EXAMPLE 3 Simplify an expression by dividing out binomials

Simplify $\frac{x^2 - 3x - 10}{x^2 + 6x + 8}$. State the excluded values.

$$\frac{x^2 - 3x - 10}{x^2 + 6x + 8} = \frac{(x-5)(x+2)}{(x+4)(x+2)}$$ Factor numerator and denominator.

$$= \frac{(x-5)\cancel{(x+2)}}{(x+4)\cancel{(x+2)}}$$ Divide out common factor.

$$= \frac{x-5}{x+4}$$ Simplify.

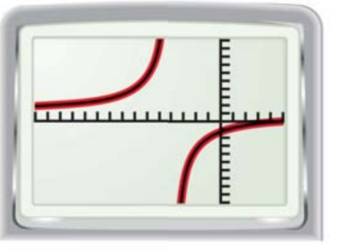
▶ The excluded values are −4 and −2.

CHECK In the graphing calculator activity on page 560, you saw how to use a graph to check a sum or difference of polynomials.

Check your simplification using a graphing calculator.

Graph $y_1 = \frac{x^2 - 3x - 10}{x^2 + 6x + 8}$ and $y_2 = \frac{x-5}{x+4}$.

The graphs coincide. So, the expressions are equivalent for all values of x other than the excluded values (−4 and −2).



INTERPRET THE GRAPH

Although the graphs of y_1 and y_2 appear to pass through (−2, −3.5), the point is not on either graph because −2 is an excluded value of both y_1 and y_2.

OPPOSITES When simplifying a rational expression, look for factors that are opposites of each other. For example, $x - 1$ and $1 - x$ are opposites, because $x - 1 = -(1 - x)$.

EXAMPLE 4 Recognize opposites

Simplify $\frac{x^2 - 7x + 12}{16 - x^2}$. State the excluded values.

$$\frac{x^2 - 7x + 12}{16 - x^2} = \frac{(x-3)(x-4)}{(4-x)(4+x)}$$ Factor numerator and denominator.

$$= \frac{(x-3)(x-4)}{-(x-4)(4+x)}$$ Rewrite $4 - x$ as $-(x - 4)$.

$$= \frac{(x-3)\cancel{(x-4)}}{-\cancel{(x-4)}(4+x)}$$ Divide out common factor.

$$= \frac{x-3}{-(4+x)} = -\frac{x-3}{x+4}$$ Simplify.

▶ The excluded values are −4 and 4.

✓ GUIDED PRACTICE for Examples 3 and 4

Simplify the rational expression. State the excluded values.

9. $\frac{x^2 + 3x + 2}{x^2 + 7x + 10}$

10. $\frac{y^2 - 64}{y^2 - 16y + 64}$

11. $\frac{5 + 4z - z^2}{z^2 - 3z - 10}$