ITERATING FUNCTIONS Iteration involves the repeated composition of a function f with itself. The result of one iteration is f(f(x)). The result of two iterations is f(f(f(x))). You can use iteration to generate a sequence recursively. Begin with an initial value x_0 , and let $x_1 = f(x_0)$, $x_2 = f(x_1) = f(f(x_0))$, and so on.

EXAMPLE 5 Iterate a function

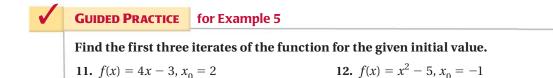
READING

An *iterate* is a number that is the result of iterating a function. Find the first three iterates x_1, x_2 , and x_3 of the function f(x) = -3x + 1 for an initial value of $x_0 = 2$.

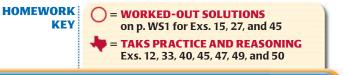
Solution

 $\begin{array}{ll} x_1 = f(\textbf{x_0}) & x_2 = f(\textbf{x_1}) & x_3 = f(\textbf{x_2}) \\ = f(\textbf{2}) & = f(\textbf{-5}) & = f(\textbf{16}) \\ = -3(\textbf{2}) + 1 & = -3(\textbf{-5}) + 1 & = -3(\textbf{16}) + 1 \\ = \textbf{-5} & = \textbf{16} & = -47 \end{array}$

▶ The first three iterates are -5, 16, and -47.



12.5 EXERCISES



Skill Practice

- **1. VOCABULARY** Copy and complete: The repeated composition of a function with itself is called <u>?</u>.
- **2. WRITING** *Explain* the difference between an explicit rule for a sequence and a recursive rule for a sequence.

EXAMPLE 1 on p. 827 for Exs. 3–12

WRITING TERMS Write the first five terms of the sequence.

	$a_1 = 1$ $a_n = a_{n-1} + 3$	4. $a_0 = 4$ $a_n = 2a_{n-1}$	5. $a_1 = -1$ $a_n = a_{n-1} - 5$	
	$a_0 = 3$ $a_n = a_{n-1} - n^2$	7. $a_1 = 2$ $a_n = (a_{n-1})^2 + 1$	8. $a_0 = 4$ $a_n = (a_{n-1})^2 - 10$	
9.	$a_1 = 2$ $a_n = n^2 + 3n - a_{n-1}$	10. $a_0 = 2, a_1 = 4$ $a_n = a_{n-1} - a_{n-2}$	11. $a_1 = 2, a_2 = 3$ $a_n = a_{n-1} \cdot a_{n-2}$	
12.	12. TAKS REASONING What are the first four terms of the sequence for which $a_1 = 1, a_2 = 4$, and $a_n = a_{n-1} \cdot a_{n-2}$?			
	(A) 1, 4, 4, 16 (B)	1, 4, 16, 64 C 1, 4, 8, 1	6 (D) 1, 4, 4, 8	