EXAMPLE 3 on p. 956 for Exs. 21–29

SIMPLIFYING EXPRESSIONS **Rewrite the expression without double angles or half angles, given that $0 < \theta < \frac{\pi}{2}$. Then simplify the expression.**

21. $\dfrac{\cos 2\theta}{1 - 2\sin^2\theta}$

22. $\dfrac{\sin 2\theta}{2\cos\theta}$

23. $(1 - \tan\theta)\tan 2\theta$

24. $\dfrac{\cos 2\theta}{\sin\theta - \cos\theta}$

25. $\dfrac{-\tan\frac{\theta}{2}}{\csc\theta}$

26. $2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

27. **TAKS REASONING** Which expression is equivalent to $\cot\theta + \tan\theta$?

(A) $\csc 2\theta$ (B) $2\csc 2\theta$ (C) $\sec 2\theta$ (D) $2\sec 2\theta$

ERROR ANALYSIS ***Describe*** **and correct the error in simplifying the expression.**

28.

$$\frac{\cos 2x}{\cos^2 x} = \frac{\cos^2 x - \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

29.

$$\sin 22.5^\circ = \sin\frac{1}{2}(45^\circ)$$

$$= 2\sin 45^\circ \cos 45^\circ$$

$$= 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= 1$$

EXAMPLE 5 on p. 958 for Exs. 30–35

VERIFYING IDENTITIES **Verify the identity.**

30. $2\cos^2\theta = 1 + \cos 2\theta$

31. $\sin 3\theta = \sin\theta\,(4\cos^2\theta - 1)$

32. $\frac{1}{2}\sin\frac{2x}{3} = \sin\frac{x}{3}\cos\frac{x}{3}$

33. $2\sin^2 x\tan\frac{x}{2} = 2\sin x - \sin 2x$

34. $-\dfrac{\cos 2\theta}{\sin\theta} = 2\sin\theta - \csc\theta$

35. $\cos 4\theta = \cos^4\theta - 6\sin^2\theta\cos^2\theta + \sin^4\theta$

EXAMPLE 6 on p. 958 for Exs. 36–41

SOLVING EQUATIONS **Solve the equation for $0 \le x < 2\pi$.**

36. $\sin\frac{x}{2} = 1$

37. $2\cos\frac{x}{2} + 1 = 0$

38. $\tan x - \tan 2x = 0$

39. $\tan\frac{x}{2} = \dfrac{2 - \sqrt{2}}{2\sin x}$

40. $\cos 2x = -2\cos^2 x$

41. $2\sin 2x\sin x = 3\cos x$

EXAMPLE 7 on p. 958 for Exs. 42–47

FINDING GENERAL SOLUTIONS **Find the general solution of the equation.**

42. $\cos\frac{x}{2} = 1$

43. $\tan\frac{x}{2} = \sin x$

44. $\sin 2x = \sin x$

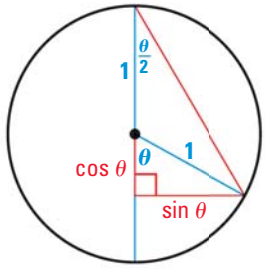
45. $\cos 2x + \cos x = 0$

46. $\cos\frac{x}{2} + \sin x = 0$

47. $\sin\frac{x}{2} + \cos x = 0$

48. **REASONING** Show that the three double-angle formulas for cosine are equivalent.

49. **CHALLENGE** Use the diagram shown at the right to derive the formulas for $\sin\frac{\theta}{2}$, $\cos\frac{\theta}{2}$, and $\tan\frac{\theta}{2}$ when θ is an acute angle.



= WORKED-OUT SOLUTIONS

= TAKS PRACTICE AND REASONING