EXAMPLE 4 Solve an equation with an extraneous solution

Solve
$$\sqrt{6-x}=x$$
.

REVIEW FACTORINGFor help with factoring,

see pp. 583, 593, 600,

and 606.

$$\sqrt{6-x}=x$$
 Write original equation.

$$(\sqrt{6-x})^2 = x^2$$
 Square each side.

$$6 - x = x^2$$
 Simplify.

$$0 = x^2 + x - 6$$
 Write in standard form.

$$0 = (x - 2)(x + 3)$$
 Factor.

$$x-2=0 \ or \ x+3=0$$
 Zero-product property

$$x = 2 \ or \qquad x = -3$$
 Solve for x.

CHECK Check 2 and -3 in the original equation.

If
$$x = 2$$
: $\sqrt{6-2} \stackrel{?}{=} 2$ If $x = -3$: $\sqrt{6-(-3)} \stackrel{?}{=} -3$ $3 = -3$

▶ Because −3 does not check in the original equation, it is an extraneous solution. The only solution of the equation is 2.

EXAMPLE 5 Solve a real-world problem

SAILING The hull speed s (in nautical miles per hour) of a sailboat can be estimated using the formula $s=1.34\sqrt{\ell}$ where ℓ is the length (in feet) of the sailboat's waterline, as shown. Find the length (to the nearest foot) of the sailboat's waterline if it has a hull speed of 8 nautical miles per hour.

Solution

$$s = 1.34\sqrt{\ell}$$
 Write original equation.

$$8 = 1.34\sqrt{\ell}$$
 Substitute 8 for s.

$$\frac{8}{1.34} = \sqrt{\ell}$$
 Divide each side by 1.34.

$$\left(\frac{8}{1.34}\right)^2 = (\sqrt{\ell})^2$$
 Square each side.

$$35.6 \approx \ell$$
 Simplify.

▶ The sailboat has a waterline length of about 36 feet.





5. Solve
$$\sqrt{3x + 4} = x$$
.

6. WHAT IF? In Example 5, suppose the sailboat's hull speed is 6.5 nautical miles per hour. Find the sailboat's waterline length to the nearest foot.