# 12.4 Investigating an Infinite Geometric Series  TEKS *a.4, a.5; P.4.A*

**MATERIALS** • scissors • paper

**QUESTION** **What is the sum of an infinite geometric series?**

You can illustrate an infinite geometric series by cutting a piece of paper into smaller and smaller pieces.

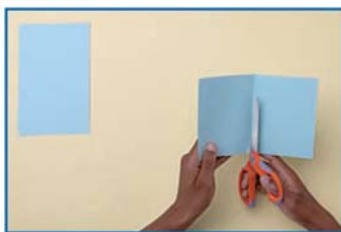**EXPLORE** **Model an infinite geometric series**

**Start with a rectangular piece of paper. Define its area to be 1 square unit.**

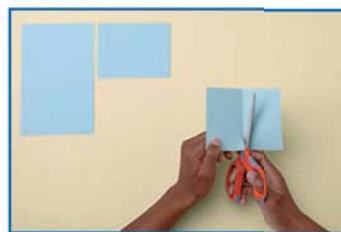**STEP 1** *Cut paper in half*

Fold the paper in half and cut along the fold. Place one half on a desktop and hold the remaining half.

**STEP 2** *Cut paper again*

Fold the piece of paper you are holding in half and cut along the fold. Place one half on the desktop and hold the remaining half.

**STEP 3** *Repeat steps*

Repeat Steps 1 and 2 until you find it too difficult to fold and cut the piece of paper you are holding.

**STEP 4** *Find areas* The first piece of paper on the desktop has an area of $\frac{1}{2}$ square unit. The second piece has an area of $\frac{1}{4}$ square unit. Write the areas of the next three pieces of paper. Explain why these areas form a geometric sequence.

**STEP 5** *Make a table*
Copy and complete the table by recording the number of pieces of paper on the desktop and the combined area of the pieces at each step.

| Number of pieces | 1 | 2 | 3 | 4 | . . . |
|---|---|---|---|---|---|
| Combined area | $\frac{1}{2}$ | $\frac{1}{2} + \frac{1}{4} = ?$ | ? | ? | . . . |

**DRAW CONCLUSIONS** **Use your observations to complete these exercises**

1. Based on your table, what number does the combined area of the pieces of paper appear to be approaching?

2. Using the formula for the sum of a finite geometric series, write and simplify a rule for the combined area $A_n$ of the pieces of paper after $n$ cuts. What happens to $A_n$ as $n \to \infty$? *Justify* your answer mathematically.