Extension

Use after Lesson 12.5

Prove Statements Using Mathematical Induction

TEKS a.2, a.6; G.3.D

GOAL Use mathematical induction to prove statements about all positive integers.

In Lesson 12.1, you saw the rule for the sum of the first n positive integers:

$$\sum_{i=1}^{n} i = 1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

You can use *mathematical induction* to prove statements about positive integers.

KEY CONCEPT — *For Your Notebook*

Mathematical Induction

To show that a statement is true for all positive integers n, perform these steps.

Basis Step: Show that the statement is true for $n = 1$.

Inductive Step: Assume that the statement is true for $n = k$ where k is any positive integer. Show that this implies the statement is true for $n = k + 1$.

EXAMPLE 1 Use mathematical induction

Use mathematical induction to prove that $1 + 2 + \cdots + n = \frac{n(n+1)}{2}$.

Solution

Basis Step: Check that the formula works for $n = 1$.

$$1 \stackrel{?}{=} \frac{1(1+1)}{2} \rightarrow 1 = 1 \checkmark$$

Inductive Step: Assume that $1 + 2 + \cdots + k = \frac{k(k+1)}{2}$.

Show that $1 + 2 + \cdots + k + (k+1) = \frac{(k+1)[(k+1)+1]}{2}$.

$$1 + 2 + \cdots + k = \frac{k(k+1)}{2}$$ **Assume true for k.**

$$1 + 2 + \cdots + k + (k+1) = \frac{k(k+1)}{2} + (k+1)$$ **Add $k + 1$ to each side.**

$$= \frac{k(k+1) + 2(k+1)}{2}$$ **Add.**

$$= \frac{(k+1)(k+2)}{2}$$ **Factor out $k + 1$.**

$$= \frac{(k+1)[(k+1)+1]}{2}$$ **Rewrite $k + 2$ as $(k + 1) + 1$.**

Therefore, $1 + 2 + \cdots + n = \frac{n(n+1)}{2}$ for all positive integers n.

UNDERSTAND INDUCTION

If you know from the basis step that a statement is true for $n = 1$, then the inductive step implies that it is true for $n = 2$, and therefore for $n = 3$, and so on for all positive integers n.