Extension
Use after Lesson 8.6

Relate Geometric Sequences to Exponential Functions

TEKS A.1.B, A.1.D, A.3.B, A.11.C

GOAL Identify, graph, and write geometric sequences.

Key Vocabulary
- **geometric sequence**
- **common ratio**

In a **geometric sequence**, the ratio of any term to the previous term is constant. This constant ratio is called the **common ratio** and is denoted by r.

A geometric sequence with first term a_1 and common ratio r has the form a_1, a_1r, a_1r^2, a_1r^3, ... For instance, if $a_1 = 5$ and $r = 2$, the sequence $5, 5 \cdot 2, 5 \cdot 2^2, 5 \cdot 2^3, \ldots$, or 5, 10, 20, 40, . . . , is geometric.

EXAMPLE 1 Identify a geometric sequence

Tell whether the sequence is *arithmetic* or *geometric*. Then write the next term of the sequence.

a. 3, 6, 9, 12, 15, . . .

b. 128, 64, 32, 16, 8, . . .

Solution

a. The first term is $a_1 = 3$. Find the ratios of consecutive terms:

$$\frac{a_2}{a_1} = \frac{6}{3} = 2 \qquad \frac{a_3}{a_2} = \frac{9}{6} = 1\frac{1}{2} \qquad \frac{a_4}{a_3} = \frac{12}{9} = 1\frac{1}{3} \qquad \frac{a_5}{a_4} = \frac{15}{12} = 1\frac{1}{4}$$

Because the ratios are not constant, the sequence is not geometric. To see if the sequence is arithmetic, find the differences of consecutive terms.

$$a_2 - a_1 = 6 - 3 = 3 \qquad a_3 - a_2 = 9 - 6 = 3$$

$$a_4 - a_3 = 12 - 9 = 3 \qquad a_5 - a_4 = 15 - 12 = 3$$

The common difference is 3, so the sequence is arithmetic. The next term of the sequence is $a_6 = a_5 + 3 = 18$.

b. The first term is $a_1 = 128$. Find the ratios of consecutive terms:

$$\frac{a_2}{a_1} = \frac{64}{128} = \frac{1}{2} \qquad \frac{a_3}{a_2} = \frac{32}{64} = \frac{1}{2} \qquad \frac{a_4}{a_3} = \frac{16}{32} = \frac{1}{2} \qquad \frac{a_5}{a_4} = \frac{8}{16} = \frac{1}{2}$$

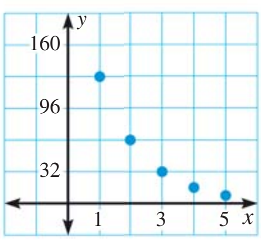
Because the ratios are constant, the sequence is geometric. The common ratio is $\frac{1}{2}$. The next term of the sequence is $a_6 = a_5 \cdot \frac{1}{2} = 4$.

REVIEW ARITHMETIC SEQUENCES
For help with identifying an arithmetic sequence and finding a common difference, see p. 309.

EXAMPLE 2 Graph a geometric sequence

To graph the sequence from part (b) of Example 1, let each term's position number in the sequence be the x-value. The term is the corresponding y-value. Then make and plot the points.

Position, x	1	2	3	4	5
Term, y	128	64	32	16	8



ANALYZE A GRAPH
Notice that the graph in Example 2 appears to be exponential.