EXAMPLE 1 Find trigonometric values

Given that $\sin\theta = \frac{4}{5}$ and $\frac{\pi}{2} < \theta < \pi$, find the values of the other five trigonometric functions of θ.

Solution

STEP 1 **Find** $\cos\theta$.

$\sin^2\theta + \cos^2\theta = 1$ — Write Pythagorean identity.

$\left(\frac{4}{5}\right)^2 + \cos^2\theta = 1$ — Substitute $\frac{4}{5}$ for $\sin\theta$.

$\cos^2\theta = 1 - \left(\frac{4}{5}\right)^2$ — Subtract $\left(\frac{4}{5}\right)^2$ from each side.

$\cos^2\theta = \frac{9}{25}$ — Simplify.

$\cos\theta = \pm\frac{3}{5}$ — Take square roots of each side.

$\cos\theta = -\frac{3}{5}$ — Because θ is in Quadrant II, $\cos\theta$ is negative.

REVIEW TRIGONOMETRY
For help with finding the sign of a trigonometric function value, see p. 866.

STEP 2 **Find** the values of the other four trigonometric functions of θ using the known values of $\sin\theta$ and $\cos\theta$.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{4}{5}}{-\frac{3}{5}} = -\frac{4}{3} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{-\frac{3}{5}}{\frac{4}{5}} = -\frac{3}{4}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{4}{5}} = \frac{5}{4} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{1}{-\frac{3}{5}} = -\frac{5}{3}$$

EXAMPLE 2 Simplify a trigonometric expression

Simplify the expression $\tan\left(\frac{\pi}{2} - \theta\right)\sin\theta$.

$\tan\left(\frac{\pi}{2} - \theta\right)\sin\theta = \cot\theta\sin\theta$ — Cofunction identity

$= \left(\frac{\cos\theta}{\sin\theta}\right)(\sin\theta)$ — Cotangent identity

$= \cos\theta$ — Simplify.

EXAMPLE 3 Simplify a trigonometric expression

Simplify the expression $\csc\theta\cot^2\theta + \frac{1}{\sin\theta}$.

$\csc\theta\cot^2\theta + \frac{1}{\sin\theta} = \csc\theta\cot^2\theta + \csc\theta$ — Reciprocal identity

$= \csc\theta(\csc^2\theta - 1) + \csc\theta$ — Pythagorean identity

$= \csc^3\theta - \csc\theta + \csc\theta$ — Distributive property

$= \csc^3\theta$ — Simplify.