EXAMPLE 2 Use mathematical induction

Let $a_n = 5a_{n-1} + 2$ with $a_1 = 2$. Use mathematical induction to prove that an explicit rule for the nth term is $a_n = \frac{5^n - 1}{2}$.

Solution

Basis Step: Check that the formula works for $n = 1$.

$$a_1 \stackrel{?}{=} \frac{5^1 - 1}{2} \rightarrow 2 = 2 \checkmark$$

Inductive Step: Assume that $a_k = \frac{5^k - 1}{2}$. Show that $a_{k+1} = \frac{5^{k+1} - 1}{2}$.

$a_{k+1} = 5a_k + 2$ — **Definition of a_n for $n = k + 1$**

$= 5\left(\frac{5^k - 1}{2}\right) + 2$ — **Substitute for a_k.**

$= \frac{5^{k+1} - 5}{2} + 2$ — **Multiply.**

$= \frac{5^{k+1} - 5 + 4}{2}$ — **Add.**

$= \frac{5^{k+1} - 1}{2}$ — **Simplify.**

Therefore, an explicit rule for the nth term is $a_n = \frac{5^n - 1}{2}$ for all positive integers n.

PRACTICE

EXAMPLES 1 and 2 on pp. 836–837 for Exs. 1–8

Use mathematical induction to prove the statement.

1. $\sum_{i=1}^{n} (2i - 1) = n^2$

2. $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

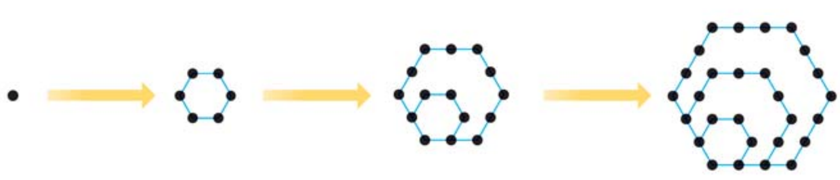
3. $\sum_{i=1}^{n} 2^{i-1} = 2^n - 1$

4. $\sum_{i=1}^{n} a_1 r^{i-1} = a_1\left(\frac{1 - r^n}{1 - r}\right)$

5. $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$

6. $\sum_{i=1}^{n} (2i)^2 = \frac{2n(n+1)(2n+1)}{3}$

7. **GEOMETRY** The numbers 1, 6, 15, 28, . . . are called *hexagonal numbers* because they represent the numbers of dots used to make hexagons, as shown below. Prove that the nth hexagonal number H_n is given by $H_n = n(2n - 1)$.



8. **REASONING** Let $f_1, f_2, \ldots, f_n, \ldots$ be the Fibonacci sequence. Prove that $f_1 + f_2 + \cdots + f_n = f_{n+2} - 1$ for all positive integers n.