14.4 Solve Trigonometric Equations

TEKS a.5, a.6, 2A.2.A; P.3.D

Before You verified trigonometric identities.

Now You will solve trigonometric equations.

Why? So you can solve surface area problems, as in Ex. 43.

Key Vocabulary
- **extraneous solution,** *p. 52*

In Lesson 14.3, you verified trigonometric identities. In this lesson, you will solve trigonometric equations. To see the difference, consider the following:

$$\sin^2 x + \cos^2 x = 1 \qquad \textbf{Equation 1}$$

$$\sin x = 1 \qquad \textbf{Equation 2}$$

Equation 1 is an identity because it is true for all real values of x. Equation 2, however, is true only for some values of x. When you find these values, you are solving the equation.

EXAMPLE 1 Solve a trigonometric equation

Solve $2 \sin x - \sqrt{3} = 0$.

Solution

First isolate $\sin x$ on one side of the equation.

$2 \sin x - \sqrt{3} = 0$ **Write original equation.**

$2 \sin x = \sqrt{3}$ **Add $\sqrt{3}$ to each side.**

$\sin x = \frac{\sqrt{3}}{2}$ **Divide each side by 2.**

One solution of $\sin x = \frac{\sqrt{3}}{2}$ in the interval $0 \le x < 2\pi$ is $x = \sin^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{3}$. The other solution in the interval is $x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$. Moreover, because $y = \sin x$ is periodic, there will be infinitely many solutions.

You can use the two solutions found above to write the general solution:

$$x = \frac{\pi}{3} + 2n\pi \quad \text{or} \quad x = \frac{2\pi}{3} + 2n\pi \quad \text{(where } n \text{ is any integer)}$$

WRITE GENERAL SOLUTION
To write the general solution of a trigonometric equation, you can add multiples of the period to all the solutions from one cycle.

CHECK You can check the answer by graphing $y = \sin x$ and $y = \frac{\sqrt{3}}{2}$ in the same coordinate plane. Then find the points where the graphs intersect. You can see that there are infinitely many such points.