Graph the function.

1. $y = \cos x + 4$ **2.** $y = 3 \sin\left(x - \dfrac{\pi}{2}\right)$ **3.** $f(x) = \sin(x + \pi) - 1$

REFLECTIONS You have graphed functions of the form $y = a \sin b(x - h) + k$ and $y = a \cos b(x - h) + k$ where $a > 0$. To see what happens when $a < 0$, consider the graphs of $y = -\sin x$ and $y = -\cos x$.
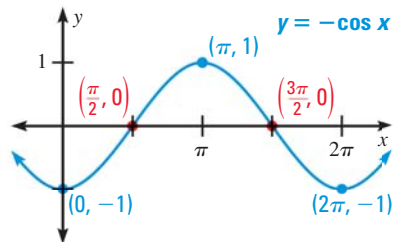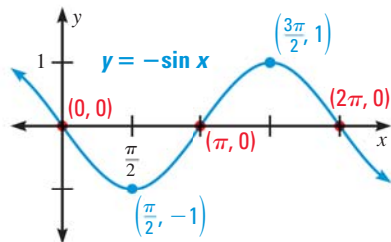
Notice that the graphs are reflections of the graphs of $y = \sin x$ and $y = \cos x$ in the x-axis. In general, when $a < 0$ the graphs of $y = a \sin b(x - h) + k$ and $y = a \cos b(x - h) + k$ are reflections of the graphs of $y = |a| \sin b(x - h) + k$ and $y = |a| \cos b(x - h) + k$, respectively, in the midline $y = k$.

EXAMPLE 4 **Combine a translation and a reflection**

Graph $y = -2 \sin \dfrac{2}{3}\left(x - \dfrac{\pi}{2}\right)$.

Solution

STEP 1 **Identify** the amplitude, period, horizontal shift, and vertical shift.

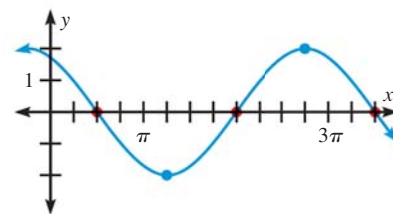Amplitude: $|a| = |-2| = 2$ Horizontal shift: $h = \dfrac{\pi}{2}$

Period: $\dfrac{2\pi}{b} = \dfrac{2\pi}{\frac{2}{3}} = 3\pi$ Vertical shift: $k = 0$

STEP 2 **Draw** the midline of the graph. Because $k = 0$, the midline is the x-axis.

STEP 3 **Find** the five key points of $y = |-2| \sin \dfrac{2}{3}\left(x - \dfrac{\pi}{2}\right)$.

On $y = k$: $\left(0 + \dfrac{\pi}{2}, 0\right) = \left(\dfrac{\pi}{2}, 0\right)$; $\left(\dfrac{3\pi}{2} + \dfrac{\pi}{2}, 0\right) = (2\pi, 0)$; $\left(3\pi + \dfrac{\pi}{2}, 0\right) = \left(\dfrac{7\pi}{2}, 0\right)$

Maximum: $\left(\dfrac{3\pi}{4} + \dfrac{\pi}{2}, 2\right) = \left(\dfrac{5\pi}{4}, 2\right)$ **Minimum:** $\left(\dfrac{9\pi}{4} + \dfrac{\pi}{2}, -2\right) = \left(\dfrac{11\pi}{4}, -2\right)$

GRAPH REFLECTIONS
The maximum and minimum of the original graph become the minimum and maximum, respectively, of the reflected graph.

STEP 4 **Reflect** the graph. Because $a < 0$, the graph is reflected in the midline $y = 0$. So, $\left(\dfrac{5\pi}{4}, 2\right)$ becomes $\left(\dfrac{5\pi}{4}, -2\right)$ and $\left(\dfrac{11\pi}{4}, -2\right)$ becomes $\left(\dfrac{11\pi}{4}, 2\right)$.

STEP 5 **Draw** the graph through the key points.

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