KEY CONCEPT — For Your Notebook

The Sum of an Infinite Geometric Series

The sum of an infinite geometric series with first term a_1 and common ratio r is given by

$$S = \frac{a_1}{1 - r}$$

provided $|r| < 1$. If $|r| \geq 1$, the series has no sum.

EXAMPLE 2 Find sums of infinite geometric series

Find the sum of the infinite geometric series.

a. $\sum_{i=1}^{\infty} 5(0.8)^{i-1}$

b. $1 - \frac{3}{4} + \frac{9}{16} - \frac{27}{64} + \cdots$

Solution

a. For this series, $a_1 = 5$ and $r = 0.8$.

$$S = \frac{a_1}{1 - r} = \frac{5}{1 - 0.8} = 25$$

b. For this series, $a_1 = 1$ and $r = -\frac{3}{4}$.

$$S = \frac{a_1}{1 - r} = \frac{1}{1 - \left(-\frac{3}{4}\right)} = \frac{4}{7}$$

TAKS

EXAMPLE 3 TAKS PRACTICE: Multiple Choice

What is the sum of the infinite geometric series $1 - 4 + 16 - 64 + \cdots$?

(A) $\frac{1}{5}$ (B) $\frac{4}{3}$ (C) 4 (D) Does not exist

Solution

You know that $a_1 = 1$ and $a_2 = -4$. So, $r = \frac{-4}{1} = -4$.

Because $|-4| \geq 1$, the sum does not exist.

▶ The correct answer is D. (A) (B) (C) (D)

AVOID ERRORS

If you substitute 1 for a_1 and -4 for r in the formula $S = \frac{a_1}{1 - r}$, you get an answer of $S = \frac{1}{5}$ for the sum. However, this answer is not correct because the sum formula does not apply when $|r| \geq 1$.

✓ GUIDED PRACTICE for Examples 1, 2, and 3

1. Consider the series $\frac{2}{5} + \frac{4}{25} + \frac{8}{125} + \frac{16}{625} + \frac{32}{3125} + \cdots$. Find and graph the partial sums S_n for $n = 1, 2, 3, 4,$ and 5. Then describe what happens to S_n as n increases.

Find the sum of the infinite geometric series, if it exists.

2. $\sum_{n=1}^{\infty} \left(-\frac{1}{2}\right)^{n-1}$

3. $\sum_{n=1}^{\infty} 3\left(\frac{5}{4}\right)^{n-1}$

4. $3 + \frac{3}{4} + \frac{3}{16} + \frac{3}{64} + \cdots$