10.5 Solve Quadratic Equations by Completing the Square

TEKS A.10.A; 2A.6.A, 2A.8.A, 2A.8.D

Before You solved quadratic equations by finding square roots.

Now You will solve quadratic equations by completing the square.

Why? So you can solve a problem about snowboarding, as in Ex. 50.

Key Vocabulary
- **completing the square**
- **perfect square trinomial,** *p. 601*

For an expression of the form $x^2 + bx$, you can add a constant c to the expression so that the expression $x^2 + bx + c$ is a perfect square trinomial. This process is called **completing the square**.

KEY CONCEPT *For Your Notebook*

Completing the Square

Words To complete the square for the expression $x^2 + bx$, add the square of half the coefficient of the term bx.

Algebra $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

EXAMPLE 1 Complete the square

Find the value of c that makes the expression $x^2 + 5x + c$ a perfect square trinomial. Then write the expression as the square of a binomial.

STEP 1 **Find** the value of c. For the expression to be a perfect square trinomial, c needs to be the square of half the coefficient of bx.

$c = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$ Find the square of half the coefficient of bx.

STEP 2 **Write** the expression as a perfect square trinomial. Then write the expression as the square of a binomial.

$x^2 + 5x + c = x^2 + 5x + \frac{25}{4}$ Substitute $\frac{25}{4}$ for c.

$= \left(x + \frac{5}{2}\right)^2$ Square of a binomial

GUIDED PRACTICE for Example 1

Find the value of c that makes the expression a perfect square trinomial. Then write the expression as the square of a binomial.

1. $x^2 + 8x + c$
2. $x^2 - 12x + c$
3. $x^2 + 3x + c$