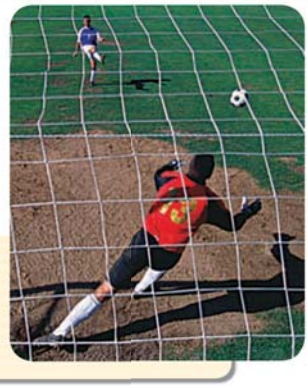
14.7 Apply Double-Angle and Half-Angle Formulas

TEKS a.2, 2A.2.A; P.3.A



Before You evaluated expressions using sum and difference formulas.

Now You will use double-angle and half-angle formulas.

Why? So you can find the distance an object travels, as in Example 4.

Key Vocabulary
- **sine,** *p. 852*
- **cosine,** *p. 852*
- **tangent,** *p. 852*

In this lesson, you will use formulas for double angles (angles of measure $2a$) and half angles $\left(\text{angles of measure } \frac{a}{2}\right)$.

KEY CONCEPT *For Your Notebook*

Double-Angle and Half-Angle Formulas

Double-Angle Formulas

$\sin 2a = 2 \sin a \cos a$

$\cos 2a = 1 - 2 \sin^2 a$

$\cos 2a = 2 \cos^2 a - 1$

$\cos 2a = \cos^2 a - \sin^2 a$

$\tan 2a = \dfrac{2 \tan a}{1 - \tan^2 a}$

Half-Angle Formulas

$\sin \dfrac{a}{2} = \pm\sqrt{\dfrac{1 - \cos a}{2}}$

$\cos \dfrac{a}{2} = \pm\sqrt{\dfrac{1 + \cos a}{2}}$

$\tan \dfrac{a}{2} = \dfrac{1 - \cos a}{\sin a}$

$\tan \dfrac{a}{2} = \dfrac{\sin a}{1 + \cos a}$

The signs of $\sin \frac{a}{2}$ and $\cos \frac{a}{2}$ depend on the quadrant in which $\frac{a}{2}$ lies.

EXAMPLE 1 Evaluate trigonometric expressions

Find the exact value of (a) cos 165° and (b) $\tan \frac{\pi}{12}$.

a. $\cos 165° = \cos \frac{1}{2}(330°)$

$= -\sqrt{\dfrac{1 + \cos 330°}{2}}$

$= -\sqrt{\dfrac{1 + \frac{\sqrt{3}}{2}}{2}}$

$= -\dfrac{\sqrt{2 + \sqrt{3}}}{2}$

b. $\tan \frac{\pi}{12} = \tan \frac{1}{2}\left(\frac{\pi}{6}\right)$

$= \dfrac{1 - \cos \frac{\pi}{6}}{\sin \frac{\pi}{6}}$

$= \dfrac{1 - \frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$= 2 - \sqrt{3}$

CHOOSE SIGNS
Because 165° is in Quadrant II and the value of cosine is negative in Quadrant II, the following formula is used:

$\cos \dfrac{a}{2} = -\sqrt{\dfrac{1 + \cos a}{2}}$