13.6 Apply the Law of Cosines

TEKS *a.4, 2A.2.A, 2A.4.C; P.3.E*

Before	You solved triangles using the law of sines.
Now	You will solve triangles using the law of cosines.
Why?	So you can find angles formed by trapeze artists, as in Ex. 43.

Key Vocabulary
- **law of cosines**

In Lesson 13.5, you solved triangles for the AAS, ASA, and SSA cases. In this lesson, you will use the **law of cosines** to solve triangles when two sides and the included angle are known (SAS), or when all three sides are known (SSS).

KEY CONCEPT *For Your Notebook*

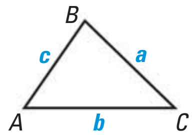
Law of Cosines

If $\triangle ABC$ has sides of length a, b, and c as shown, then:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

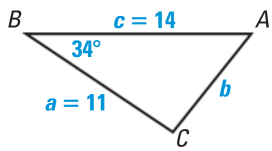
$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$



EXAMPLE 1 Solve a triangle for the SAS case

Solve $\triangle ABC$ with $a = 11$, $c = 14$, and $B = 34°$.



Solution

Use the law of cosines to find side length b.

$b^2 = a^2 + c^2 - 2ac\cos B$ **Law of cosines**

$b^2 = 11^2 + 14^2 - 2(11)(14)\cos 34°$ **Substitute for *a, c,* and *B*.**

$b^2 \approx 61.7$ **Simplify.**

$b \approx \sqrt{61.7} \approx 7.85$ **Take positive square root.**

ANOTHER WAY
When you know all three sides and one angle, you can use the law of cosines *or* the law of sines to find the measure of a second angle.

Use the law of sines to find the measure of angle A.

$\frac{\sin A}{a} = \frac{\sin B}{b}$ **Law of sines**

$\frac{\sin A}{11} = \frac{\sin 34°}{7.85}$ **Substitute for *a, b,* and *B*.**

$\sin A = \frac{11 \sin 34°}{7.85} \approx 0.7836$ **Multiply each side by 11 and simplify.**

$A \approx \sin^{-1} 0.7836 \approx 51.6°$ **Use inverse sine.**

The third angle C of the triangle is $C \approx 180° - 34° - 51.6° = 94.4°$.

▶ In $\triangle ABC$, $b \approx 7.85$, $A \approx 51.6°$, and $C \approx 94.4°$.