13.3 Evaluate Trigonometric Functions of Any Angle

TEKS *a.4, a.5, 2A.2.A; P.3.A*

Before You evaluated trigonometric functions of an acute angle.

Now You will evaluate trigonometric functions of any angle.

Why? So you can calculate distances involving rotating objects, as in Ex. 37.

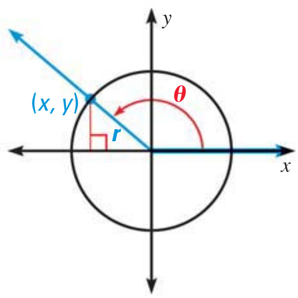
Key Vocabulary
- **unit circle**
- **quadrantal angle**
- **reference angle**

You can generalize the right-triangle definitions of trigonometric functions from Lesson 13.1 so that they apply to *any* angle in standard position.

KEY CONCEPT — *For Your Notebook*

General Definitions of Trigonometric Functions

Let θ be an angle in standard position, and let (x, y) be the point where the terminal side of θ intersects the circle $x^2 + y^2 = r^2$. The six trigonometric functions of θ are defined as follows:



$\sin\theta = \frac{y}{r}$ $\qquad$ $\csc\theta = \frac{r}{y}, y \neq 0$

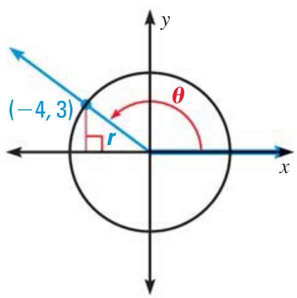
$\cos\theta = \frac{x}{r}$ $\qquad$ $\sec\theta = \frac{r}{x}, x \neq 0$

$\tan\theta = \frac{y}{x}, x \neq 0$ $\qquad$ $\cot\theta = \frac{x}{y}, y \neq 0$

These functions are sometimes called *circular functions.*

EXAMPLE 1 Evaluate trigonometric functions given a point

Let $(-4, 3)$ be a point on the terminal side of an angle θ in standard position. Evaluate the six trigonometric functions of θ.



Solution

Use the Pythagorean theorem to find the value of r.

$$r = \sqrt{x^2 + y^2} = \sqrt{(-4)^2 + 3^2} = \sqrt{25} = 5$$

Using $x = -4$, $y = 3$, and $r = 5$, you can write the following:

$\sin\theta = \frac{y}{r} = \frac{3}{5}$ $\qquad$ $\cos\theta = \frac{x}{r} = -\frac{4}{5}$ $\qquad$ $\tan\theta = \frac{y}{x} = -\frac{3}{4}$

$\csc\theta = \frac{r}{y} = \frac{5}{3}$ $\qquad$ $\sec\theta = \frac{r}{x} = -\frac{5}{4}$ $\qquad$ $\cot\theta = \frac{x}{y} = -\frac{4}{3}$