12.5 Use Recursive Rules with Sequences and Functions

TEKS a.1, a.5, a.6; P.4.A

Before You used explicit rules for sequences.

Now You will use recursive rules for sequences.

Why? So you can model evaporation from a pool, as in Ex. 44.

Key Vocabulary
- explicit rule
- recursive rule
- iteration

So far in this chapter you have worked with *explicit rules* for the *n*th term of a sequence, such as $a_n = 3n - 2$ and $a_n = 3(2)^n$. An **explicit rule** gives a_n as a function of the term's position number n in the sequence.

In this lesson you will learn another way to define a sequence—by a *recursive rule*. A **recursive rule** gives the beginning term or terms of a sequence and then a *recursive equation* that tells how a_n is related to one or more preceding terms.

EXAMPLE 1 Evaluate recursive rules

Write the first six terms of the sequence.

a. $a_0 = 1, a_n = a_{n-1} + 4$

b. $a_1 = 1, a_n = 3a_{n-1}$

Solution

a. $a_0 = 1$

$a_1 = a_0 + 4 = 1 + 4 = 5$

$a_2 = a_1 + 4 = 5 + 4 = 9$

$a_3 = a_2 + 4 = 9 + 4 = 13$

$a_4 = a_3 + 4 = 13 + 4 = 17$

$a_5 = a_4 + 4 = 17 + 4 = 21$

b. $a_1 = 1$

$a_2 = 3a_1 = 3(1) = 3$

$a_3 = 3a_2 = 3(3) = 9$

$a_4 = 3a_3 = 3(9) = 27$

$a_5 = 3a_4 = 3(27) = 81$

$a_6 = 3a_5 = 3(81) = 243$

ARITHMETIC AND GEOMETRIC SEQUENCES In part (a) of Example 1, observe that the *differences* of consecutive terms of the sequence are constant, so the sequence is arithmetic. In part (b), the *ratios* of consecutive terms are constant, so the sequence is geometric. In general, rules for arithmetic and geometric sequences can be written recursively as follows.

KEY CONCEPT *For Your Notebook*

Recursive Equations for Arithmetic and Geometric Sequences

Arithmetic Sequence

$a_n = a_{n-1} + d$ where d is the common difference

Geometric Sequence

$a_n = r \cdot a_{n-1}$ where r is the common ratio