14.2 Translate and Reflect Trigonometric Graphs

Before	You graphed sine, cosine, and tangent functions.
Now	You will translate and reflect trigonometric graphs.
Why?	So you can model predator-prey populations, as in Ex. 54.

Key Vocabulary
- **translation,** *p. 123*
- **reflection,** *p. 124*
- **amplitude,** *p. 908*
- **period,** *p. 908*

KEY CONCEPT   *For Your Notebook*

Translations of Sine and Cosine Graphs

To graph $y = a \sin b(x - h) + k$ or $y = a \cos b(x - h) + k$ where $a > 0$ and $b > 0$, follow these steps:

STEP 1 **Identify** the amplitude a, the period $\frac{2\pi}{b}$, the horizontal shift h, and the vertical shift k of the graph.

STEP 2 **Draw** the horizontal line $y = k$, called the *midline* of the graph.

STEP 3 **Find** the five key points by translating the key points of $y = a \sin bx$ or $y = a \cos bx$ horizontally h units and vertically k units.

STEP 4 **Draw** the graph through the five translated key points.

EXAMPLE 1 Graph a vertical translation

Graph $y = 2 \sin 4x + 3$.

Solution

STEP 1 **Identify** the amplitude, period, horizontal shift, and vertical shift.

Amplitude: $a = 2$ Horizontal shift: $h = 0$

Period: $\frac{2\pi}{b} = \frac{2\pi}{4} = \frac{\pi}{2}$ Vertical shift: $k = 3$

STEP 2 **Draw** the midline of the graph, $y = 3$.

FIND KEY POINTS
Because the graph is shifted up 3 units, the y-coordinates of the five key points will be increased by 3.

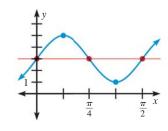**STEP 3** **Find** the five key points.

On $y = k$: $(0, 0 + 3) = (0, 3)$;

$$\left(\frac{\pi}{4}, 0 + 3\right) = \left(\frac{\pi}{4}, 3\right); \left(\frac{\pi}{2}, 0 + 3\right) = \left(\frac{\pi}{2}, 3\right)$$

Maximum: $\left(\frac{\pi}{8}, 2 + 3\right) = \left(\frac{\pi}{8}, 5\right)$

Minimum: $\left(\frac{3\pi}{8}, -2 + 3\right) = \left(\frac{3\pi}{8}, 1\right)$

STEP 4 **Draw** the graph through the key points.