EXAMPLE 5 Find the sum of a geometric series

Find the sum of the geometric series $\sum_{i=1}^{16} 4(3)^{i-1}$.

$a_1 = 4(3)^{1-1} = 4$ **Identify first term.**

$r = 3$ **Identify common ratio.**

$S_{16} = a_1\left(\frac{1 - r^{16}}{1 - r}\right)$ **Write rule for S_{16}.**

$= 4\left(\frac{1 - 3^{16}}{1 - 3}\right)$ **Substitute 4 for a_1 and 3 for r.**

$= 86{,}093{,}440$ **Simplify.**

▶ The sum of the series is 86,093,440.

EXAMPLE 6 Use a geometric sequence and series in real life

MOVIE REVENUE In 1990, the total box office revenue at U.S. movie theaters was about \$5.02 billion. From 1990 through 2003, the total box office revenue increased by about 5.9% per year.

a. Write a rule for the total box office revenue a_n (in billions of dollars) in terms of the year. Let $n = 1$ represent 1990.

b. What was the total box office revenue at U.S. movie theaters for the entire period 1990–2003?

Solution

a. Because the total box office revenue increased by the same percent each year, the total revenues from year to year form a geometric sequence. Use $a_1 = 5.02$ and $r = 1 + 0.059 = 1.059$ to write a rule for the sequence.

$a_n = 5.02(1.059)^{n-1}$ **Write a rule for a_n.**

b. There are 14 years in the period 1990–2003, so find S_{14}.

$$S_{14} = a_1\left(\frac{1 - r^{14}}{1 - r}\right) = 5.02\left(\frac{1 - (1.059)^{14}}{1 - 1.059}\right) \approx 105$$

▶ The total movie box office revenue for the period 1990–2003 was about \$105 billion.

✓ GUIDED PRACTICE for Examples 5 and 6

7. Find the sum of the geometric series $\sum_{i=1}^{8} 6(-2)^{i-1}$.

8. **MOVIE REVENUE** Use the rule in part (a) of Example 6 to estimate the total box office revenue at U.S. movie theaters in 2000.