

ITERATING FUNCTIONS Iteration involves the repeated composition of a function f with itself. The result of one iteration is $f(f(x))$. The result of two iterations is $f(f(f(x)))$. You can use iteration to generate a sequence recursively. Begin with an initial value x_0 , and let $x_1 = f(x_0)$, $x_2 = f(x_1) = f(f(x_0))$, and so on.

EXAMPLE 5 Iterate a function

READING

An *iterate* is a number that is the result of iterating a function.

Find the first three iterates x_1 , x_2 , and x_3 of the function $f(x) = -3x + 1$ for an initial value of $x_0 = 2$.

Solution

$$\begin{array}{lll} x_1 = f(x_0) & x_2 = f(x_1) & x_3 = f(x_2) \\ = f(2) & = f(-5) & = f(16) \\ = -3(2) + 1 & = -3(-5) + 1 & = -3(16) + 1 \\ = -5 & = 16 & = -47 \end{array}$$

▶ The first three iterates are -5 , 16 , and -47 .



GUIDED PRACTICE for Example 5

Find the first three iterates of the function for the given initial value.

11. $f(x) = 4x - 3$, $x_0 = 2$

12. $f(x) = x^2 - 5$, $x_0 = -1$

12.5 EXERCISES

HOMWORK KEY

- = **WORKED-OUT SOLUTIONS**
on p. WS1 for Exs. 15, 27, and 45
- ➔ = **TAKS PRACTICE AND REASONING**
Exs. 12, 33, 40, 45, 47, 49, and 50

SKILL PRACTICE

- VOCABULARY** Copy and complete: The repeated composition of a function with itself is called ?.
- WRITING** Explain the difference between an explicit rule for a sequence and a recursive rule for a sequence.

EXAMPLE 1

on p. 827
for Exs. 3–12

WRITING TERMS Write the first five terms of the sequence.

- | | | |
|--|--|--|
| 3. $a_1 = 1$
$a_n = a_{n-1} + 3$ | 4. $a_0 = 4$
$a_n = 2a_{n-1}$ | 5. $a_1 = -1$
$a_n = a_{n-1} - 5$ |
| 6. $a_0 = 3$
$a_n = a_{n-1} - n^2$ | 7. $a_1 = 2$
$a_n = (a_{n-1})^2 + 1$ | 8. $a_0 = 4$
$a_n = (a_{n-1})^2 - 10$ |
| 9. $a_1 = 2$
$a_n = n^2 + 3n - a_{n-1}$ | 10. $a_0 = 2$, $a_1 = 4$
$a_n = a_{n-1} - a_{n-2}$ | 11. $a_1 = 2$, $a_2 = 3$
$a_n = a_{n-1} \cdot a_{n-2}$ |
12. ➔ **TAKS REASONING** What are the first four terms of the sequence for which $a_1 = 1$, $a_2 = 4$, and $a_n = a_{n-1} \cdot a_{n-2}$?

- (A) 1, 4, 4, 16 (B) 1, 4, 16, 64 (C) 1, 4, 8, 16 (D) 1, 4, 4, 8