FACTORIAL In Example 1, you evaluated the expression $4 \cdot 3 \cdot 2 \cdot 1$. This expression can be written as 4! and is read "4 *factorial*." For any positive integer n, the product of the integers from 1 to n is called ***n* factorial** and is written as $n!$. The value of 0! is defined to be 1.

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1 \text{ and } 0! = 1$$

In Example 1, you also found the permutations of four objects taken two at a time. You can find the number of permutations using the formulas below.

KEY CONCEPT *For Your Notebook*

Permutations

Formulas	Examples
The number of permutations of n objects is given by: $${}_nP_n = n!$$	The number of permutations of 4 objects is: $${}_4P_4 = 4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$
The number of permutations of n objects taken r at a time, where $r \leq n$, is given by: $${}_nP_r = \frac{n!}{(n-r)!}$$	The number of permutations of 4 objects taken 2 at a time is: $${}_4P_2 = \frac{4!}{(4-2)!} = \frac{4 \cdot 3 \cdot 2!}{2!} = 12$$

EXAMPLE 2 Use a permutations formula

CD RECORDING Your band has written 12 songs and plans to record 9 of them for a CD. In how many ways can you arrange the songs on the CD?

Solution

To find the number of permutations of 9 songs chosen from 12, find ${}_{12}P_9$.

$${}_{12}P_9 = \frac{12!}{(12-9)!}$$ **Permutations formula**

$$= \frac{12!}{3!}$$ **Subtract.**

$$= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3!}{3!}$$ **Expand factorials. Divide out common factor, 3!.**

$$= 79{,}833{,}600$$ **Multiply.**

▶ There are 79,833,600 ways to arrange 9 songs out of 12.

DIVIDE COMMON FACTORS
When you divide out common factors, remember that 3! is a factor of 12!.

GUIDED PRACTICE for Example 2

3. WHAT IF? In Example 2, suppose your band has written 15 songs. You will record 9 of them for a CD. In how many ways can you arrange the songs on the CD?