EXAMPLE 2 Write recursive rules

Write a recursive rule for the sequence.

a. 3, 13, 23, 33, 43, . . . **b.** 16, 40, 100, 250, 625, . . .

Solution

a. The sequence is arithmetic with first term $a_1 = 3$ and common difference $d = 13 - 3 = 10$.

AVOID ERRORS
A recursive *equation* for a sequence does not include the initial term. To write a recursive *rule* for a sequence, the initial term must be included.

$a_n = a_{n-1} + d$ **General recursive equation for a_n**

$= a_{n-1} + 10$ **Substitute 10 for d.**

▶ So, a recursive rule for the sequence is $a_1 = 3$, $a_n = a_{n-1} + 10$.

b. The sequence is geometric with first term $a_1 = 16$ and common ratio $r = \frac{40}{16} = 2.5$.

$a_n = r \cdot a_{n-1}$ **General recursive equation for a_n**

$= 2.5a_{n-1}$ **Substitute 2.5 for r.**

▶ So, a recursive rule for the sequence is $a_1 = 16$, $a_n = 2.5a_{n-1}$.

✓ **GUIDED PRACTICE**   for Examples 1 and 2

Write the first five terms of the sequence.

1. $a_1 = 3$, $a_n = a_{n-1} - 7$ **2.** $a_0 = 162$, $a_n = 0.5a_{n-1}$

3. $a_0 = 1$, $a_n = a_{n-1} + n$ **4.** $a_1 = 4$, $a_n = 2a_{n-1} - 1$

Write a recursive rule for the sequence.

5. 2, 14, 98, 686, 4802, . . .   **6.** 19, 13, 7, 1, −5, . . .

7. 11, 22, 33, 44, 55, . . .   **8.** 324, 108, 36, 12, 4, . . .

RECURSIVE RULES FOR SPECIAL SEQUENCES For some sequences, it is difficult to write an explicit rule but relatively easy to write a recursive rule.

EXAMPLE 3 Write recursive rules for special sequences

Write a recursive rule for the sequence.

a. 1, 1, 2, 3, 5, . . .   **b.** 1, 1, 2, 6, 24, . . .

NAME SEQUENCES
The sequence in part (a) of Example 3 is called the *Fibonacci sequence.* The sequence in part (b) of Example 3 lists the factorial numbers you studied in Chapter 10.

Solution

a. Beginning with the third term in the sequence, each term is the sum of the two previous terms.

▶ So, a recursive rule is $a_1 = 1$, $a_2 = 1$, $a_n = a_{n-2} + a_{n-1}$.

b. Denote the first term by $a_0 = 1$. Then note that $a_1 = 1 = \mathbf{1} \cdot a_0$, $a_2 = 2 = \mathbf{2} \cdot a_1$, $a_3 = 6 = \mathbf{3} \cdot a_2$, and so on.

▶ So, a recursive rule is $a_0 = 1$, $a_n = n \cdot a_{n-1}$.