Using ALTERNATIVE METHODS

TEKS *a.4, 2A.3.A, 2A.3.B; P.3.E*

Another Way to Solve Example 4, page 891

MULTIPLE REPRESENTATIONS In Example 4 on page 891, you found the area of a triangle given the lengths of its sides by using Heron's formula. You can also find the area of the triangle by writing and solving a system of equations.

PROBLEM

URBAN PLANNING The intersection of three streets forms a piece of land called a traffic triangle. Find the area of the traffic triangle shown.

$a = 170$ yd $b = 240$ yd $c = 350$ yd
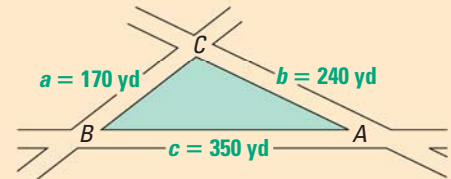
METHOD

Using a System of Equations Use a system of quadratic equations to find the triangle's height h. Then find the area of the triangle using the formula $A = \frac{1}{2}bh$.

STEP 1 **Draw** a new diagram of the triangle as shown. Let h be the height of the triangle. The altitude labeled by h divides $\overline{AB}$ into two segments of length x and $350 - x$.
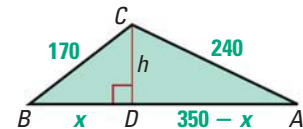
STEP 2 **Use** the Pythagorean theorem to write a system of quadratic equations.

$$h^2 + x^2 = 170^2$$
$$h^2 + (350 - x)^2 = 240^2$$

STEP 3 **Solve** the first equation for h^2 to get $h^2 = 170^2 - x^2$. Substitute this expression for h^2 in the second equation, and solve for x.

$$170^2 - x^2 + (350 - x)^2 = 240^2$$
$$28{,}900 - x^2 + 122{,}500 - 700x + x^2 = 57{,}600$$
$$-700x = -93{,}800$$
$$x = 134$$

STEP 4 **Use** the Pythagorean theorem to find that $h = \sqrt{170^2 - 134^2} \approx 104.6$.

So the area of the triangle is $A = \frac{1}{2}bh \approx \frac{1}{2}(350)(104.6) \approx 18{,}300$.

▶ The area of the triangle is about 18,300 square yards.

PRACTICE

FINDING AREAS Use the method above to find the area of △*ABC* with the given side lengths.

1. $a = 12$, $b = 17$, $c = 26$

2. $a = 63$, $b = 92$, $c = 87$

3. $a = 101$, $b = 94$, $c = 153$

4. **WHAT IF?** Suppose $a = 200$ yd in the problem above. Find the area of the triangle.

5. **GARDEN AREA** A triangular garden has sides with lengths 50 feet, 38 feet, and 43 feet. Use the method above to find the area of the garden.