EXTRANEOUS SOLUTIONS When solving a trigonometric equation, it is possible to obtain extraneous solutions. So, you should always check your solutions in the original equation.

EXAMPLE 6 Solve an equation with an extraneous solution

Solve $1 + \cos x = \sin x$ in the interval $0 \le x < 2\pi$.

$1 + \cos x = \sin x$	Write original equation.
$(1 + \cos x)^2 = (\sin x)^2$	Square both sides.
$1 + 2\cos x + \cos^2 x = \sin^2 x$	Multiply.
$1 + 2\cos x + \cos^2 x = 1 - \cos^2 x$	Pythagorean identity
$2\cos^2 x + 2\cos x = 0$	Quadratic form
$2\cos x(\cos x + 1) = 0$	Factor out 2 cos *x*.
$2\cos x = 0$ or $\cos x + 1 = 0$	Zero product property
$\cos x = 0$ or $\cos x = -1$	Solve for cos *x*.

REVIEW FOIL METHOD
For help multiplying binomials, see p. 245.

On the interval $0 \le x < 2\pi$, $\cos x = 0$ has two solutions: $x = \frac{\pi}{2}$ or $x = \frac{3\pi}{2}$.

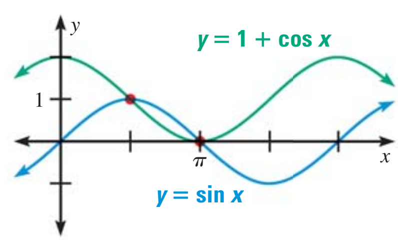
On the interval $0 \le x < 2\pi$, $\cos x = -1$ has one solution: $x = \pi$.

Therefore, $1 + \cos x = \sin x$ has three possible solutions: $x = \frac{\pi}{2}$, π, and $\frac{3\pi}{2}$.

CHECK To check the solutions, substitute them into the original equation and simplify.

$1 + \cos x = \sin x$	$1 + \cos x = \sin x$	$1 + \cos x = \sin x$
$1 + \cos\frac{\pi}{2} \stackrel{?}{=} \sin\frac{\pi}{2}$	$1 + \cos\pi \stackrel{?}{=} \sin\pi$	$1 + \cos\frac{3\pi}{2} \stackrel{?}{=} \sin\frac{3\pi}{2}$
$1 + 0 \stackrel{?}{=} 1$	$1 + (-1) \stackrel{?}{=} 0$	$1 + 0 \stackrel{?}{=} -1$
$1 = 1$ ✓	$0 = 0$ ✓	$1 \ne -1$

▶ The apparent solution $x = \frac{3\pi}{2}$ is extraneous because it does not check in the original equation. The only solutions in the interval $0 \le x < 2\pi$ are $x = \frac{\pi}{2}$ and $x = \pi$. Graphs of each side of the original equation confirm the solutions.



GUIDED PRACTICE for Examples 4, 5, and 6

Find the general solution of the equation.

4. $\sin^3 x - \sin x = 0$

5. $1 - \cos x = \sqrt{3}\sin x$

Solve the equation in the interval $0 \le x \le \pi$.

6. $2\sin x = \csc x$

7. $\tan^2 x - \sin x\tan^2 x = 0$