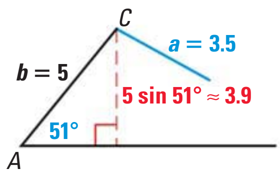
EXAMPLE 3 Examine the SSA case with no solution

Solve $\triangle ABC$ with $A = 51°$, $a = 3.5$, and $b = 5$.

Solution

Begin by drawing a horizontal line. On one end form a 51° angle (A) and draw a segment 5 units long ($\overline{AC}$, or b). At vertex C, draw a segment 3.5 units long (a). You can see that a needs to be at least $5 \sin 51° \approx 3.9$ units long to reach the horizontal side and form a triangle. So, it is not possible to draw the indicated triangle.



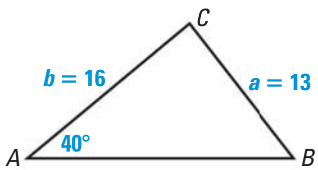
EXAMPLE 4 Solve the SSA case with two solutions

Solve $\triangle ABC$ with $A = 40°$, $a = 13$, and $b = 16$.

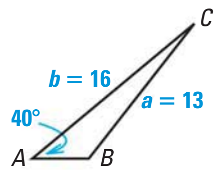
Solution

First make a sketch. Because $b \sin A = 16 \sin 40° \approx 10.3$, and $10.3 < 13 < 16$ ($h < a < b$), two triangles can be formed.

Triangle 1



Triangle 2



Use the law of sines to find the possible measures of B.

$$\frac{\sin B}{16} = \frac{\sin 40°}{13}$$ **Law of sines**

$$\sin B = \frac{16 \sin 40°}{13} \approx 0.7911$$ **Use a calculator.**

There are two angles B between 0° and 180° for which $\sin B \approx 0.7911$. One is acute and the other is obtuse. Use your calculator to find the acute angle: $\sin^{-1} 0.7911 \approx 52.3°$.

The obtuse angle has 52.3° as a reference angle, so its measure is $180° - 52.3° = 127.7°$. Therefore, $B \approx 52.3°$ or $B \approx 127.7°$.

Now find the remaining angle C and side length c for each triangle.

Triangle 1

$C \approx 180° - 40° - 52.3° = 87.7°$

$$\frac{c}{\sin 87.7°} = \frac{13}{\sin 40°}$$

$$c = \frac{13 \sin 87.7°}{\sin 40°} \approx 20.2$$

▶ In Triangle 1, $B \approx 52.3°$, $C \approx 87.7°$, and $c \approx 20.2$.

Triangle 2

$C \approx 180° - 40° - 127.7° = 12.3°$

$$\frac{c}{\sin 12.3°} = \frac{13}{\sin 40°}$$

$$c = \frac{13 \sin 12.3°}{\sin 40°} \approx 4.3$$

▶ In Triangle 2, $B \approx 127.7°$, $C \approx 12.3°$, and $c \approx 4.3$.

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