EXAMPLE 4 Write a rule given two terms

Two terms of a geometric sequence are $a_3 = -48$ and $a_6 = 3072$. Find a rule for the *n*th term.

Solution

STEP 1 **Write** a system of equations using $a_n = a_1 r^{n-1}$ and substituting 3 for n (Equation 1) and then 6 for n (Equation 2).

$a_3 = a_1 r^{3-1}$ → $-48 = a_1 r^2$ **Equation 1**

$a_6 = a_1 r^{6-1}$ → $3072 = a_1 r^5$ **Equation 2**

STEP 2 **Solve** the system.

$\frac{-48}{r^2} = a_1$ **Solve Equation 1 for a_1.**

$3072 = \frac{-48}{r^2}(r^5)$ **Substitute for a_1 in Equation 2.**

$3072 = -48r^3$ **Simplify.**

$-4 = r$ **Solve for r.**

$-48 = a_1(-4)^2$ **Substitute for r in Equation 1.**

$-3 = a_1$ **Solve for a_1.**

STEP 3 **Find** a rule for a_n.

$a_n = a_1 r^{n-1}$ **Write general rule.**

$a_n = -3(-4)^{n-1}$ **Substitute for a_1 and r.**

✓ GUIDED PRACTICE for Examples 2, 3, and 4

Write a rule for the *n*th term of the geometric sequence. Then find a_8.

4. 3, 15, 75, 375, . . .

5. $a_6 = -96$, $r = 2$

6. $a_2 = -12$, $a_4 = -3$

GEOMETRIC SERIES The expression formed by adding the terms of a geometric sequence is called a **geometric series**. The sum of the first n terms of a geometric series is denoted by S_n. You can develop a rule for S_n as follows.

$$\begin{array}{rcl} S_n &=& a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \cdots + a_1 r^{n-1} \\ -rS_n &=& \quad\; - a_1 r - a_1 r^2 - a_1 r^3 - \cdots - a_1 r^{n-1} - a_1 r^n \\ \hline S_n(1-r) &=& a_1 + 0 + 0 + 0 + \cdots + 0 - a_1 r^n \end{array}$$

So, $S_n(1 - r) = a_1(1 - r^n)$. If $r \neq 1$, you can divide each side of this equation by $1 - r$ to obtain the following rule for S_n.

KEY CONCEPT *For Your Notebook*

The Sum of a Finite Geometric Series

The sum of the first n terms of a geometric series with common ratio $r \neq 1$ is:

$$S_n = a_1\left(\frac{1 - r^n}{1 - r}\right)$$