SOLVING TRIGONOMETRIC EQUATIONS Solve the equation for $0 \leq x<2 \pi$.
33. $\cos \left(x+\frac{\pi}{6}\right)-1=\cos \left(x-\frac{\pi}{6}\right)$
34. $\sin \left(x+\frac{\pi}{4}\right)+\sin \left(x-\frac{\pi}{4}\right)=0$
35. $\sin \left(x+\frac{5 \pi}{6}\right)+\sin \left(x-\frac{5 \pi}{6}\right)=1$
36. $\tan (x+\pi)+\cos \left(x+\frac{\pi}{2}\right)=0$
37. $\tan (x+\pi)+2 \sin (x+\pi)=0$
38. $\sin (x+\pi)+\cos (x+\pi)=0$
39. Challenge Consider a complex number $z=a+b i$ in the complex plane shown. Let $r$ be the length of the line segment joining $z$ and the origin, and let $\theta$ be the angle that this segment makes with the positive real axis, as shown.
a. Explain why $a=r \cos \theta$ and $b=r \sin \theta$, so that $z=(r \cos \theta)+i(r \sin \theta)$.
b. Use the result from part (a) to show the following: $z^{2}=r^{2}[(\cos \theta \cos \theta-\sin \theta \sin \theta)+i(\sin \theta \cos \theta+\cos \theta \sin \theta)]$

c. Use the sum and difference formulas to show that the equation in part (b) can be written as $z^{2}=r^{2}(\cos 2 \theta+i \sin 2 \theta)$.

## Problem Solving

## EXAMPLE 5

 ......................... on p. 951for Exs. 40-41
40. METEOROLOGY The number $h$ of hours of daylight for Rome, Italy, and Miami, Florida, can be approximated by the equations below, where $t$ is the time in days and $t=0$ represents January 1.

Rome: $h_{1}=2.7 \sin \left(\frac{\pi t}{182}-4.94\right)+12.1 \quad$ Miami: $h_{2}=-1.6 \cos \frac{\pi t}{182}+12.1$
On which days of the year will the cities have the same amount of daylight?
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41. CLOCK TOWER The heights $m$ and $h$ (in feet) of a clock tower's minute hand and hour hand, respectively, can be approximated by
$m=182.5-11.5 \sin \left(\frac{\pi t}{30}-\frac{\pi}{2}\right) \quad$ and $\quad h=182.5-7 \sin \left(\frac{\pi t}{360}\right)$
where $t$ is the time in minutes and $t=0$ represents 3:00 P.M. Use a graphing calculator to find how long it takes for the height of the minute hand to equal the height of the hour hand.
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42. PHYSICAL SCIENCE When a wave travels through a taut string, the displacement $y$ of each point on the string depends on the time $t$ and the point's position $x$. The equation of a standing wave can be obtained by adding the displacements of two waves traveling in opposite directions. Suppose two waves can be modeled by these equations:

$$
y_{1}=A \cos \left(\frac{2 \pi t}{3}-\frac{2 \pi x}{5}\right) \quad y_{2}=A \cos \left(\frac{2 \pi t}{3}+\frac{2 \pi x}{5}\right)
$$

Show that $y_{1}+y_{2}=2 A \cos \left(\frac{2 \pi t}{3}\right) \cos \left(\frac{2 \pi x}{5}\right)$.

