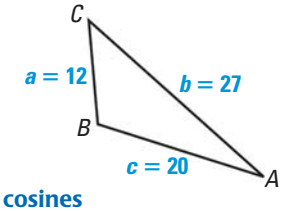
EXAMPLE 2 Solve a triangle for the SSS case

Solve $\triangle ABC$ with $a = 12$, $b = 27$, and $c = 20$.



Solution

AVOID ERRORS
In Example 2, the largest angle is found first to make sure that the other two angles are acute. This way, when you use the law of sines to find another angle measure, you will know that it is between 0° and 90°.

First find the angle opposite the longest side, $\overline{AC}$. Use the law of cosines to solve for B.

$$b^2 = a^2 + c^2 - 2ac\cos B \qquad \text{Law of cosines}$$

$$27^2 = 12^2 + 20^2 - 2(12)(20)\cos B \qquad \text{Substitute.}$$

$$\frac{27^2 - 12^2 - 20^2}{-2(12)(20)} = \cos B \qquad \text{Solve for cos } B.$$

$$-0.3854 \approx \cos B \qquad \text{Simplify.}$$

$$B \approx \cos^{-1}(-0.3854) \approx 112.7° \qquad \text{Use inverse cosine.}$$

Now use the law of sines to find A.

$$\frac{\sin A}{a} = \frac{\sin B}{b} \qquad \text{Law of sines}$$

$$\frac{\sin A}{12} = \frac{\sin 112.7°}{27} \qquad \text{Substitute for } a, b, \text{ and } B.$$

$$\sin A = \frac{12 \sin 112.7°}{27} \approx 0.4100 \qquad \text{Multiply each side by 12 and simplify.}$$

$$A \approx \sin^{-1} 0.4100 \approx 24.2° \qquad \text{Use inverse sine.}$$

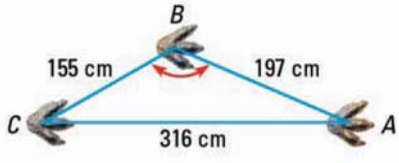
The third angle C of the triangle is $C \approx 180° - 24.2° - 112.7° = 43.1°$.

▶ In $\triangle ABC$, $A \approx 24.2°$, $B \approx 112.7°$, and $C \approx 43.1°$.

EXAMPLE 3 Use the law of cosines in real life

SCIENCE Scientists can use a set of footprints to calculate an organism's *step angle*, which is a measure of walking efficiency. The closer the step angle is to 180°, the more efficiently the organism walked.

The diagram at the right shows a set of footprints for a dinosaur. Find the step angle B.



Solution

$$b^2 = a^2 + c^2 - 2ac\cos B \qquad \text{Law of cosines}$$

$$316^2 = 155^2 + 197^2 - 2(155)(197)\cos B \qquad \text{Substitute.}$$

$$\frac{316^2 - 155^2 - 197^2}{-2(155)(197)} = \cos B \qquad \text{Solve for cos } B.$$

$$-0.6062 \approx \cos B \qquad \text{Simplify.}$$

$$B \approx \cos^{-1}(-0.6062) \approx 127.3° \qquad \text{Use inverse cosine.}$$

▶ The step angle B is about 127.3°.