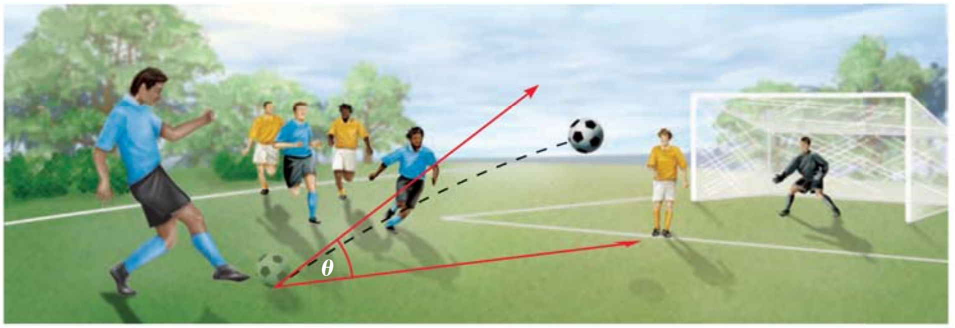
PATH OF A PROJECTILE The path traveled by an object that is projected at an initial height of h_0 feet, an initial speed of v feet per second, and an initial angle θ is given by

$$y = -\frac{16}{v^2 \cos^2 \theta}x^2 + (\tan \theta)x + h_0$$

where x is the horizontal distance (in feet) and y is the vertical distance (in feet). (This model neglects air resistance.)

EXAMPLE 4 Derive a trigonometric model

SOCCER Write an equation for the horizontal distance traveled by a soccer ball kicked from ground level ($h_0 = 0$) at speed v and angle θ.



Solution

$$-\frac{16}{v^2 \cos^2 \theta}x^2 + (\tan \theta)x + 0 = 0 \quad \textbf{Let } h_0 = 0.$$

$$-x\left(\frac{16}{v^2 \cos^2 \theta}x - \tan \theta\right) = 0 \quad \textbf{Factor.}$$

$$\frac{16}{v^2 \cos^2 \theta}x - \tan \theta = 0 \quad \textbf{Zero product property}$$

$$\frac{16}{v^2 \cos^2 \theta}x = \tan \theta \quad \textbf{Add } \tan\theta \textbf{ to each side.}$$

$$x = \frac{1}{16}v^2 \cos^2 \theta \tan \theta \quad \textbf{Multiply each side by } \frac{1}{16}v^2\cos^2\theta.$$

$$x = \frac{1}{16}v^2 \cos \theta \sin \theta \quad \textbf{Use } \cos\theta\tan\theta = \sin\theta.$$

$$x = \frac{1}{32}v^2 (2 \cos \theta \sin \theta) \quad \textbf{Rewrite } \frac{1}{16} \textbf{ as } \frac{1}{32} \cdot 2.$$

$$x = \frac{1}{32}v^2 \sin 2\theta \quad \textbf{Use a double-angle formula.}$$

USE ZERO PRODUCT PROPERTY
One solution of this equation is $x = 0$, which corresponds to the point where the ball leaves the ground. This solution is ignored in later steps, because the problem requires finding where the ball *lands*.

✓ GUIDED PRACTICE for Example 4

9. **WHAT IF?** Suppose you kick a soccer ball from ground level with an initial speed of 70 feet per second. Can you make the ball travel 200 feet?

10. **REASONING** Use the equation $x = \frac{1}{32}v^2 \sin 2\theta$ to explain why the projection angle that maximizes the distance a soccer ball travels is $\theta = 45°$.