EXAMPLE 5 Verify a trigonometric identity

Verify the identity $\cos 3x = 4\cos^3 x - 3\cos x$.

$\cos 3x = \cos(2x + x)$	**Rewrite $\cos 3x$ as $\cos(2x + x)$.**
$= \cos 2x \cos x - \sin 2x \sin x$	**Use a sum formula.**
$= (2\cos^2 x - 1)\cos x - (2\sin x \cos x)\sin x$	**Use double-angle formulas.**
$= 2\cos^3 x - \cos x - 2\sin^2 x \cos x$	**Multiply.**
$= 2\cos^3 x - \cos x - 2(1 - \cos^2 x)\cos x$	**Use a Pythagorean identity.**
$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$	**Distributive property**
$= 4\cos^3 x - 3\cos x$	**Combine like terms.**

EXAMPLE 6 Solve a trigonometric equation

Solve $\sin 2x + 2\cos x = 0$ for $0 \le x < 2\pi$.

Solution

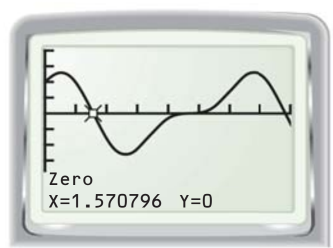
$\sin 2x + 2\cos x = 0$	**Write original equation.**
$2\sin x \cos x + 2\cos x = 0$	**Use a double-angle formula.**
$2\cos x(\sin x + 1) = 0$	**Factor.**

Set each factor equal to 0 and solve for x.

$2\cos x = 0$	$\sin x + 1 = 0$
$\cos x = 0$	$\sin x = -1$
$x = \frac{\pi}{2}, \frac{3\pi}{2}$	$x = \frac{3\pi}{2}$

CHECK Graph the function $y = \sin 2x + 2\cos x$ on a graphing calculator. Then use the *zero* feature to find the x-values on the interval $0 \le x < 2\pi$ for which $y = 0$. The two x-values are:

$$x = \frac{\pi}{2} \approx 1.57 \quad \text{and} \quad x = \frac{3\pi}{2} \approx 4.71$$



EXAMPLE 7 Find a general solution

Find the general solution of $2\sin\frac{x}{2} = 1$.

$2\sin\frac{x}{2} = 1$	**Write original equation.**
$\sin\frac{x}{2} = \frac{1}{2}$	**Divide each side by 2.**
$\frac{x}{2} = \frac{\pi}{6} + 2n\pi$ or $\frac{5\pi}{6} + 2n\pi$	**General solution for $\frac{x}{2}$**
$x = \frac{\pi}{3} + 4n\pi$ or $\frac{5\pi}{3} + 4n\pi$	**General solution for x**

SOLVE EQUATIONS
As seen in Example 7, some equations that involve double or half angles can be solved without resorting to double- or half-angle formulas.