EXAMPLE 2 Evaluate trigonometric expressions

Given $\cos a = \frac{5}{13}$ with $\frac{3\pi}{2} < a < 2\pi$, find (a) $\sin 2a$ and (b) $\sin \frac{a}{2}$.

Solution

a. Using a Pythagorean identity gives $\sin a = -\frac{12}{13}$.

$$\sin 2a = 2 \sin a \cos a = 2\left(-\frac{12}{13}\right)\left(\frac{5}{13}\right) = -\frac{120}{169}$$

b. Because $\frac{a}{2}$ is in Quadrant II, $\sin \frac{a}{2}$ is positive.

$$\sin \frac{a}{2} = \sqrt{\frac{1 - \cos a}{2}} = \sqrt{\frac{1 - \frac{5}{13}}{2}} = \sqrt{\frac{4}{13}} = \frac{2\sqrt{13}}{13}$$

MULTIPLY AN INEQUALITY

In part (b), you can multiply through the inequality $\frac{3\pi}{2} < a < 2\pi$ by $\frac{1}{2}$ to get $\frac{3\pi}{4} < \frac{a}{2} < \pi$. So, $\frac{a}{2}$ is in Quadrant II.

EXAMPLE 3 TAKS PRACTICE: Multiple Choice

Which expression is equivalent to $\frac{1 - \cos 2\theta}{\sin 2\theta}$?

Ⓐ $\cos \theta$ Ⓑ $\tan \theta$ Ⓒ $\cot \theta$ Ⓓ $\sin \theta$

Solution

$$\frac{1 - \cos 2\theta}{\sin 2\theta} = \frac{1 - (1 - 2\sin^2 \theta)}{2 \sin \theta \cos \theta}$$ Use double-angle formulas.

$$= \frac{2 \sin^2 \theta}{2 \sin \theta \cos \theta}$$ Simplify numerator.

$$= \frac{\sin \theta}{\cos \theta}$$ Divide out common factor 2 sin θ.

$$= \tan \theta$$ Use tangent identity.

▶ The correct answer is B. Ⓐ Ⓑ Ⓒ Ⓓ

✓ GUIDED PRACTICE for Examples 1, 2, and 3

Find the exact value of the expression.

1. $\tan \frac{\pi}{8}$ **2.** $\sin \frac{5\pi}{8}$ **3.** $\cos 15°$

4. Given $\sin a = \frac{\sqrt{2}}{2}$ with $0 < a < \frac{\pi}{2}$, find $\cos 2a$ and $\tan \frac{a}{2}$.

5. Given $\cos a = -\frac{3}{5}$ with $\pi < a < \frac{3\pi}{2}$, find $\sin 2a$ and $\sin \frac{a}{2}$.

Simplify the expression.

6. $\frac{\cos 2\theta}{\sin \theta + \cos \theta}$ **7.** $\frac{\tan 2x}{\tan x}$ **8.** $\sin 2x \tan \frac{x}{2}$