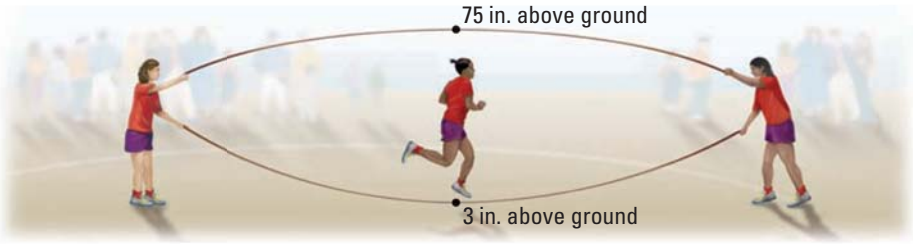
EXAMPLE 2 Model circular motion

JUMP ROPE At a Double Dutch competition, two people swing jump ropes as shown in the diagram below. The highest point of the middle of each rope is 75 inches above the ground, and the lowest point is 3 inches. The rope makes 2 revolutions per second. Write a model for the height h (in feet) of a rope as a function of the time t (in seconds) if the rope is at its lowest point when $t = 0$.



Solution

STEP 1 **Find** the maximum and minimum values of the function. A rope's maximum height is 75 inches, so $M = 75$. A rope's minimum height is 3 inches, so $m = 3$.

STEP 2 **Identify** the vertical shift. The vertical shift for the model is:

$$k = \frac{M + m}{2} = \frac{75 + 3}{2} = \frac{78}{2} = 39$$

STEP 3 **Decide** whether the height should be modeled by a sine or cosine function. When $t = 0$, the height is at its minimum. So, use a cosine function whose graph is a reflection in the x-axis with no horizontal shift ($h = 0$).

STEP 4 **Find** the amplitude and period.

The amplitude is $|a| = \frac{M - m}{2} = \frac{75 - 3}{2} = 36$.

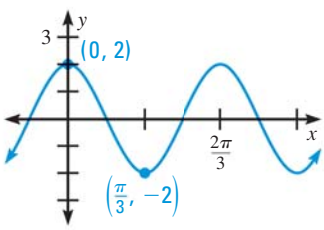
Because the graph is a reflection, $a < 0$. So, $a = -36$. Because a rope is rotating at a rate of 2 revolutions per second, one revolution is completed in 0.5 second. So, the period is $\frac{2\pi}{b} = 0.5$, and $b = 4\pi$.

▶ A model for the height of a rope is $h = -36 \cos 4\pi t + 39$.

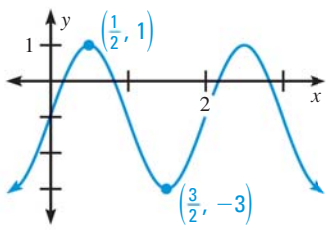
GUIDED PRACTICE for Examples 1 and 2

Write a function for the sinusoid.

1.



2.



3. WHAT IF? *Describe* how the model in Example 2 would change if the lowest point of a rope is 5 inches above the ground and the highest point is 70 inches above the ground.