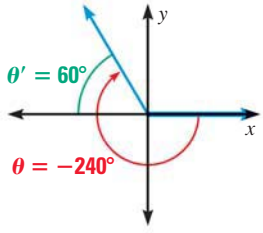
EXAMPLE 4 Use reference angles to evaluate functions

Evaluate (a) $\tan(-240°)$ and (b) $\csc\frac{17\pi}{6}$.

Solution

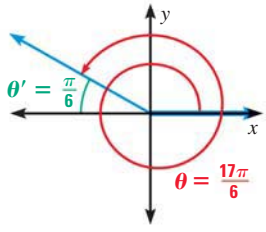
a. The angle $-240°$ is coterminal with $120°$. The reference angle is $\theta' = 180° - 120° = 60°$. The tangent function is negative in Quadrant II, so you can write:

$$\tan(-240°) = -\tan 60° = -\sqrt{3}$$



b. The angle $\frac{17\pi}{6}$ is coterminal with $\frac{5\pi}{6}$. The reference angle is $\theta' = \pi - \frac{5\pi}{6} = \frac{\pi}{6}$. The cosecant function is positive in Quadrant II, so you can write:

$$\csc\frac{17\pi}{6} = \csc\frac{\pi}{6} = 2$$



GUIDED PRACTICE for Examples 3 and 4

Sketch the angle. Then find its reference angle.

5. $210°$ **6.** $-260°$ **7.** $-\frac{7\pi}{9}$ **8.** $\frac{15\pi}{4}$

9. Evaluate $\cos(-210°)$ without using a calculator.

EXAMPLE 5 Calculate horizontal distance traveled

ROBOTICS The "frogbot" is a robot designed for exploring rough terrain on other planets. It can jump at a $45°$ angle and with an initial speed of 16 feet per second. On Earth, the horizontal distance d (in feet) traveled by a projectile launched at an angle θ and with an initial speed v (in feet per second) is given by:

Frogbot

$$d = \frac{v^2}{32}\sin 2\theta$$

INTERPRET MODELS
This model neglects air resistance and assumes that the projectile's starting and ending heights are the same.

How far can the frogbot jump on Earth?

Solution

$d = \frac{v^2}{32}\sin 2\theta$ — Write model for horizontal distance.

$= \frac{16^2}{32}\sin(2 \cdot 45°)$ — Substitute 16 for v and $45°$ for θ.

$= 8$ — Simplify.

▸ The frogbot can jump a horizontal distance of 8 feet on Earth.