✓ GUIDED PRACTICE for Examples 1, 2, and 3

1. Find the general solution of the equation $2 \sin x + 4 = 5$.
2. Solve the equation $3 \csc^2 x = 4$ in the interval $0 \le x < 2\pi$.
3. **OCEANOGRAPHY** In Example 3, at what time(s) is the water depth 63 feet?

TAKS EXAMPLE 4 TAKS PRACTICE: Multiple Choice

What is the general solution of $\sin^3 x - 9 \sin x = 0$?

Ⓐ $x = \frac{\pi}{2} + 2n\pi$ or $x = \frac{3\pi}{2} + 2n\pi$

Ⓑ $x = \frac{\pi}{2} + 2n\pi$ or $x = \pi + 2n\pi$

Ⓒ $x = \pi + 2n\pi$

Ⓓ $x = 2n\pi$ or $x = \pi + 2n\pi$

Solution

$\sin^3 x - 9 \sin x = 0$ — Write original equation.

$\sin x (\sin^2 x - 9) = 0$ — Factor out sin *x*.

$\sin x (\sin x + 3)(\sin x - 3) = 0$ — Factor difference of squares.

Set each factor equal to 0 and solve for *x*, if possible.

$\sin x = 0$	$\sin x + 3 = 0$	$\sin x - 3 = 0$
$x = 0$ or $x = \pi$	$\sin x = -3$	$\sin x = 3$

ELIMINATE SOLUTIONS
Because sin *x* is never less than −1 or greater than 1, there are no solutions of $\sin x = -3$ and $\sin x = 3$.

The only solutions in the interval $0 \le x < 2\pi$ are $x = 0$ and $x = \pi$.

The general solution is $x = 2n\pi$ or $x = \pi + 2n\pi$ where n is any integer.

▶ The correct answer is D. Ⓐ Ⓑ Ⓒ Ⓓ

EXAMPLE 5 Use the quadratic formula

Solve $\cos^2 x - 5 \cos x + 2 = 0$ in the interval $0 \le x \le \pi$.

Solution

Because the equation is in the form $au^2 + bu + c = 0$ where $u = \cos x$, you can use the quadratic formula to solve for cos *x*.

$\cos^2 x - 5 \cos x + 2 = 0$ — Write original equation.

$\cos x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(2)}}{2(1)}$ — Quadratic formula

$= \frac{5 \pm \sqrt{17}}{2}$ — Simplify.

≈ 4.56 or 0.44 — Use a calculator.

$x = \cos^{-1} 4.56$	$x = \cos^{-1} 0.44$	Use inverse cosine.
No solution	≈ 1.12	Use a calculator, if possible.

▶ In the interval $0 \le x \le \pi$, the only solution is $x \approx 1.12$.