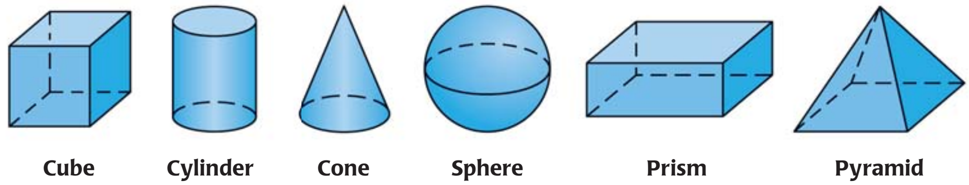
12. **JEWELRY** You are making a necklace out of glass beads. You use one bead that is $1\frac{1}{2}$ inches long and smaller beads that are each $\frac{3}{4}$ inch long. The necklace is 18 inches long. How many smaller beads do you need?

1 TAKS PREPARATION

TEXAS
TAKS Obj. 7
TEKS 8.7.B,
TAKS Obj. 8
TEKS 8.12.C,
8.13.B

REVIEWING SURFACE AREA AND VOLUME PROBLEMS

Some solids are shown below.



You can use the formulas below to find surface areas and volumes of solids. The lateral surface area of a cylinder or cone is the area of the curved surface.

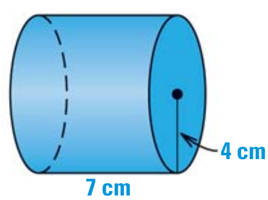
KEY CONCEPT — *For Your Notebook*

Surface Area Formulas	**Volume Formulas**
Cube: $S = 6s^2$	**Prism:** $V = Bh$*
Cylinder (lateral): $S = 2\pi rh$	**Cylinder:** $V = Bh$*
Cylinder (total): $S = 2\pi rh + 2\pi r^2$	**Pyramid:** $V = \frac{1}{3}Bh$*
Cone (lateral): $S = \pi rl$	**Cone:** $V = \frac{1}{3}Bh$*
Cone (total): $S = \pi rl + \pi r^2$	**Sphere:** $V = \frac{4}{3}\pi r^3$
Sphere: $S = 4\pi r^2$	

* *B represents the area of the Base of a solid figure.*

EXAMPLE

Find the volume and the lateral surface area of the cylinder shown.



Solution

$$V = Bh \qquad S = 2\pi rh$$
$$= (\pi r^2)h \qquad = 2\pi(4)(7)$$
$$= \pi(4^2)(7) \qquad \approx 175.93$$
$$\approx 351.86$$

▶ So, the cylinder has a volume of about 351.86 cubic centimeters and a surface area of about 175.93 square centimeters.