

EXAMPLE 2 Solve a trigonometric equation in an interval

Solve $9 \tan^2 x + 2 = 3$ in the interval $0 \leq x < 2\pi$.

$$9 \tan^2 x + 2 = 3 \quad \text{Write original equation.}$$

$$9 \tan^2 x = 1 \quad \text{Subtract 2 from each side.}$$

$$\tan^2 x = \frac{1}{9} \quad \text{Divide each side by 9.}$$

$$\tan x = \pm \frac{1}{3} \quad \text{Take square roots of each side.}$$

REVIEW INVERSE FUNCTIONS

For help with inverse trigonometric functions, see p. 875.

Using a calculator, you find that $\tan^{-1} \frac{1}{3} \approx 0.322$ and $\tan^{-1} \left(-\frac{1}{3}\right) \approx -0.322$.

Therefore, the general solution of the equation is:

$$x \approx 0.322 + n\pi \quad \text{or} \quad x \approx -0.322 + n\pi \quad (\text{where } n \text{ is any integer})$$

► The specific solutions in the interval $0 \leq x < 2\pi$ are:

$$x \approx 0.322 \qquad x \approx -0.322 + \pi \approx 2.820$$

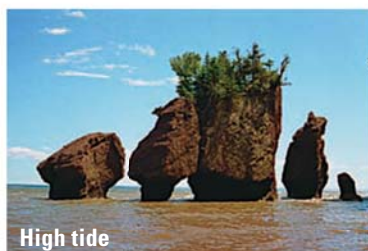
$$x \approx 0.322 + \pi \approx 3.464 \qquad x \approx -0.322 + 2\pi \approx 5.961$$

EXAMPLE 3 Solve a real-life trigonometric equation

OCEANOGRAPHY The water depth d for the Bay of Fundy can be modeled by

$$d = 35 - 28 \cos \frac{\pi}{6.2}t$$

where d is measured in feet and t is the time in hours. If $t = 0$ represents midnight, at what time(s) is the water depth 7 feet?



High tide



Low tide

ANOTHER WAY

For alternative methods for solving the problem in Example 3, turn to page 938 for the **Problem Solving Workshop**.

Solution

Substitute 7 for d in the model and solve for t .

$$35 - 28 \cos \frac{\pi}{6.2}t = 7 \quad \text{Substitute 7 for } d.$$

$$-28 \cos \frac{\pi}{6.2}t = -28 \quad \text{Subtract 35 from each side.}$$

$$\cos \frac{\pi}{6.2}t = 1 \quad \text{Divide each side by } -28.$$

$$\frac{\pi}{6.2}t = 2n\pi \quad \text{cos } \theta = 1 \text{ when } \theta = 2n\pi.$$

$$t = 12.4n \quad \text{Solve for } t.$$

► On the interval $0 \leq t \leq 24$ (representing one full day), the water depth is 7 feet when $t = 12.4(0) = 0$ (that is, at midnight) and when $t = 12.4(1) = 12.4$ (that is, at 12:24 P.M.).