METHOD 2

Using Algebra Another approach is to use an algebraic method to determine what happens to the number of members over time.

STEP 1 **Write** the recursive rule.

$$a_1 = 50{,}000, \; a_n = 0.8a_{n-1} + 5000.$$

STEP 2 **Assume** that the sequence has a limit L, which is the value that the sequence approaches as n becomes large.

STEP 3 **Consider** what happens to the equation $a_n = 0.8a_{n-1} + 5000$ as n becomes large. The value of a_n (the left-hand side) approaches L while the value of $0.8a_{n-1} + 5000$ (the right-hand side) approaches $0.8L + 5000$. So, you can conclude that $L = 0.8L + 5000$.

STEP 4 **Solve** the equation $L = 0.8L + 5000$ for L.

$L = 0.8L + 5000$	**Write equation.**
$0.2L = 5000$	**Subtract 0.8L from each side.**
$L = 25{,}000$	**Divide each side by 0.2.**

▶ The sequence approaches the limit $L = 25{,}000$ as n becomes large. So, over time the number of members of the music service approaches 25,000.

PRACTICE

Describe **what happens to the terms of the sequence as n becomes large.**

1. $a_1 = 3000, \; a_n = 0.25a_{n-1} + 300$

2. $a_1 = 1700, \; a_n = 0.38a_{n-1} + 512$

3. **WHAT IF?** Suppose the online music service in the problem on page 834 loses 8% of its current members and adds 1200 new members each year. Use the graphing method and the algebraic method to determine what happens to the number of members over time.

4. **TOWN LIBRARY** A town library initially has 54,000 books in its collection. Each year 2% of the books are lost or discarded. The library can afford to purchase 1150 new books each year. Write a recursive rule for the number a_n of books in the library at the beginning of the nth year. Use the graphing method and the algebraic method to determine what happens to the number of books in the library over time.

5. **ERROR ANALYSIS** A student attempted to solve the problem in Exercise 4 as shown below. *Describe* and correct the error in the student's work.

> $a_1 = 54{,}000, \; a_n = 0.02a_{n-1} + 1150$
> Let L be the limit of the sequence. Then:
> $$L = 0.02L + 1150$$
> $$0.98L = 1150$$
> $$L \approx 1173$$
> So, over time the number of books in the library approaches about 1173.

6. **REASONING** Give an example of a real-life situation which you can represent with a recursive rule that does not approach a limit. Write a recursive rule that represents the situation.