EXAMPLE 2 Graph a cosine function

SKETCH A GRAPH
After you have drawn one complete cycle of the graph in Example 2 on the interval $0 \le x \le 1$, you can extend the graph by copying the cycle as many times as desired to the left and right of $0 \le x \le 1$.

Graph $y = \frac{1}{2} \cos 2\pi x$.

Solution

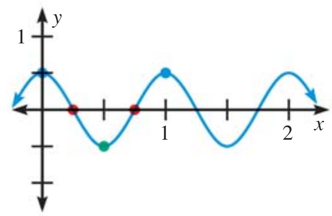
The amplitude is $a = \frac{1}{2}$ and the period is $\frac{2\pi}{b} = \frac{2\pi}{2\pi} = 1$.

Intercepts: $\left(\frac{1}{4} \cdot 1, 0\right) = \left(\frac{1}{4}, 0\right)$;

$\left(\frac{3}{4} \cdot 1, 0\right) = \left(\frac{3}{4}, 0\right)$

Maximums: $\left(0, \frac{1}{2}\right)$; $\left(1, \frac{1}{2}\right)$

Minimum: $\left(\frac{1}{2} \cdot 1, -\frac{1}{2}\right) = \left(\frac{1}{2}, -\frac{1}{2}\right)$



MODELING WITH TRIGONOMETRIC FUNCTIONS The periodic nature of trigonometric functions is useful for modeling *oscillating* motions or repeating patterns that occur in real life. Some examples are sound waves, the motion of a pendulum, and seasons of the year. In such applications, the reciprocal of the period is called the **frequency**, which gives the number of cycles per unit of time.

EXAMPLE 3 Model with a sine function

AUDIO TEST A sound consisting of a single frequency is called a pure tone. An *audiometer* produces pure tones to test a person's auditory functions. Suppose an audiometer produces a pure tone with a frequency f of 2000 hertz (cycles per second). The maximum pressure P produced from the pure tone is 2 millipascals. Write and graph a sine model that gives the pressure P as a function of the time t (in seconds).

Solution

STEP 1 **Find** the values of a and b in the model $P = a \sin bt$. The maximum pressure is 2, so $a = 2$. You can use the frequency f to find b.

$$\text{frequency} = \frac{1}{\text{period}} \longrightarrow 2000 = \frac{b}{2\pi} \longrightarrow 4000\pi = b$$

The pressure P as a function of time t is given by $P = 2 \sin 4000\pi t$.

STEP 2 **Graph** the model. The amplitude is $a = 2$ and the period is $\frac{1}{f} = \frac{1}{2000}$.

Intercepts: $(0, 0)$;

$\left(\frac{1}{2} \cdot \frac{1}{2000}, 0\right) = \left(\frac{1}{4000}, 0\right)$; $\left(\frac{1}{2000}, 0\right)$

Maximum: $\left(\frac{1}{4} \cdot \frac{1}{2000}, 2\right) = \left(\frac{1}{8000}, 2\right)$

Minimum: $\left(\frac{3}{4} \cdot \frac{1}{2000}, -2\right) = \left(\frac{3}{8000}, -2\right)$