13.4 Investigating Inverse Trigonometric Functions

MATERIALS • paper and pencil

TEKS a.3, 2A.1.A, 2A.4.C; P.3.A

QUESTION Do the sine and cosine functions have inverse functions?

EXPLORE Determine if a trigonometric function has an inverse function

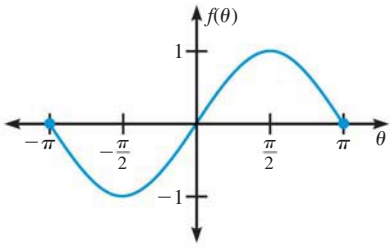
STEP 1 **Make a table** Copy and complete the table to find the values of $f(\theta) = \sin\theta$ and $g(\theta) = \cos\theta$ for each of the given values of θ.

θ	$-\pi$	$-\frac{3\pi}{4}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$f(\theta) = \sin\theta$	?	?	?	?	?	?	?	?	?
$g(\theta) = \cos\theta$	?	?	?	?	?	?	?	?	?

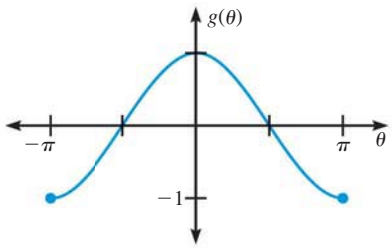
STEP 2 **Analyze sine** Use the table to explain why $f(\theta) = \sin\theta$ does not have an inverse function on the domain $-\pi \le \theta \le \pi$.

STEP 3 **Analyze cosine** Does $g(\theta) = \cos\theta$ have an inverse function on the domain $-\pi \le \theta \le \pi$? Explain why or why not.

STEP 4 **Use graphs** The graphs of $f(\theta) = \sin\theta$ and $g(\theta) = \cos\theta$ are shown for the domain $-\pi \le \theta \le \pi$. Explain how the graphs justify your answers in Steps 2 and 3.



$f(\theta) = \sin\theta$



$g(\theta) = \cos\theta$

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Use the graph of $f(\theta) = \sin\theta$ in Step 4 to choose a restricted domain for which the sine function does have an inverse function. *Explain* how you made your choice.
2. Give a restricted domain for which $g(\theta) = \cos\theta$ has an inverse function. *Explain* how you chose the domain.
3. Are the domains that you wrote in Exercises 1 and 2 the *only* domains for which the trigonometric functions have inverse functions? *Explain*.