Extension

Use after Lesson 11.2

Derive the Quadratic Formula

TEKS *a.6, A.10.A; 2A.8.B*

GOAL Solve quadratic equations and check solutions.

In Lesson 10.6, you learned how to find solutions of quadratic equations using the quadratic formula. You can use the method of completing the square and the quotient property of radicals to derive the quadratic formula.

$ax^2 + bx + c = 0$	**Write standard form of a quadratic equation.**
$ax^2 + bx = -c$	**Subtract c from each side.**
$x^2 + \frac{b}{a}x = -\frac{c}{a}$	**Divide each side by a, $a \neq 0$.**
$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$	**Add $\left(\frac{b}{2a}\right)^2$ to each side to complete the square.**
$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$	**Write left side as the square of a binomial.**
$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$	**Simplify right side.**
$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$	**Take square roots of each side.**
$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$	**Quotient property of radicals**
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**Subtract $\frac{b}{2a}$ from each side.**

SOLVING QUADRATIC EQUATIONS You can use the quadratic formula and properties of radicals to solve quadratic equations.

EXAMPLE 1 Solve an equation

Solve $x^2 - 6x + 3 = 0$.

Solution

$x^2 - 6x + 3 = 0$	**Identify $a = 1$, $b = -6$, and $c = 3$.**
$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(3)}}{2(1)}$	**Substitute values in the quadratic formula.**
$= \frac{6 \pm \sqrt{24}}{2}$	**Simplify.**
$= \frac{6 \pm \sqrt{4 \cdot 6}}{2}$	**Product property of radicals**
$= \frac{6 \pm 2\sqrt{6}}{2} = 3 \pm \sqrt{6}$	**Simplify.**

▶ The solutions of the equation are $3 + \sqrt{6}$ and $3 - \sqrt{6}$.