

SIMPLIFYING RADICAL EXPRESSIONS Simplify the expression.

46. $\sqrt{75m^2np^4}$

47. $\sqrt{512rs^6} \cdot \sqrt{t^3}$

48. $\sqrt{\frac{600a}{4b^3}}$

49. $\sqrt{\frac{50gh^2}{125f^3}}$

50. $\frac{4}{\sqrt{3}} + \frac{7}{\sqrt{12}}$

51. $\frac{2\sqrt{6}}{\sqrt{30}} - \frac{3}{\sqrt{20}}$

52. $\frac{7}{\sqrt{x}} + \frac{3}{2\sqrt{x}}$

53. $\frac{3}{\sqrt{x^3}} + \frac{4}{\sqrt{x}}$

54. $\frac{6m}{\sqrt{m^3}} - \frac{8}{\sqrt{m}}$

CONJUGATES In Exercises 55–62, use the example to simplify the expression.

EXAMPLE Rationalize the denominator using conjugates

Simplify $\frac{9}{2 - \sqrt{3}}$.

Solution

The binomials $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$ are called *conjugates*. They differ only by the sign of one term. The product of two conjugates $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$ does not contain a radical:
 $(2 + \sqrt{3})(2 - \sqrt{3}) = 2^2 - (\sqrt{3})^2 = 4 - 3 = 1$. You can use conjugates to simplify the expression.

$$\frac{9}{2 - \sqrt{3}} = \frac{9}{2 - \sqrt{3}} \cdot \frac{2 + \sqrt{3}}{2 + \sqrt{3}} \quad \text{Multiply the numerator and denominator by the conjugate of the denominator.}$$

$$= \frac{9(2 + \sqrt{3})}{(2 - \sqrt{3})(2 + \sqrt{3})} \quad \text{Multiply fractions.}$$

$$= \frac{18 + 9\sqrt{3}}{4 - 3} \quad \text{Simplify numerator and denominator.}$$

$$= 18 + 9\sqrt{3} \quad \text{Simplify.}$$

55. $\frac{1}{\sqrt{7} + 1}$

56. $\frac{2}{5 - \sqrt{3}}$

57. $\frac{\sqrt{10}}{7 - \sqrt{2}}$

58. $\frac{\sqrt{5}}{6 + \sqrt{5}}$

59. $\frac{3}{\sqrt{7} + \sqrt{6}}$

60. $\frac{11}{\sqrt{11} - \sqrt{7}}$

61. $\frac{\sqrt{6}}{\sqrt{2} - \sqrt{3}}$

62. $\frac{\sqrt{7} + 1}{\sqrt{7} + \sqrt{2}}$

63. **REASONING** Multiply the binomials $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$ to show that the product does not contain a radical.

64. **WRITING** According to the definition of square root, a number b is a square root of a number a if $b^2 = a$. How can you use the definition to show that $\sqrt{x^2} = x$? *Explain.*

65. **MULTIPLYING FUNCTIONS** Let $f(x) = \sqrt{x} - \sqrt{4x}$, and let $g(x) = \sqrt{x}$. Find $h(x) = f(x) \cdot g(x)$.

66. **CHALLENGE** Consider the expression $\sqrt{2^m}$. Assume m is a positive integer. For what values of m will the expression contain a radical when simplified? For what values of m will the expression contain no radical when simplified? *Explain.*