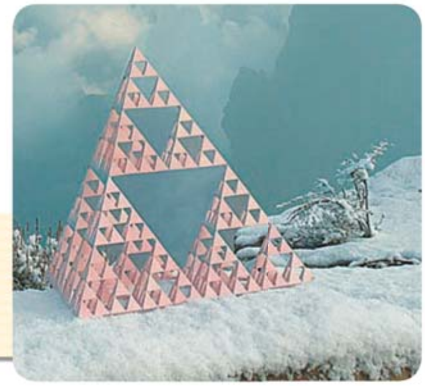
12.4 Find Sums of Infinite Geometric Series

TEKS a.2, 2A.2.A; P.4.B, P.4.D



Before You found the sums of finite geometric series.

Now You will find the sums of infinite geometric series.

Why? So you can analyze a fractal, as in Ex. 42.

Key Vocabulary
- **partial sum**

The sum S_n of the first n terms of an infinite series is called a **partial sum**. The partial sums of an infinite geometric series may approach a limiting value.

EXAMPLE 1 Find partial sums

Consider the infinite geometric series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots$. Find and graph the partial sums S_n for $n = 1, 2, 3, 4,$ and 5. Then describe what happens to S_n as n increases.

Solution

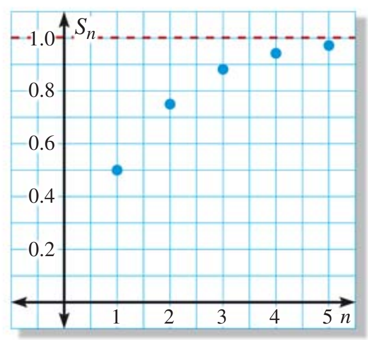
$$S_1 = \frac{1}{2} = 0.5$$

$$S_2 = \frac{1}{2} + \frac{1}{4} = 0.75$$

$$S_3 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \approx 0.88$$

$$S_4 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \approx 0.94$$

$$S_5 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} \approx 0.97$$



From the graph, S_n appears to approach 1 as n increases.

Animated Algebra at classzone.com

SUMS OF INFINITE SERIES In Example 1, you can understand why S_n approaches 1 as n increases by considering the rule for S_n:

$$S_n = a_1\left(\frac{1 - r^n}{1 - r}\right) = \frac{1}{2}\left(\frac{1 - \left(\frac{1}{2}\right)^n}{1 - \frac{1}{2}}\right) = 1 - \left(\frac{1}{2}\right)^n$$

As n increases, $\left(\frac{1}{2}\right)^n$ approaches 0, so S_n approaches 1. Therefore, 1 is defined to be the sum of the infinite geometric series in Example 1. More generally, as n increases for *any* infinite geometric series with common ratio r between -1 and 1, the value of $S_n = a_1\left(\frac{1 - r^n}{1 - r}\right) \approx a_1\left(\frac{1 - 0}{1 - r}\right) = \frac{a_1}{1 - r}$.