

12.5 Exploring Recursive Rules TEKS a.1, a.5, a.6; P.4.A

MATERIALS • computer with spreadsheet program

QUESTION How can you evaluate a recursive rule for a sequence?

A *recursive rule* for a sequence gives the beginning term or terms of the sequence and then an equation relating the n th term a_n to one or more preceding terms.

For example, the rule $a_1 = 4$, $a_n = a_{n-1} + 7$ defines a sequence recursively.

EXPLORE Find terms of a sequence given by a recursive rule

Find the first eight terms of the sequence defined by $a_1 = 4$, $a_n = a_{n-1} + 7$.
What type of sequence does this rule represent?

STEP 1 Enter first term

Enter the value of a_1 into cell A1.

| A1 | 4 | | |
|----|---|---|---|
| | A | B | C |
| 1 | 4 | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 8 | | | |

STEP 2 Enter recursive equation

Enter the formula “=A1+7” into cell A2.

| A2 | =A1+7 | | |
|----|-------|---|---|
| | A | B | C |
| 1 | 4 | | |
| 2 | 11 | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |
| 6 | | | |
| 7 | | | |
| 8 | | | |

STEP 3 Fill cells

Use the *fill down* command to copy the recursive equation into the rest of column A.

| A8 | =A7+7 | | |
|----|-------|---|---|
| | A | B | C |
| 1 | 4 | | |
| 2 | 11 | | |
| 3 | 18 | | |
| 4 | 25 | | |
| 5 | 32 | | |
| 6 | 39 | | |
| 7 | 46 | | |
| 8 | 53 | | |

STEP 4 Identify terms and type of sequence

The first eight terms of the sequence are 4, 11, 18, 25, 32, 39, 46, and 53. This sequence is an arithmetic sequence because the difference of consecutive terms is always 7.

DRAW CONCLUSIONS Use your observations to complete these exercises

- Find the first eight terms of the sequence defined by $a_1 = 4$, $a_n = 7a_{n-1}$.
What type of sequence does this rule represent?
- Write a recursive rule for the sequence 15, 11, 7, 3, -1, -5, ...
- Write a recursive rule for the sequence 81, 27, 9, 3, 1, $\frac{1}{3}$, ...
- What equation relates the n th term a_n to the preceding term a_{n-1} for an arithmetic sequence with common difference d ? for a geometric sequence with common ratio r ?