TANGENT FUNCTIONS Graphing tangent functions using translations and reflections is similar to graphing sine and cosine functions.

EXAMPLE 5 Combine a translation and a reflection

Graph $y = -3 \tan x + 5$.

Solution

STEP 1 **Identify** the period, horizontal shift, and vertical shift.

Period: π Horizontal shift: $h = 0$ Vertical shift: $k = 5$

STEP 2 **Draw** the midline of the graph, $y = 5$.

STEP 3 **Find** the asymptotes and key points of $y = |-3| \tan x + 5$.

Asymptotes: $x = -\frac{\pi}{2 \cdot 1} = -\frac{\pi}{2}$; $x = \frac{\pi}{2 \cdot 1} = \frac{\pi}{2}$

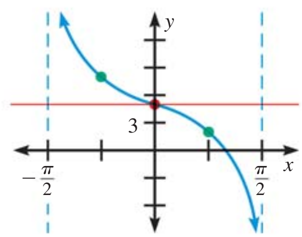
On $y = k$: $(0, 0 + 5) = (0, 5)$

Halfway points: $\left(-\frac{\pi}{4}, -3 + 5\right) = \left(-\frac{\pi}{4}, 2\right)$; $\left(\frac{\pi}{4}, 3 + 5\right) = \left(\frac{\pi}{4}, 8\right)$

FIND ASYMPTOTES
Notice that the asymptotes are not shifted. This is because there is no horizontal shift.

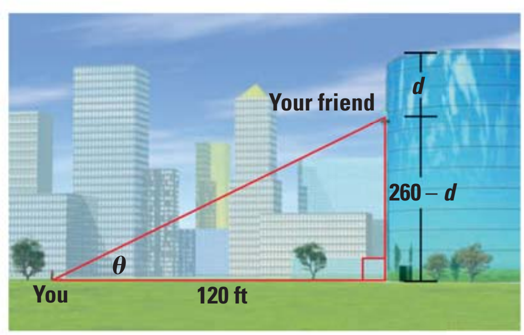
STEP 4 **Reflect** the graph. Because $a < 0$, the graph is reflected in the midline $y = 5$. So, $\left(-\frac{\pi}{4}, 2\right)$ becomes $\left(-\frac{\pi}{4}, 8\right)$ and $\left(\frac{\pi}{4}, 8\right)$ becomes $\left(\frac{\pi}{4}, 2\right)$.

STEP 5 **Draw** the graph through the key points.



EXAMPLE 6 Model with a tangent function

GLASS ELEVATOR You are standing 120 feet from the base of a 260 foot building. You watch your friend go down the side of the building in a glass elevator. Write and graph a model that gives your friend's distance d (in feet) from the top of the building as a function of the angle of elevation θ.



Solution

Use a tangent function to write an equation relating d and θ.

$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{260 - d}{120}$	**Definition of tangent**
$120 \tan \theta = 260 - d$	**Multiply each side by 120.**
$120 \tan \theta - 260 = -d$	**Subtract 260 from each side.**
$-120 \tan \theta + 260 = d$	**Solve for *d*.**

The graph of $d = -120 \tan \theta + 260$ is shown at the right.