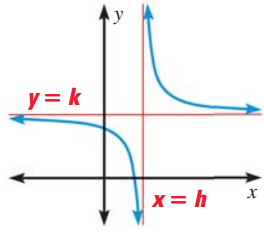
GRAPHING RATIONAL FUNCTIONS You can graph a rational function of the form $y = \frac{a}{x-h} + k$ $(a \neq 0)$ by using the values of a, k, and h.

KEY CONCEPT *For Your Notebook*

Graph of $y = \frac{a}{x-h} + k$

The graph of $y = \frac{a}{x-h} + k$ is a hyperbola that has the following characteristics:



- If $|a| > 1$, the graph is a vertical stretch of the graph of $y = \frac{1}{x}$. If $0 < |a| < 1$, the graph is a vertical shrink of the graph of $y = \frac{1}{x}$. If $a < 0$, the graph is a reflection in the x-axis of the graph of $y = \frac{1}{x}$.
- The horizontal asymptote is $y = k$. The vertical asymptote is $x = h$.

The domain of the function is all real numbers except $x = h$. The range is all real numbers except $y = k$.

EXAMPLE 4 Graph $y = \frac{a}{x-h} + k$

Graph $y = \frac{2}{x+1} - 3$.

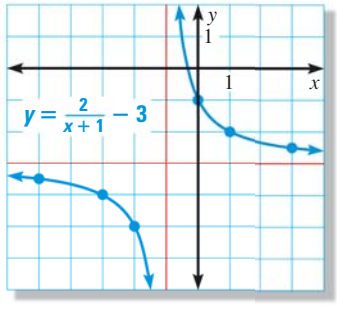
Solution

AVOID ERRORS
The asymptotes are used to help you draw a hyperbola. They are *not* part of the hyperbola.

STEP 1 **Identify** the asymptotes of the graph. The vertical asymptote is $x = -1$. The horizontal asymptote is $y = -3$.

STEP 2 **Plot** several points on each side of the vertical asymptote.

STEP 3 **Graph** two branches that pass through the plotted points and approach the asymptotes.



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GUIDED PRACTICE for Example 4

5. Graph $y = \frac{4}{x-5} + 6$.

6. For which function is the domain all real numbers except -3 and the range all real numbers except 7?

Ⓐ $y = \frac{2}{x-3} + 7$ Ⓑ $y = \frac{2}{x-3} - 7$ Ⓒ $y = \frac{2}{x+3} + 7$ Ⓓ $y = \frac{2}{x+3} - 7$