

EXAMPLE 1 Evaluate inverse trigonometric functions

Evaluate the expression in both radians and degrees.

a. $\cos^{-1} \frac{\sqrt{3}}{2}$

b. $\sin^{-1} 2$

c. $\tan^{-1}(-\sqrt{3})$

Solutiona. When $0 \leq \theta \leq \pi$, or $0^\circ \leq \theta \leq 180^\circ$, the angle whose cosine is $\frac{\sqrt{3}}{2}$ is:

$$\theta = \cos^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{6} \quad \text{or} \quad \theta = \cos^{-1} \frac{\sqrt{3}}{2} = 30^\circ$$

b. There is no angle whose sine is 2. So, $\sin^{-1} 2$ is undefined.c. When $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, or $-90^\circ < \theta < 90^\circ$, the angle whose tangent is $-\sqrt{3}$ is:

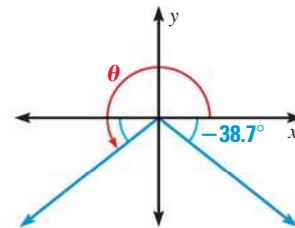
$$\theta = \tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3} \quad \text{or} \quad \theta = \tan^{-1}(-\sqrt{3}) = -60^\circ$$

EXAMPLE 2 Solve a trigonometric equationSolve the equation $\sin \theta = -\frac{5}{8}$ where $180^\circ < \theta < 270^\circ$.**Solution****STEP 1** Use a calculator to determine that in the interval $-90^\circ \leq \theta \leq 90^\circ$, the angle whose sine is $-\frac{5}{8}$ is $\sin^{-1}\left(-\frac{5}{8}\right) \approx -38.7^\circ$. This angle is in Quadrant IV, as shown.**STEP 2** Find the angle in Quadrant III (where $180^\circ < \theta < 270^\circ$) that has the same sine value as the angle in Step 1. The angle is:

$$\theta \approx 180^\circ + 38.7^\circ = 218.7^\circ$$

CHECK Use a calculator to check the answer.

$$\sin 218.7^\circ \approx -0.625 = -\frac{5}{8} \checkmark$$

**USE A CALCULATOR**

On most calculators, you can evaluate inverse trigonometric functions using the keys **2nd** **SIN** for inverse sine, **2nd** **COS** for inverse cosine, and **2nd** **TAN** for inverse tangent.

**GUIDED PRACTICE** for Examples 1 and 2

Evaluate the expression in both radians and degrees.

1. $\sin^{-1} \frac{\sqrt{2}}{2}$

2. $\cos^{-1} \frac{1}{2}$

3. $\tan^{-1}(-1)$

4. $\sin^{-1}\left(-\frac{1}{2}\right)$

Solve the equation for θ .

5. $\cos \theta = 0.4$; $270^\circ < \theta < 360^\circ$

6. $\tan \theta = 2.1$; $180^\circ < \theta < 270^\circ$

7. $\sin \theta = -0.23$; $270^\circ < \theta < 360^\circ$

8. $\tan \theta = 4.7$; $180^\circ < \theta < 270^\circ$

9. $\sin \theta = 0.62$; $90^\circ < \theta < 180^\circ$

10. $\cos \theta = -0.39$; $180^\circ < \theta < 270^\circ$